Module 3 : Cables

Lecture 2 : The General Cable Theorem

Objectives

In this course you will learn the following

- Statement and derivation for the general cable theorem.

3.2 The General Cable Theorem

The *general cable theorem* helps us determine the shape of a cable supported at two ends when it is acted upon by vertical forces. It can be stated as: " At any point on a cable acted upon by vertical loads, the product of the horizontal component of cable tension and the vertical distance from that point to the cable chord equals the moment which would occur at that section if the loads carried by the cable were acting on an simply-supported beam of the same span as that of the cable."

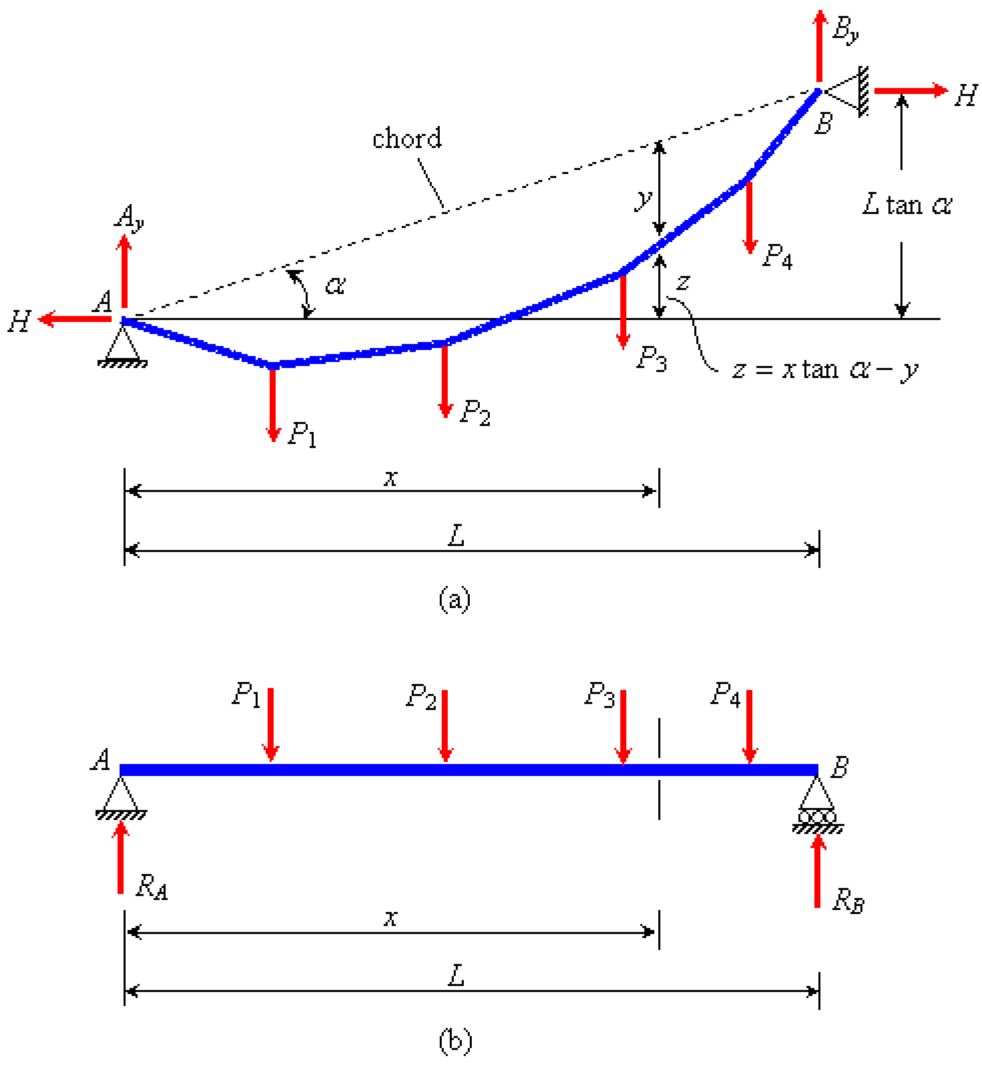

Figure 3.4 Explanation of the general cable theorem: (a) Cable under vertical loads, and (b) Simply supported beam with equal span under the same set of loads

To explain, let us consider the cable *AB* in Figure 3.4a, which is acted upon by the vertical loads P_1, P_2, P_3 and P_4 at known locations. The line *AB* joining the two supports is known as the *chord* of the cable and the horizontal distance between the supports is known as its *span* . The vertical distance between the chord and the cable at any cross section is known as the *dip* . This is vertical distance that is mentioned in the *general* *cable theorem* . The cable in Figure 3.4a has a span *L* and the dip at a distance *x* from *A* is *y* . The horizontal reactions at supports *A* and *B* have to be equal to satisfy static equilibrium, and let it be *H* . The vertical reactions at supports *A* and *B* are A_y and B_y, respectively. Figure 3.4b shows a simply-supported beam *AB* of same span (*L*) and acted upon by the same set of forces as the cable *AB* in Figure 3.4a.

For moment equilibrium about support *B* for the cable:

$$A_y L + HL \tan \alpha = \sum M_{BP} \qquad \textbf{(3.1)}$$

where, $\sum M_{BP}$ is the summation of moments due to external forces (P_1, P_2, P_3 and P_4) about point *B* . Since the cable is totally flexible against bending, bending moment at any cross-section is zero. By equating bending moment at a distance *x* from *A* to zero, we get:

$$A_y x + Hz = \sum M_{xP} \qquad \textbf{(3.2a)}$$

or,

$$A_y x + H(x \tan \alpha - y) = \sum M_{xP} \qquad \textbf{(3.2b)}$$

where, $\sum M_{xP}$ is the summation of moments due to external forces (P_1, P_2, and P_3 to the left of *x*) about section *x* . Substituting $A_y x$ from Equations 3.1 and 3.2b:

$$Hy = \left(\frac{x}{L}\right)\sum M_{BP} - \sum M_{xP} \qquad \textbf{(3.3)}$$

Now, let us consider the simply-supported beam in Figure 3.4b. From moment equilibrium about support *B* , we get the vertical reaction at support *A* :

$$R_A = \frac{\sum M_{BP}}{L} \qquad \textbf{(3.4)}$$

So, the bending moment at a distance *x* from *A* is:

$$\text{Moment at } x \text{ for the be am} = \left(\frac{\sum M_{BP}}{L}\right)x - \sum M_{xP} \qquad \textbf{(3.5)}$$

which, is same as the right side of Equation 3.3. Therefore:

$Hy =$ Moment at *x* for the simply-supported beam **(3.6)**

which is the claim as per the *general cable theorem* .

Note that the horizontal component of the axial force at any section of a cable (under vertical external forces only) is same as the horizontal reaction (*H*) at the end supports. This can be proved considering the equilibrium of horizontal forces on any segment of the cable.

We can solve internal forces in a cable using the general cable theorem, and also we can obtain for the shape of the cable. If the cable length (not the span) is known to us, we can express this length in terms of the dip *y* . Using this information along with the general cable theorem we can solve for both the unknowns *H* and *y* . Alternatively, the dip at a certain point, instead of the total length of the cable, may be known to us. This information, along with the general cable theorem helps us solve for both *H* and *y* .

Recap

In this course you have learnt the following

- Statement and derivation for the general cable theorem.