Module – 4

Strong Ground Motion
Attenuation Relationship
Spatial variability of ground motions

The ground motion parameters at any site depend upon the magnitude of earthquake and the distance of the site from epicenter.

The ground motion parameters measured at a site have been used to develop empirical relationships to predict the parameters as functions of earthquake magnitude and source-to-site distance. But these predictions are not accurate.

For structures that extend over considerable distance (such as bridges and pipelines), the ground motion parameters will be different at different part of the structure, causing differential movement of the supports. Local variation of ground motion parameters need to be considered for the design of such structures.

Ref: Kramer (1996)
Amplitude Parameters - Estimation

Predictive relationships for parameters (like peak acceleration, peak velocity) which decrease with increase in distance are called attenuation relationships.

**Peak Acceleration**

Campbell (1981) developed attenuation relationship for mean PHA for sites within 50 km of fault rupture in magnitude 5.0 to 7.7 earthquakes:

\[
\ln PHA(g) = -4.141 + 0.868M - 1.09 \ln [R + 0.0606 \exp(0.7M)]
\]

Where \( M = M_L \) for magnitude < 6 or \( M_s \) for magnitude > 6, \( R \) is the closest distance to fault rupture in km.

Latest mostly used relationship in western North America is given by Boore et al. (1993)

*Ref: Kramer (1996)*
Amplitude Parameters - Estimation

Attenuation relationship in western North America is given by Boore et al. (1993)

*(From North American Earthquakes (magnitude 5-7.7) within 100 km of surface projection of fault)*

\[
\text{Log PHA}(g) = b_1 + b_2 (M_w - 6) + b_3 (M_w - 6)^2 + b_4 R + b_5 \log R + b_6 G_b + b_7 G_c
\]

\(R = (d^2 + h^2)^{1/2}, \ d = \text{closest distance to the surface projection of the fault in km.}\)

\(|G_b| = 0 \text{ for site class A} \quad |G_c| = 0 \text{ for site class A}\)

\(|G_b| = 1 \text{ for site class B} \quad |G_c| = 0 \text{ for site class B}\)

\(|G_b| = 0 \text{ for site class C} \quad |G_c| = 1 \text{ for site class C}\)

*(Site classes are defined next slide on the basis of the avg. } V_s \text{ in the upper 30 m).*
Definitions of Site Classes for Boore et al. (1993) Attenuation Relationship

<table>
<thead>
<tr>
<th>Site Class</th>
<th>( \bar{v}_s ) in Upper 30 m (100 ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>&gt; 750 m/sec (2500 ft/sec)</td>
</tr>
<tr>
<td>B</td>
<td>360–750 m/sec (1200–2500 ft/sec)</td>
</tr>
<tr>
<td>C</td>
<td>180–360 m/sec (600–1200 ft/sec)</td>
</tr>
</tbody>
</table>

Coefficients for Attenuation Relationships of Boore et al. (1993)

<table>
<thead>
<tr>
<th>Component</th>
<th>( b_1 )</th>
<th>( b_2 )</th>
<th>( b_3 )</th>
<th>( b_4 )</th>
<th>( b_5 )</th>
<th>( b_6 )</th>
<th>( b_7 )</th>
<th>( h )</th>
<th>( \sigma_{\log PHA} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random</td>
<td>-0.105</td>
<td>0.229</td>
<td>0.0</td>
<td>0.0</td>
<td>0.778</td>
<td>0.162</td>
<td>0.251</td>
<td>5.57</td>
<td>0.230</td>
</tr>
<tr>
<td>Larger</td>
<td>-0.038</td>
<td>0.216</td>
<td>0.0</td>
<td>0.0</td>
<td>0.777</td>
<td>0.158</td>
<td>0.254</td>
<td>5.48</td>
<td>0.205</td>
</tr>
</tbody>
</table>
Attenuation Relationship for peak horizontal rock acceleration by Toro et al., 1994 (for mid continent of North America)

\[
\ln \text{PHA (g)} = 2.2 + 0.81(M_w - 6) - 1.27 \ln R_m + 0.11 \max[\ln (R_m/100), 0] - 0.0021R_m
\]

\[
\sigma_{\ln \text{PHA}} = (\sigma_m^2 + \sigma_r^2)^{1/2}
\]

Where \( R_m = (R^2 + 9.3^2)^{1/2} \), \( R \) being closest horizontal distance to earthquake rupture (in km), \( \sigma_m = 0.36 + 0.07(M_w - 6) \), and

\[
\begin{align*}
\sigma_r &= 0.54 - 0.0227(R - 5) \text{ for } 5 \text{ km} \leq R \leq 20 \text{ km} \\
&= 0.2 \text{ for } R > 20 \text{ km}
\end{align*}
\]

