

## Module 6 : Influence Lines

### Lecture 2 :Construction of Influence Lines using Equilibrium Methods

#### Objectives

In this course you will learn the following

- Construction of influence lines using equilibrium conditions.
- Some examples following this method.

#### 6.2 Construction of Influence Lines using Equilibrium Methods

The most basic method of obtaining influence line for a specific response parameter is to solve the static equilibrium equations for various locations of the unit load. The general procedure for constructing an influence line is described below.

1. Define the positive direction of the response parameter under consideration through a free body diagram of the whole system.

2. For a particular location of the unit load, solve for the equilibrium of the whole system and if required, as in the case of an internal force, also for a part of the member to obtain the response parameter for that location of the unit load. This gives the ordinate of the influence line at that particular location of the load.

3. Repeat this process for as many locations of the unit load as required to determine the shape of the influence line for the whole length of the member. It is often helpful if we can consider a generic location (or several locations)  $x$  of the unit load.

4. Joining ordinates for different locations of the unit load throughout the length of the member, we get the influence line for that particular response parameter.

The following three examples show how to construct influence lines for a support reaction, a shear force and a bending moment for the simply supported beam  $AB$ .

**Example 6.1** Draw the influence line for  $R_A$  (vertical reaction at  $A$ ) of beam  $AB$  in Fig. E6.1.

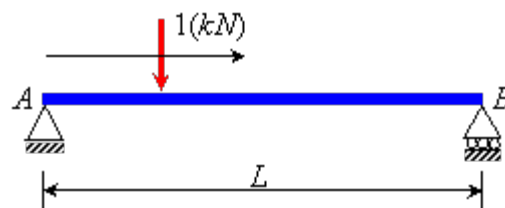
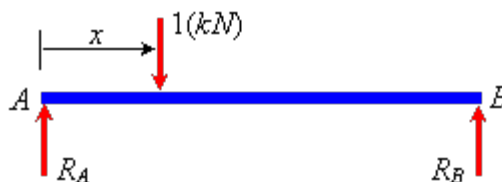


Fig. E6.1

#### Solution:

Free body diagram of  $AB$  :

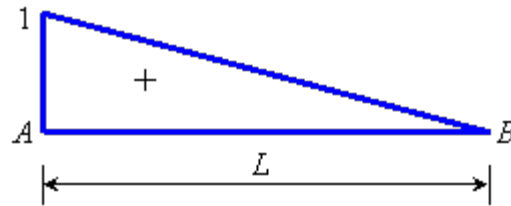


$$\sum F_y = 0 \Rightarrow R_A = 1 - R_B$$

$$\sum M(\text{about } B) = 0 \Rightarrow R_A(L) = 1(L - x)$$

$$\Rightarrow R_A = 1 - \frac{x}{L}$$

So the influence line of  $R_A$ :



**Example 6.2** Draw the influence line for  $V_C$  (shear force at mid point) of beam  $AB$  in Fig. E6.2.

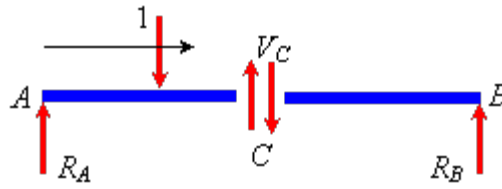
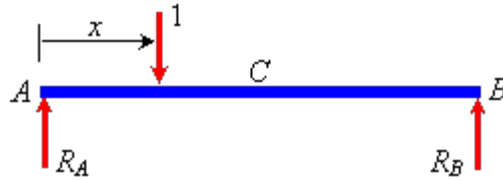


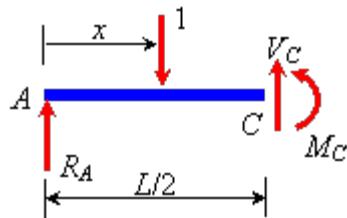
Fig. E6.2

**Solution:**



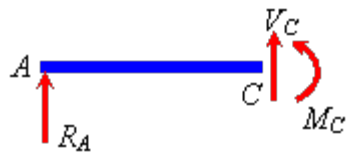
$$\sum M(\text{about } B) = 0 \Rightarrow R_A = 1 - \frac{x}{L}$$

