Module 6 : Influence Lines

Lecture 5 : Müller-Breslau Principle

Objectives

In this course you will learn the following

- The Müller-Breslau principle for influence lines.
- Derivation of the principle for different types of internal forces.
- Example of application of this principle.

6.5 Müller-Breslau Principle

The Müller-Breslau principle uses Betti's law of virtual work to construct influence lines. To illustrate the method let us consider a structure *AB* (Figure 6.7a). Let us apply a unit downward force at a distance *x* from *A* , at point *C* . Let us assume that it creates the vertical reactions R_A and R_B at supports *A* and *B* , respectively (Figure 6.7b). Let us call this condition "System 1." In "System 2" (figure 6.7c), we have the same structure with a unit deflection applied in the direction of R_A. Here Δ is the deflection at point *C* .

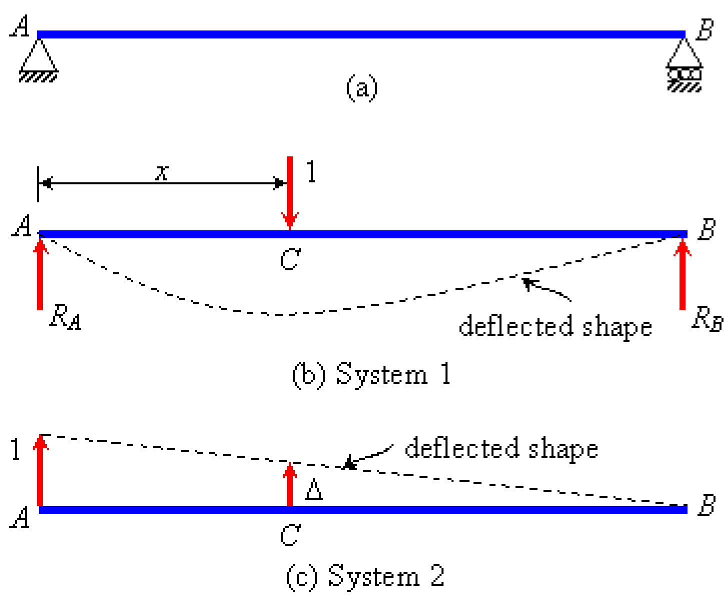


Figure 6.7 (a) Given system *AB* , (b) System 1, structure under a unit load, (c) System 2, structure with a unit deflection corresponding to R_A

According to Betti's law, the virtual work done by the forces in System 1 going through the corresponding displacements in System 2 should be equal to the virtual work done by the forces in System 2 going through the corresponding displacements in System 1. For these two systems, we can write:

$$(R_A)(1) + (1)(-\Delta) = 0$$

The right side of this equation is zero, because in System 2 forces can exist only at the supports, corresponding to which the displacements in System 1 (at supports *A* and *B*) are zero. The negative sign before Δ accounts for the fact that it acts against the unit load in System 1. Solving this equation we get:

$R_A = \Delta$

In other words, the reaction at support *A* due to a unit load at point *C* is equal to the displacement at point *C* when the structure is subjected to a unit displacement corresponding to the positive direction of support reaction at *A* . Similarly, we can place the unit load at any other point and obtain the support reaction due to that from System 2. Thus the deflection pattern in System 2 represents the influence line for R_A.

Following the same general procedure, we can obtain the influence line for any other response parameter as well. Let us consider the shear force at point *C* of a simply-supported beam *AB* (Figure 6.8a). We apply a unit downward force at some point *D* as shown in System 1 (Figure 6.8b). In system 2 (Figure 6.8c), we apply a unit deflection corresponding to the shear force, V_C. Note that the displacement at point *C* is applied in a way such that there is no relative rotation between *AC* and *CB* . This will avoid any virtual work done by the bending moment at *C* (M_C) going through the rotation in System 2. Now, according to Betti's law:

$$(V_C)(\Delta_{CA} + \Delta_{CB}) + (1)(-\Delta_D) = 0$$

$$(V_C)(1) - \Delta_D = 0$$

$$V_C = \Delta_D$$

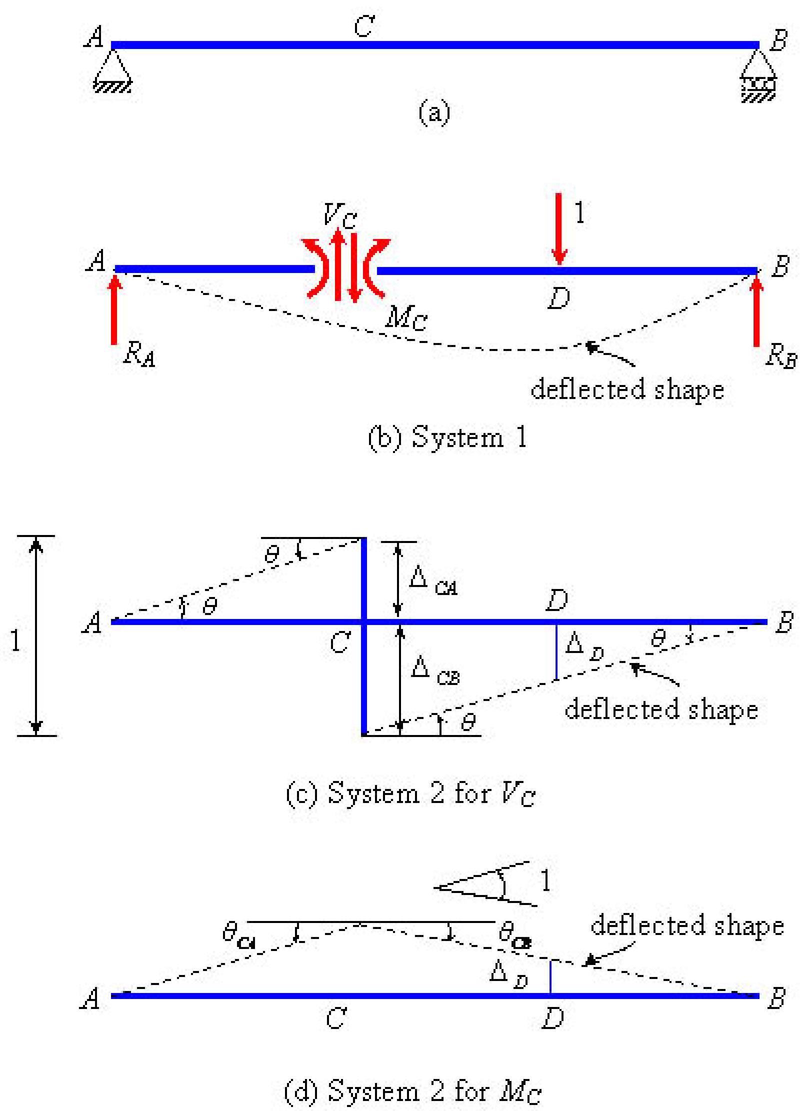


Figure 6.8 (a) Given system *AB* , (b) System 1, structure under a unit load, (c) System 2, structure with a

unit deflection corresponding to V_C, (d) System 2, structure with a unit deflection corresponding to M_C

Thus, the deflected shape in System 2 represents the influence line for shear force V_C. Similarly, if we want to find the influence line for bending moment M_C, we obtain System 2 (Figure 6.8d) by applying a unit rotation at point *C* (that is, a unit relative rotation between *AC* and *CB*). However, we do not want any relative displacement (between *AC* and *CB*) at point *C* in order to avoid any virtual work done by V_C going through the displacements in System 2. Betti's law provides the virtual work equation:

$$(M_C)(\theta_{CA}+\theta_{CB})+(1)(-\Delta_D)=0$$

$$(M_C)(1)-\Delta_D=0$$

$$M_C=\Delta_D$$

So, as we have seen earlier, the displaced shape in System 2 represents the influence line for the response parameter M_C.

Construction of System 2 for a given response function is the most important part in applying the Müller-Breslau principle. One must take care that other than the concerned response function no other force (or moment) in System 1 should do any virtual work going through the corresponding displacements in System 2. So we make all displacements in System 2 corresponding to other response functions equal to zero. For example, in Figure 6.8c, displacements corresponding to R_A, R_B and M_C are equal to zero. Example 6.4 illustrates the construction of influence lines using Müller-Breslau principle.

Example 6.4 Construct influence lines for R_A, R_E, V_D and M_B for the beam *AB* in Fig. E6.4.

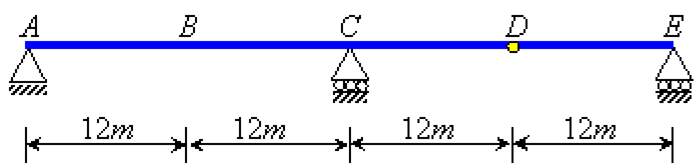
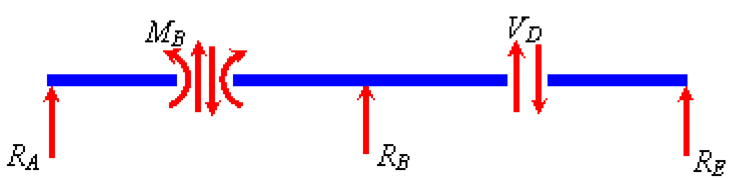
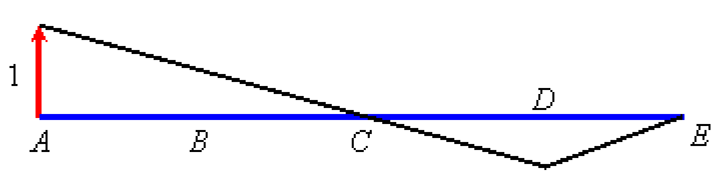


Fig. E6.4

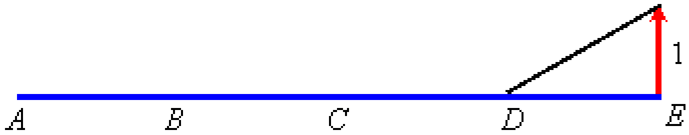
Solution:



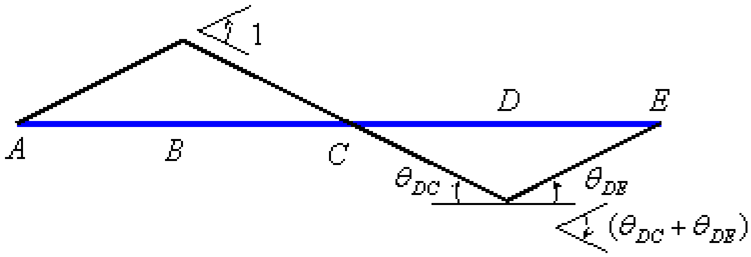
System 2 for R_A: (Note that there is no bending moment at *D* , i.e. $M_D=0$)



System 2 for R_E:



System 2 for M_B: (Note that only M_B contributes to virtual work because even though there is rotation at point *D* $(=\theta_{DC}+\theta_{DE})$, $M_D=0$)



The deflected shape in each system 2 provides the influence line for the corresponding response function.

Recap

In this course you have learnt the following

- The Müller-Breslau principle for influence lines.
- Derivation of the principle for different types of internal forces.
- Example of application of this principle.