Attenuation relationship for subduction zone (Youngs et al., 1988)

\[
\ln \text{PHA (g)} = 19.16 + 1.045M_w - 4.738 \ln [R + 205.5 \exp(0.0968M_w)] + 0.54 Z_t
\]

\[
\sigma_{\ln \text{PHA}} = 1.55 - 0.125M_w, \text{ R = closest distance to the zone of rupture in km and } Z_t = 0 \text{ for interface and 1 for intraslab events}
\]
Peak Velocity Attenuation Relationships (Joyner and Boore, 1988)

(for earthquake magnitudes 5-7.7)

\[
\log \text{PHV (cm/sec)} = j_1 + j_2(M-6) + j_3(M-6)^2 + j_4 \log R + j_5 R + j_6
\]

Where PHV can be selected as randomly oriented or larger horizontal component

\[ R = (r_0^2 + j_7^2)^{1/2}, \] and \( r_0 \) is the shortest distance (km) from the site to the vertical projection of the EQ fault rupture on the surface of the earth.

The coefficients \( j_i \) are given in the table below:

<table>
<thead>
<tr>
<th>Component</th>
<th>( j_1 )</th>
<th>( j_2 )</th>
<th>( j_3 )</th>
<th>( j_4 )</th>
<th>( j_5 )</th>
<th>( j_6 )</th>
<th>( j_7 )</th>
<th>( \sigma_{\log \text{PHV}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random</td>
<td>2.09</td>
<td>0.49</td>
<td>0.0</td>
<td>-1.0</td>
<td>-0.0026</td>
<td>0.17</td>
<td>4.0</td>
<td>0.33</td>
</tr>
<tr>
<td>Larger</td>
<td>2.17</td>
<td>0.49</td>
<td>0.0</td>
<td>-1.0</td>
<td>-0.0026</td>
<td>0.17</td>
<td>4.0</td>
<td>0.33</td>
</tr>
</tbody>
</table>
Amplitude Parameters - Estimation

A. Patwardhan et al. (1978):

\[ \ln y = \ln A + B M_s + E \ln [R + d \exp(f M_s)] \]

Where, \( y \) in cm/s², \( d = 0.864 \) and \( f = 0.463 \)

<table>
<thead>
<tr>
<th>Path</th>
<th>( A ) (for median)</th>
<th>( A ) (for mean)</th>
<th>( B )</th>
<th>( E )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Path A (rock)</td>
<td>157</td>
<td>186</td>
<td>1.04</td>
<td>-1.90</td>
</tr>
<tr>
<td>Path A (stiff soil)</td>
<td>191</td>
<td>224</td>
<td>0.823</td>
<td>-1.56</td>
</tr>
<tr>
<td>Path B (stiff soil)</td>
<td>284</td>
<td>363</td>
<td>0.587</td>
<td>-1.05</td>
</tr>
</tbody>
</table>

Path A: Shallow focus earthquakes (California, Japan, Nicaragua and India), 63 records

Path B: Subduction (Benioff) zone earthquakes (Japan & South America) 23 earthquakes, 5.3 \( \leq M_s \leq 7.8 \), 32 records
Considered:

For Path A: 1. Rock sites = 21 records and 2. Stiff soil = 42 records

Use only stiff soil records for deriving subduction zone equation

Most earthquakes for path A have $5 \leq M_s \leq 6.7$

All data corrected. PGA for corrected Japanese and South American records much higher than uncorrected PGA.
Aptikaev & Kopnichev (1980)

\[ \log Ae = a_1 M + a_2 \log R + a_3 \]

<table>
<thead>
<tr>
<th>$\text{Ae (cm/s}^2\text{)}$</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\geq 160$</td>
<td>0.28</td>
<td>-0.8</td>
<td>1.70</td>
</tr>
<tr>
<td>$&lt; 160$</td>
<td>0.80</td>
<td>-2.3</td>
<td>0.80</td>
</tr>
</tbody>
</table>

- PGA corresponds to S-wave
- Used five source mechanism categories (about 70 records, 59 earthquakes from W. N. America including Hawaii, Guatemala, Nicaragua, Chile, Peru, Argentina, Italy, Greece, Romania, central Asia, India and Japan):
  1. Contraction faulting (uplift and thrust), about 16 earthquakes
  2. Contraction faulting with strike-slip component, about 6 earthquakes
3. Strike-slip, about 17 earthquakes

4. Strike-slip with dip-slip component, about 6 earthquakes

5. Dip-slip, about 9 earthquakes

Use these approximately 70 records to derive ratios of mean measured, $A_0$, to predicted PGA, $A_e$, $\log(A_0/A_e)$, and for ratios of mean horizontal to vertical PGA, $\log(A_h/A_v)$, for each type of faulting. Use every earthquake with equal weight independent of number of records for each earthquake.

