Module 4 : Deflection of Structures
Lecture 5 : Bending Deflection Due to Temperature Variation

Objectives

In this course you will learn the following

- Bending deflection of beams due to temperature variation.

4.6 Bending Deflection due to Temperature Variation

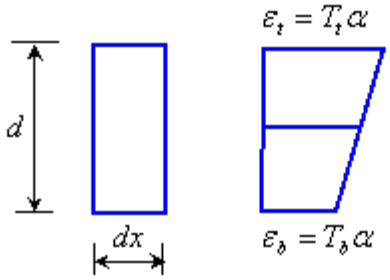
Consider a beam member (refer Figure 4.29) subjected to temperature gradient ΔT over the depth of beam such that

$$\Delta T = T_t - T_b \qquad \textbf{(4.22)}$$

where T_t = temperature at the top of the beam; and T_b = temperature at the bottom of the beam.

(a) Beam under temperature variation

(b) Deflection of beam due to temperature variation



(c) Strain diagram

Figure 4.29

The deflection of the beam due to temperature variation is shown in Figure 4.29(b). It is assumed that temperature varies linearly through the depth, *d* and α is the coefficient of thermal expansion of the material.

Consider a small element of length *dx* . The strain at top and bottom of the small elements are

$$\varepsilon_t = \frac{T_t\,\alpha\,dx}{dx} = T_t\alpha \qquad \textbf{(4.23a)}$$

$$\varepsilon_b = \frac{T_b\,\alpha\,dx}{dx} = T_b\alpha \qquad \textbf{(4.23b)}$$

The curvature of the beam is given by

$$\frac{1}{\rho} = \frac{\varepsilon_m}{d} = \frac{\alpha(T_t - T_b)}{d}$$

$$\frac{d^2v}{dx^2} = \frac{\alpha(T_t - T_b)}{d} \qquad \textbf{(4.24)}$$

The equation (4.24) can be used for finding out the bending deflection in beams due to temperature variation. If the beam is restrained from rotation, the moment induced in the beam will be given by

$$M_T = \frac{EI\alpha(T_t - T_b)}{d} \qquad \textbf{(4.25)}$$

The equation (4.25) is obtained by equating the right hand side of equation (4.24) to $\frac{M_T}{EI}$ from the simple bending theory.

Temperature deflections of a cantilever beam:

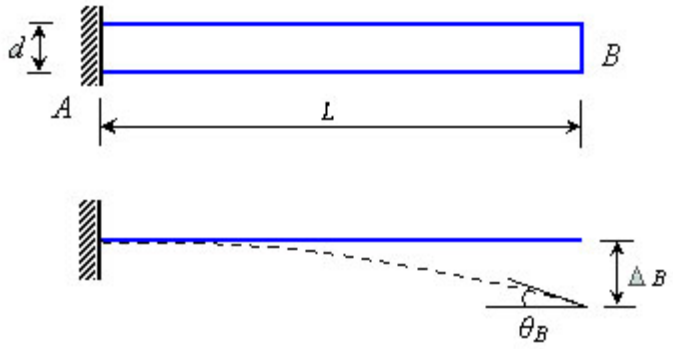


Figure 4.30

Consider a cantilever beam as shown in Figure 4.30 subjected to temperature gradient $\Delta T = T_t - T_b$ over the depth. Integrating the equation (4.24)

$$\frac{dv}{dx} = \frac{\alpha(T_t - T_b)}{d}x + C_1 \qquad \textbf{(4.26)}$$

$$v = \frac{\alpha(T_t - T_b)}{d}\frac{x^2}{2} + C_1 x + C_2 \qquad \textbf{(4.27)}$$

Boundary conditions: At x = 0, $\frac{dv}{dx} = 0$ and v = 0 will give the values of arbitrary constants as $C_1 = C_2 = 0$.

The slope and deflection of the free end of the cantilever beam are

$$\theta_B = \frac{\alpha(T_t - T_b)L}{d} \qquad \textbf{(4.28a)}$$

$$\Delta_B = \frac{\alpha(T_t - T_b)L^2}{2d} \qquad \textbf{(4.28a)}$$

Recap

In this course you have learnt the following

- Bending deflection of beams due to temperature variation.