Module 2 : Analysis of Statically Determinate Structures

Lecture 7 : Internal Force as a Function of *x*

Objectives

In this course you will learn the following

- How to express an internal force as a function of the distance measured along the length of the member.
- Use of this function in obtaining internal force diagrams.

2.7 Internal Force as a Function of *x*

Another alternative of studying internal force variations in a structural member is to express the internal force as a mathematical function of the longitudinal dimension (x). Thus, the axial force, shear force and bending moment at a section are expressed as $P(x)$, $V(x)$ and $M(x)$, respectively, where x is the distance measured along the primary dimension from one end of the member (Figure 2.13). For this course, we will consider the left end of the member as origin unless otherwise specified. Note that equations involving these internal forces change if the direction for positive x or its origin changes.

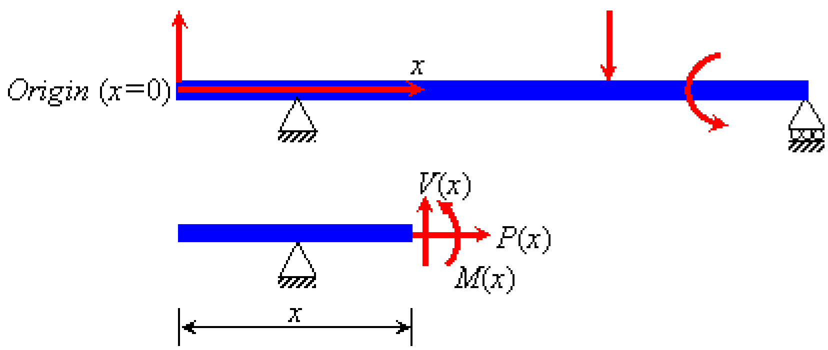

Figure 2.13. Internal forces at a distance *x* from the origin

Considering the example of Figure 2.7 again, let us obtain these internal force functions for the whole length. After obtaining the support reactions, we can investigate internal forces at different sections. Let us first consider the portion $x = 0 \longrightarrow 6\ m$. Since no force or moment is acting between these two points, the internal force functions will be continuous in this section. We draw the free body diagram of the beam upto a distance x from the left end of the beam (Figure 2.14a). Using equilibrium equations, we can find the internal forces:

$$\sum F_x = P(x) + 2kN = 0$$

$$\sum F_y = V(x) + 1.5kN = 0$$

$$\sum M_z(about\ A) = M(x) - 1.5kN(x) = 0$$

$$P(x) = -2kN$$

$$V(x) = -1.5kN$$

$$M(x) = 1.5kNm$$

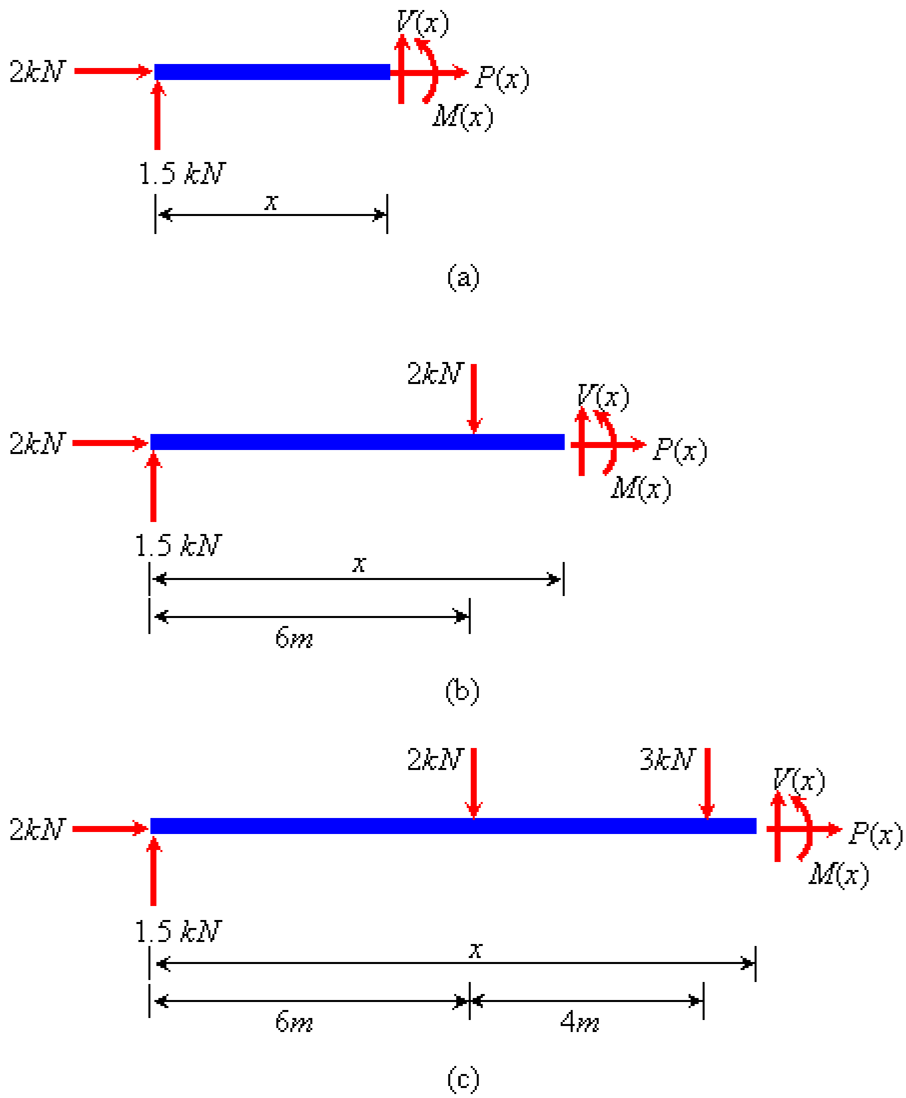

Figure 2.14 Free body diagrams upto a distance *x* from the origin

Similarly, we can find out the internal forces in the portions x = 6 $m \longrightarrow$ 10m (Figure 2.14b) and x = 10 $m \longrightarrow$ 12 m (Figure 2.14c). For x = 6 $m \longrightarrow$ 10 m :

$$P(x) = -2kN$$
$$V(x) = 0.5kN$$
$$M(x) = 1.5x - 2(x - 6)$$

and for x = 10 $m \longrightarrow$ 12 m :

$$P(x) = -2kN$$
$$V(x) = 3.5kN$$
$$M(x) = 1.5x - 2(x - 6) - 3(x - 10)$$

If we look at these expressions carefully, we see that:

•We measure x always from the same origin and in the same direction. As noted earlier, it is not absolutely necessary to follow this convention, but it is easier this way.

•The internal force expressions change at points where concentrated forces/moments (including support reactions) act. We will see later that these forces also change if a distributed force changes its distribution. Using *singularity functions* , we can combine different expressions for different segments of the beam together into a single expression, which we will discuss later.

•We need to obtain mathematical expressions of internal forces first in order to plot the force variation diagrams. Although these expressions provide adequate information on variation of internal forces, a pictorial representation is always very useful.

Recap

In this course you have learnt the following

- How to express an internal force as a function of the distance measured along the length of the member.

- Use of this function in obtaining internal force diagrams.