- Results are:

<table>
<thead>
<tr>
<th></th>
<th>Category 1</th>
<th>Category 2</th>
<th>Category 3</th>
<th>Category 4</th>
<th>Category 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\log(A_0/A_e)$</td>
<td>$0.35 \pm 0.13$ (16)</td>
<td>$0.11 \pm 0.17$ (5)</td>
<td>$0.22 \pm 0.08$ (17)</td>
<td>$0.06 \pm 0.13$ (6)</td>
<td>$-0.06 \pm 0.20$ (9)</td>
</tr>
<tr>
<td>$\log(A_h/A_v)$</td>
<td>$0.32 \pm 0.13$ (12)</td>
<td>$0.32 \pm 0.08$ (5)</td>
<td>$0.27 \pm 0.07$ (12)</td>
<td>$0.18 \pm 0.10$ (5)</td>
<td>$0.17 \pm 0.11$ (5)</td>
</tr>
</tbody>
</table>

where ± gives 0.7 confidence intervals and number in brackets is number of earthquakes used.

\[
\ln(a) = C_1 + C_2 M + C_3 \ln[R + C_4 \exp(C_5 M)]
\]

Where \( a \) is in g, \( C_1 = -1.17, C_2 = 0.587, C_3 = -1.26, C_4 = 2.13, C_5 = 0.25 \)
and \( \sigma = 0.543 \)

Used data from Italy (6 records, 6 earthquakes), USA (18 records, earthquakes), Greece (13 records, 9 earthquakes), Iran (3 records, 3 earthquakes), Pakistan (3 records, 1 earthquake), Yugoslavia (3 records, 1 earthquake), USSR (1 record, 1 earthquake), Nicaragua (1 record, 1 earthquake), India (1 record, 1 earthquake) and Atlantic Ocean (1 record, 1 earthquake).

Gaull (1988)

\[
\log\text{PGA} = \left[\frac{(a_1 \log R + a_2)/a_3}{a_3}\right](M_L - a_4) - a_5 \log R - a_6 R + a_7
\]

where PGA is in m/s², \(a_1 = 5\), \(a_2 = 3\), \(a_3 = 20\), \(a_4 = 6\), \(a_5 = 0.77\), \(a_6 = 0.0045\) and \(a_7 = 1.2\)

- Considered earthquakes of magnitudes about 3 and most from distances below about 20 km
- Adds 4 near source (\(5 \leq R \leq 10\)km) records from US, Indian and New Zealand earthquakes with magnitudes between 6.3 and 6.7 to supplement high magnitude range


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Singh et al. (1996)

\[ \log_{10}^a = 1.14 + 0.31M + 0.65\log_{10}^R \]

\[ \log_{10}^V = 0.571 + 0.41M + 0.768\log_{10}^R \]

Where, \( a = \text{PHA (cm/s}^2) \)
\( V = \text{PHV (cm/s)} \)

Data considered from 1986 – 1993

R= Hypocentral distance in km

M = M_B (5.7 to 7.2)

The following generalized predictive attenuation relationship has been proposed for Peninsular India by Iyengar and Raghukanth (2004)

\[ \ln Y = C_1 + C_2 (M-6) + C_3 (M-6)^2 - \ln R - C_4 R + \ln \varepsilon \]

where \( Y \), \( M \), and \( R \) refer to PGA(g), moment magnitude, and hypocentral distance, respectively

- **Koyna-Warna Region:**
  \[ C_1 = 1.7615; \; C_2 = 0.9325; \; C_3 = -0.0706; \; C_4 = 0.0086; \; \sigma(\ln \varepsilon) = 0.3292 \]

- **Western-Central Region:**
  \[ C_1 = 1.7236; \; C_2 = 0.9453; \; C_3 = -0.0740; \; C_4 = 0.0064; \; \sigma(\ln \varepsilon) = 0.3439 \]

- **Southern Region:**
  \[ C_1 = 1.7816; \; C_2 = 0.9205; \; C_3 = -0.0673; \; C_4 = 0.0035; \; \sigma(\ln \varepsilon) = 0.3136 \]
Attenuation relationships for India

The following predictive attenuation relationship for peak vertical ground accelerations has been proposed for Himalayan Region of India by Sharma (2000).

\[
\log(A) = -2.87 + 0.634 M - 1.16 \log(X + e^{0.62M})
\]

where \(A\), \(M\), and \(X\) refer to PGA(g), moment magnitude, and hypocentral distance, respectively.