For  $x < \frac{L}{2}$



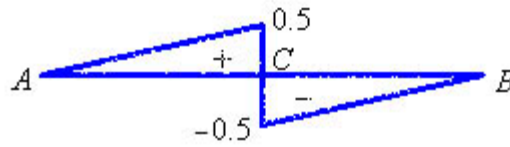
$$\sum F_y = 0 \Rightarrow V_C = 1 - R_A = \frac{x}{L}$$

For  $x > \frac{L}{2}$



$$\sum F_y = 0 \Rightarrow V_C = -R_A = \frac{x}{L} - 1$$

So the influence line for  $V_C$ :



**Example 6.3** Draw the influence line for  $M_D$  (bending moment at  $x = 2L/3$ ) for beam  $AB$  in Fig. E6.3.

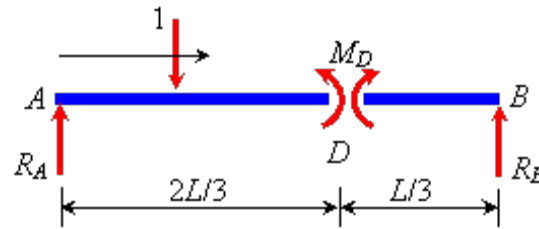
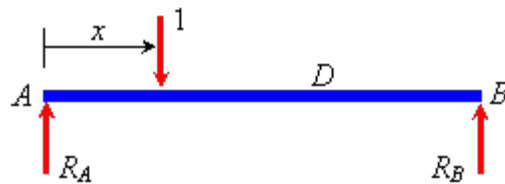


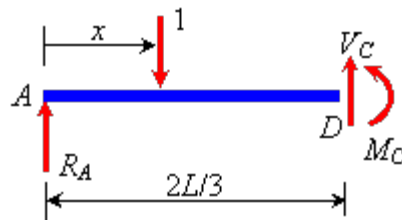
Fig. E6.3

**Solution:**



$$\sum M(\text{about } B) = 0 \Rightarrow R_A = 1 - \frac{x}{L}$$

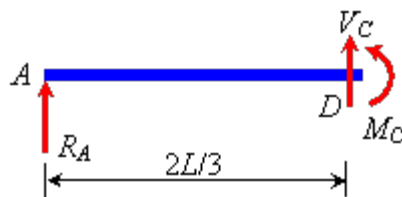
For  $x < \frac{2L}{3}$



$$\sum M(\text{about } D) = 0$$

$$\begin{aligned} \Rightarrow M_D &= R_A \left( \frac{2L}{3} \right) - 1 \left( \frac{2L}{3} - x \right) = \left( 1 - \frac{x}{L} \right) \left( \frac{2L}{3} \right) - \frac{2L}{3} + x \\ &= -\frac{2x}{3} + x = \frac{x}{3} \end{aligned}$$

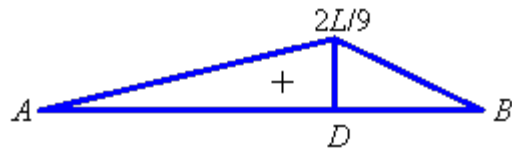
For  $x > \frac{2L}{3}$



$$\sum M(\text{about } D) = 0 \Rightarrow M_D = R_A \left( \frac{2L}{3} \right) = \frac{2L}{3} - \frac{2x}{3}$$

So, the influence of :

$M_D$



Similarly, influence lines can be constructed for any other support reaction or internal force in the beam. However, one should note that equilibrium equations will not be sufficient to obtain influence lines in indeterminate structures, because we cannot solve for the internal forces/support reactions using only equilibrium conditions for such structures.

### Recap

In this course you have learnt the following

- Construction of influence lines using equilibrium conditions.
- Some examples following this method.