The database consisting of 66 peak ground vertical accelerations from five earthquakes recorded by Strong Motion Arrays in India have been used to develop the relationship.

Vertical to horizontal acceleration ratio can be estimated based on hypocentral distance as,

\[
\log(A) = -1.072 + 0.3903 M - 1.21 \log(X + e^{0.5873M})
\]
The following predictive attenuation relationship for peak horizontal ground accelerations was used by Shiuly and Narayan (2012) for Kolkata city. It was developed by Abrahamson and Litehiser (1989).

\[
\log (a) = -0.62 + 0.177M - 0.982 \log (r + e^{0.284M}) + 0.132F - 0.0008Er
\]

where ‘a’ is peak horizontal acceleration, ‘r’ is the closest distance (in km) from site to the zone of energy release, ‘M’ is the magnitude, F is dummy variable that is ‘1’ for reverse or reverse oblique fault otherwise ‘0’, and ‘E’ is a dummy variable that is ‘1’ for interplate and ‘0’ for intraplate events.

Attenuation relationships for India

The following predictive attenuation relationship for intensity has been proposed for India by Martin and Szeliga (2010):

$log N = a + b (I - 2)$

where $N$ is the cumulative no. of observations per year and $I$ is the intensity value.

<table>
<thead>
<tr>
<th>City</th>
<th>$a$</th>
<th>$b$</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mumbai</td>
<td>-0.81</td>
<td>-0.27</td>
<td>42</td>
<td>78</td>
<td>145</td>
</tr>
<tr>
<td>Delhi</td>
<td>-0.66</td>
<td>-0.18</td>
<td>16</td>
<td>24</td>
<td>36</td>
</tr>
<tr>
<td>Bangalore</td>
<td>-1.07</td>
<td>-0.28</td>
<td>81</td>
<td>155</td>
<td>295</td>
</tr>
<tr>
<td>Kolkata</td>
<td>-0.72</td>
<td>-0.14</td>
<td>14</td>
<td>19</td>
<td>26</td>
</tr>
<tr>
<td>Chennai</td>
<td>-1.04</td>
<td>-0.20</td>
<td>44</td>
<td>69</td>
<td>110</td>
</tr>
</tbody>
</table>

Attenuation relationships for India

The following predictive attenuation relationship for peak horizontal ground acceleration \((A)\) for south India has been proposed for Peninsular India by Dunbar et al.:

\[
\log A = -1.902 + 0.249M_w
\]

where \(M_w\) is the moment magnitude. The relationship between intensity \((I_o)\) and peak horizontal ground acceleration were also obtained given as:

\[
M_s = M_w = 0.605I_o + 1.376
\]

\[
\log A = -1.902 + 0.249 \times (0.605I_o + 1.376)
\]

\[
\log A = -1.56 + 0.15I_o
\]

Reference: P.K. DUNBAR, R.G. BILHAM, M.J. LAITURI “Earthquake Loss Estimation for India Based on Macroeconomic Indicators”
Attenuation relationships for India

The following predictive attenuation relationship for pseudo spectral velocity (PSV) for North East India has been proposed by Das et al.:

$$\log[PSV(T)] = c_1(T) + c_2(T)M + c_3(T)h + c_4(T)\log(\sqrt{R^2 + h^2}) + c_5(T)v$$

where $M$ is the earthquake magnitude, $R$ is the epicentral distance, $h$ is the focal depth, $T$ is the time-period of single-degree-of-freedom (SDOF) oscillator and $v = 0$ and $1$ for horizontal and vertical motions, respectively.

Authors used 261 accelerograms recorded on stiff soil/rock sites for six earthquake events.

<table>
<thead>
<tr>
<th>Period $T$</th>
<th>$c_1' (T)$</th>
<th>$c_2' (T)$</th>
<th>$c_3' (T)$</th>
<th>$c_4' (T)$</th>
<th>$c_5' (T)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.040</td>
<td>-0.5402</td>
<td>0.3140</td>
<td>0.0039</td>
<td>-0.9001</td>
<td>-0.4251</td>
</tr>
<tr>
<td>0.048</td>
<td>-0.4277</td>
<td>0.3097</td>
<td>0.0038</td>
<td>-0.8873</td>
<td>-0.4164</td>
</tr>
<tr>
<td>0.055</td>
<td>-0.3065</td>
<td>0.3042</td>
<td>0.0039</td>
<td>-0.8854</td>
<td>-0.4107</td>
</tr>
<tr>
<td>0.065</td>
<td>-0.1401</td>
<td>0.2974</td>
<td>0.0039</td>
<td>-0.8854</td>
<td>-0.4055</td>
</tr>
<tr>
<td>0.080</td>
<td>0.0614</td>
<td>0.2913</td>
<td>0.0040</td>
<td>-0.8845</td>
<td>-0.4057</td>
</tr>
<tr>
<td>0.095</td>
<td>02108</td>
<td>0.2878</td>
<td>0.0040</td>
<td>-0.8854</td>
<td>-0.4157</td>
</tr>
<tr>
<td>0.110</td>
<td>0.3427</td>
<td>0.2852</td>
<td>0.0042</td>
<td>-0.9009</td>
<td>-0.4334</td>
</tr>
<tr>
<td>0.130</td>
<td>0.4989</td>
<td>0.2849</td>
<td>0.0044</td>
<td>-0.9326</td>
<td>-0.4607</td>
</tr>
<tr>
<td>0.150</td>
<td>0.6054</td>
<td>0.2912</td>
<td>0.0046</td>
<td>-0.9684</td>
<td>-0.4896</td>
</tr>
<tr>
<td>0.180</td>
<td>0.6374</td>
<td>0.3101</td>
<td>0.0047</td>
<td>-1.0019</td>
<td>-0.5288</td>
</tr>
<tr>
<td>0.220</td>
<td>0.5375</td>
<td>0.3301</td>
<td>0.0046</td>
<td>-0.9870</td>
<td>-0.5578</td>
</tr>
<tr>
<td>0.260</td>
<td>0.4110</td>
<td>0.3421</td>
<td>0.0046</td>
<td>-0.9472</td>
<td>-0.5716</td>
</tr>
<tr>
<td>0.300</td>
<td>0.2716</td>
<td>0.3498</td>
<td>0.0045</td>
<td>-0.8965</td>
<td>-0.5741</td>
</tr>
<tr>
<td>0.360</td>
<td>0.0446</td>
<td>0.3608</td>
<td>0.0044</td>
<td>-0.8207</td>
<td>-0.5639</td>
</tr>
<tr>
<td>0.420</td>
<td>0.1583</td>
<td>0.3738</td>
<td>0.0043</td>
<td>-0.7682</td>
<td>-0.5479</td>
</tr>
<tr>
<td>0.500</td>
<td>0.2913</td>
<td>0.3912</td>
<td>0.0040</td>
<td>-0.7505</td>
<td>-0.5364</td>
</tr>
<tr>
<td>0.600</td>
<td>0.3369</td>
<td>0.4145</td>
<td>0.0032</td>
<td>-0.7672</td>
<td>-0.5375</td>
</tr>
<tr>
<td>0.700</td>
<td>0.4101</td>
<td>0.4418</td>
<td>0.0024</td>
<td>-0.7797</td>
<td>-0.5397</td>
</tr>
<tr>
<td>0.850</td>
<td>0.6807</td>
<td>0.4854</td>
<td>0.0011</td>
<td>-0.7384</td>
<td>-0.5378</td>
</tr>
<tr>
<td>1.000</td>
<td>-1.1532</td>
<td>0.5225</td>
<td>-0.0002</td>
<td>-0.5955</td>
<td>-0.5285</td>
</tr>
</tbody>
</table>

Reference: Das et al.
Attenuation relationships for India

The following predictive attenuation relationship for peak horizontal ground acceleration ($PGA$) for Guwahati city has been proposed by Nath et al. (2009):

$$\ln(PGA) = 9.143 + 0.247M - 0.014(10-M)^3 - 2.697 \ln(r_{rup} + 32.9458 \exp(0.0663M))$$

where $M$ is the earthquake moment magnitude, $r_{rup}$ is the rupture distance (km) and $PGA$ is in $g$.

Attenuation relationships for India

The following predictive attenuation relationship for peak horizontal ground acceleration \((PGA)\) for Bhuj has been proposed by Iyengar and Raghukanth (2002)

\[
(PGA/g) = \frac{38.82}{R^{1.12}}
\]

where \(R\) is the hypocentral distance (km)

Attenuation relationships for India

Jain et al. (2000) proposed following model for attenuation relationship for India using database from four regions,

\[ \ln(\text{PGA}) = b_1 + b_2 M + b_3 R + b_4 \ln(R) \]

where PGA is in g, for central Himalayan earthquakes \( b_1 = -4.135, b_2 = 0.647, b_3 = -0.00142, b_4 = -0.753 \) and \( \sigma = 0.59 \) and for non-subduction earthquakes in N.E. India \( b_1 = -3.443, b_2 = 0.706, b_3 = 0, b_4 = -0.828 \) and \( \sigma = 0.44 \) (coefficients of other equations not given here because they are for a particular earthquake).

- a. Central Himalayan earthquakes (thrust): (32 SMA records, 117 SRR records), 3 earthquakes with \( 5.5 \leq M \leq 7.0 \), focal depths \( 10 \leq h \leq 33 \text{ km} \) and \( 2 \leq R \leq 322 \text{ km} \).
- b. Non-subduction earthquakes in NE India (thrust): (43 SMA records, 0 SRR records), 3 earthquakes with \( 5.2 \leq M \leq 5.9 \), focal depths \( 33 \leq h \leq 49 \text{ km} \) and \( 6 \leq R \leq 243 \text{ km} \).
- c. Subduction earthquakes in NE India: (33 SMA records, 104 SRR records), 1 earthquake with \( M = 7.3 \), focal depth \( h = 90 \text{ km} \) and \( 39 \leq R \leq 772 \text{ km} \).
- d. Bihar-Nepal earthquake in Indo-Gangetic plains (strike-slip): (0 SMA records, 38 SRR records), 1 earthquake with \( M = 6.8 \), focal depth \( h = 57 \text{ km} \) and \( 42 \leq R \leq 337 \text{ km} \).

Ref: Douglas (2001), ESEE Report
M.L. Sharma et al. (2009)

\[
\log A = b_1 + b_2 M_W - b_3 \log \sqrt{R_{JB}^2 + b_4^2 + b_5 S + b_6 H}
\]

Where, \( R_{JB} \) = distance to surface projection of the rupture
\( \eta = 5\% \),
No. of events considered = 201 (58 from India and 143 from Iran)
Periods = 0.04 – 2.5 sec
\( b_1, b_2, b_3, b_4, b_5 \) & \( b_6 \) are regression coefficients
\( A \) = Spectral acceleration in terms of m/sec\(^2\)
\( S = 1 \) (for rock sites) & 0 (for others)
\( H = 1 \) (for strike slip mechanism) & 0 (for a reverse mechanism)
Max magnitude = 7 and Distance = 232 km.
Mandal et al. (2009)

\[
\ln(Y) = -7.9527 + 1.4043M_w - \ln \left( r_{jb}^2 + 19.82 \right)^{1/2} - 0.06
\]

Where, \( r_{jb} \) = distance to surface projection of the rupture

For \( 3.1 < M_w \leq 7.7 \)

Standard deviation = \( \pm 0.8243 \)

Gupta (2010)

\[
\log(Y) = C_1 + C_2 M + C_3 h + C_4 R - g \log R + C_5 \frac{s}{S_C} + C_6 \frac{s}{S_D} + C_7 \frac{s}{S_E}
\]

Where, \( Y = \text{PHA (cm/s}^2\)\)

\( h = \text{focal depth in km (limited to 100km for deeper events) } \)

\[
R = \sqrt{D_{\text{fault}}^2 + D^2}
\]

\( g = \text{Geometric attenuation factor} \)

Attenuation relationships for India

Szeliga et al. (2010) proposed intensity attenuation relationship for India considering earthquakes felt since 1762 as follows,

\( I = a + b M_w + cR + d \log(R) \)

(here out of 570 earthquakes about 100 were instrumented to record magnitudes and among these common set of 29 earthquakes were finally used for prediction)

Here, \( R \) is hypocentral distance in km and \( M_w \) is moment magnitude. Constants \( a, b, c \) and \( d \) need to be calculated using following table.

<table>
<thead>
<tr>
<th>Province</th>
<th>Number of Events</th>
<th>( a' )</th>
<th>( b' )</th>
<th>( c' )</th>
<th>( d' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>India</td>
<td>29</td>
<td>5.57 ± 0.58</td>
<td>1.06 ± 0.07</td>
<td>−0.0010 ± 0.0004</td>
<td>−3.37 ± 0.25</td>
</tr>
<tr>
<td>Craton</td>
<td>17</td>
<td>3.67 ± 0.79</td>
<td>1.28 ± 0.10</td>
<td>−0.0017 ± 0.0006</td>
<td>−2.83 ± 0.30</td>
</tr>
<tr>
<td>Himalaya</td>
<td>12</td>
<td>6.05 ± 0.94</td>
<td>1.11 ± 0.10</td>
<td>−0.0006 ± 0.0006</td>
<td>−3.91 ± 0.38</td>
</tr>
</tbody>
</table>
Functional Form of Attenuation Relationship (Bozorgnia et al. 2010)

Ground Motion Prediction Equation (GMPE) can be expressed commonly as,

\[ \ln Y = f_{mag} + f_{dis} + f_{flt} + f_{hng} + f_{site} + f_{sed} + \epsilon \]

where the magnitude term is given by the expression

\[ f_{mag} = \begin{cases} c_0 + c_1 M; & M \leq 5.5 \\ c_0 + c_1 M + c_2 (M - 5.5); & 5.5 < M \leq 6.5 \\ c_0 + c_1 M + c_2 (M - 5.5) + c_3 (M - 6.5); & M > 6.5 \end{cases} \]

the distance term is given by the expression

\[ f_{dis} = (c_4 + c_5 M) \ln(\sqrt{R_{rup}^2 + c_6^2}) \]

the style-of-faulting (fault mechanism) term is given by the expressions

\[ f_{flt} = c_7 F_{RF} f_{flt,Z} + c_8 F_{NM} \]

\[ f_{flt,Z} = \begin{cases} Z_{TOR}; & Z_{TOR} < 1 \\ 1; & Z_{TOR} \geq 1 \end{cases} \]

the hanging-wall term is given by the expressions
Functional Form of Attenuation Relationship (Bozorgnia et al. 2010)

\[ f_{hng} = c_{hng,R} f_{hng,M} f_{hng,Z} f_{hng,\delta} \]

\[ f_{hng,R} = \begin{cases} 
1; & R_{JB} = 0 \\
\frac{\max(R_{RUP}, \sqrt{R_{JB}^2 + 1}) - R_{JB}}{\max(R_{RUP}, \sqrt{R_{JB}^2 + 1})}; & R_{JB} > 0, Z_{TOR} < 1 \\
\frac{R_{RUP} - R_{JB}}{R_{RUP}}; & R_{JB} > 0, Z_{TOR} \geq 1 
\end{cases} \]

\[ f_{hng,M} = \begin{cases} 
0; & M \leq 6.0 \\
2(M - 6.0); & 6.0 < M < 6.5 \\
1; & M \geq 6.5 
\end{cases} \]

\[ f_{hng,Z} = \begin{cases} 
0; & Z_{TOR} \geq 20 \\
\frac{20 - Z_{TOR}}{20}; & 0 \leq Z_{TOR} < 20 
\end{cases} \]

\[ f_{hng,\delta} = \begin{cases} 
1; & \delta \leq 70 \\
\frac{90 - \delta}{20}; & \delta > 70 
\end{cases} \]
Functional Form of Attenuation Relationship (Bozorgnia et al. 2010)

The shallow site response term is given by the expression:

\[
f_{site} = \begin{cases} 
  c_{10} \ln \left( \frac{V_{S30}}{k_1} \right) + k_2 \ln \left[ A_{1100} + c \left( \frac{V_{S30}}{k_1} \right)^n \right] - \ln [A_{1100} + c] ; & V_{S30} < k_1 \\
  (c_{10} + k_2 n) \ln \left( \frac{V_{S30}}{k_1} \right) ; & k_1 \leq V_{S30} < 1100 \\
  (c_{10} + k_2 n) \ln \left( \frac{1100}{k_1} \right) ; & V_{S30} \geq 1100 
\end{cases}
\]

And the basin (sediment depth) response term is given by the expression:

\[
f_{sed} = \begin{cases} 
  c_{11} (Z_{2.5} - 1) ; & Z_{2.5} < 1 \\
  0 ; & 1 \leq Z_{2.5} \leq 3 \\
  c_{12} k_3 e^{-0.75 \left[ 1 - e^{-0.25 (Z_{2.5} - 3)} \right]} ; & Z_{2.5} > 3 
\end{cases}
\]
Functional Form of Attenuation Relationship (Bozorgnia et al. 2010)

where $Y$ is the estimate of the geometric mean horizontal component of “seismic coefficient” $C_{Y} = \frac{F_{y}}{W}$ for an inelastic SDF system; $M$ is moment magnitude; $R_{RUP}$ is the closest distance to the coseismic rupture plane (km); $R_{JB}$ is the closest distance to the surface projection of the coseismic rupture plane (km); $F_{RV}$ is an indicator variable representing reverse and reverse-oblique faulting, $F_{NM}$ is an indicator variable representing normal and normal-oblique faulting, $Z_{TOR}$ is the depth to the top of the coseismic rupture plane (km); $\delta$ is the dip of the rupture plane (°); $V_{S30}$ is the time-averaged shear-wave velocity in the top 30 m of the site profile (m/s); $A_{1100}$ is the median estimate of PGA (in units of “g”) on a reference rock outcrop $V_{S30} = 1100$ m/s; $Z_{2.5}$ is the depth to the 2.5 km/s shear-wave velocity horizon, typically referred to as basin or sediment depth (km); and $\epsilon$ is a random error term with zero mean and total standard deviation $\sigma_{\ln Y}$ given by the equation,

\[
\sigma_{\ln Y} = \sqrt{\sigma^2 + \tau^2}
\]

where $\sigma$ is the intra-event, or within-earthquake, standard deviation and $\tau$ is the interevent, or between-earthquake, standard deviation.
If $f_{\text{max}}$ is assumed constant for a given geographic region (15 Hz and 20 Hz are typical values for Western & Eastern North America respectively), then the spectra for different quakes are functions of seismic moments $M_0$ and $f_c$ which can be related (Brune, 1970 & 1971) thus:

\[
f_c = 4.9 \times 10^6 v_s \left( \frac{\Delta \sigma}{M_0} \right)^{1/3}
\]

Where $v_s$ is in km/sec, $M_0$ is in dyne-cm, and $\Delta \sigma$ is referred to as stress parameter or stress drop in bars. Values of 50 bars and 100 bars are common for stress parameters of Western & Eastern North America respectively.

Ref: Kramer (1996)
**Ratio $\frac{v_{\text{max}}}{a_{\text{max}}}$**

This ratio is proportional to the magnitude and distance dependencies proposed by McGuire (1978) as shown in the table below:

<table>
<thead>
<tr>
<th>Site Conditions</th>
<th>Magnitude Dependence</th>
<th>Distance Dependence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rock sites</td>
<td>$e^{0.40M}$</td>
<td>$R^{0.12}$</td>
</tr>
<tr>
<td>Soil sites</td>
<td>$e^{0.15M}$</td>
<td>$R^{0.23}$</td>
</tr>
</tbody>
</table>

Ref: Kramer (1996)
Estimation of other parameters

RMS Acceleration

Hanks and McGuire (1981) used a database of California earthquake of local magnitude 4.0 to 7.0 to develop an attenuation relationship for RMS acceleration for hypocentral distances between 10 and 100 km,

\[ a_{rms} = 0.119 \sqrt{f_{max} / f_c} \frac{\text{max}}{R} \]

where \( f_c \) is the corner frequency, \( f_{max} \) is the cutoff frequency, and \( R \) is in kilometer.

Ref: Kramer (1996)
Kavazanjian et al. (1985) used the definition of duration proposed by Vanmarcke and Lai (1980) with a database of 83 strong motion records from 18 different earthquakes to obtain

\[ a_{rms} = 0.472 + 0.268M_w + 0.129 \log \left( \frac{0.966}{R^2} + \frac{0.255}{R} \right) - 0.1167R \]

where \( R \) is the distance to the closest point of rupture on the fault. The database was restricted to \( M_w > 5 \), \( R < 110 \) km (68 mi), rupture depths less than 30 km (19 mi), and soil thicknesses greater than 10 m (33 ft).

Ref: Kramer (1996)
Arias Intensity

Campbell and Duke (1974) used data from California earthquakes to predict the variation of Arias intensity within 15 to 110 km (9 to 68 mi) of magnitude 4.5 to 8.5 events.

\[
I_a (m/sec) = 313 \frac{e^{M_s(0.33M_s-1.47)}}{R^{3.79}} S
\]

where

<table>
<thead>
<tr>
<th>$S$</th>
<th>$0.57R^{0.46}$</th>
<th>for basement rock</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.02R^{0.51}$</td>
<td></td>
<td>for sedimentary rock</td>
</tr>
<tr>
<td>$0.37R^{0.81}$</td>
<td></td>
<td>for alluvium $\leq 60$ft thick</td>
</tr>
<tr>
<td>$0.65R^{0.74}$</td>
<td></td>
<td>for alluvium $&gt; 60$ft thick</td>
</tr>
</tbody>
</table>

and $R$ is the distance from the center of the energy release in kilometers.

Ref: Kramer (1996)
Wilson (1993) analyzed strong motion records from California to develop an attenuation relation which, using the Arias intensity definition of equation which can be expressed as

\[ \log I_a (m/\text{sec}) = M_w - 2 \log R - kR - 3.990 + 0.365(1 - P) \]

where 
\[ R = \sqrt{D^2 + h^2} \]

\( D \) is the minimum horizontal distance to the vertical projection of the fault plane, \( h \) is a correction factor (with a default value of 7.5 km (4.7 mi)), \( k \) is a coefficient of inelastic absorption (with default value of zero), and \( P \) is the exceedance probability.

Ref: Kramer (1996)
End of Module – 4