Module 2 : Analysis of Statically Determinate Structures

Lecture 5 : Obtaining Internal Forces in a System: General Procedure

Objectives

In this course you will learn the following

- How to obtain internal forces using equilibrium conditions.

2.5 Obtaining Internal Forces in a System: General Procedure

The general method of obtaining internal forces at certain cross-section of a system under a given loading (and support) condition is by applying the concepts of equilibrium (Lecture 2). To illustrate, let us consider the beam-column *AB* in Figure 2.7 for which we have to find the internal forces at section *a* - *a* . As we have learned earlier, equilibrium conditions are best studied through free body diagrams. We can find the reactions at supports *A* and *B* using a free body diagram of the whole beam-column *AB* (Figure 2.8). We solve the three equations for static equilibrium for this free body:

$$\sum F_x = A_x - 2kN = 0$$

$$\sum F_y = A_y - 2kN - 3kN + B_y = 0$$

$$\sum M_z (about\ A) = B_y(12m) - 3kN(10m) - 2kN(6m) = 0$$

$$A_x = 2kN$$

$$A_y = 1.5kN$$

$$B_y = 3.5kN$$

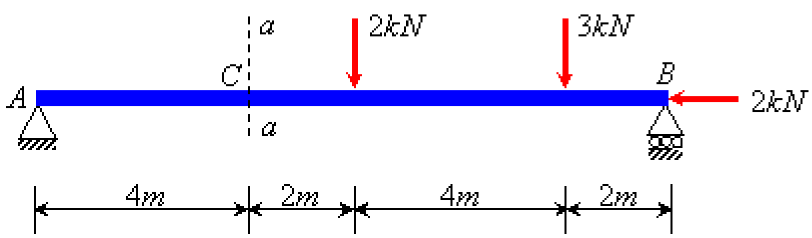

Figure 2.7 Loading and support conditions for planar beam-column system *AB*

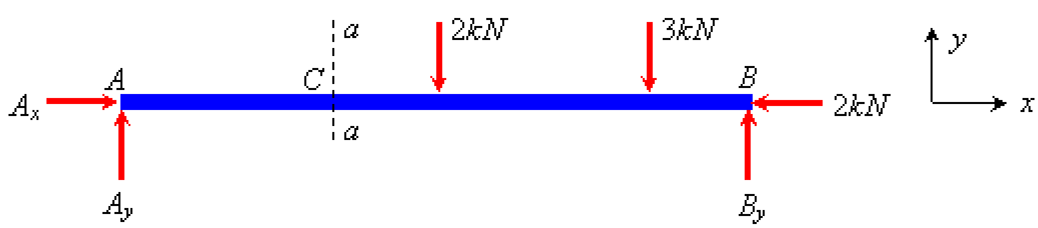

Figure 2.8 Free body diagram of *AB*

If a system is in static equilibrium condition, then every segment of it is also in equilibrium. So, we can consider the equilibrium for each of *AC* or *CB* independently. Let us consider the equilibrium of part *AC* , and draw its free body diagram (Figure 2.9). In addition to externally applied forces and the support reaction A_x and A_y, this free body is acted upon by forces *P* , *V* and *M* on the surface *a* - *a* . These are nothing but the internal forces (axial force, shear force and bending moment, respectively) acting at the cross-section *a* - *a* of *AB* . Note that these forces are drawn in their respective positive directions in order to avoid sign confusion. Solving the three static equilibrium equations for *AC* we find these internal forces:

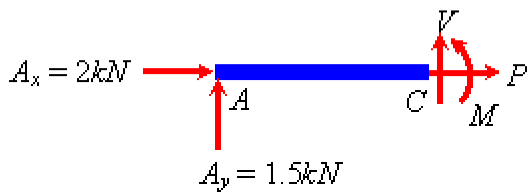

Figure 2.9 Free body diagram of part *AC*

$$\sum F_x = P + 2kN = 0$$
$$\sum F_y = V + 1.5kN = 0$$
$$\sum M_z(about\,C) = M - 1.5kN(4m) = 0$$
$$P = -2kN$$
$$V = -1.5kN$$
$$M = 6kNm$$

Thus we obtain the internal forces at section *a - a* . These could also be obtained by considering the equilibrium of the part at the other side of section *a - a* , that is of part *CB* . Figure 2.10 shows the free body diagram of *CB* . Again, the internal forces are drawn in their positive directions on surface *a - a* , which is a negative x-surface for this free body. Solving the three equations we find the values for these internal forces:

$$\sum F_x = P + 2kN = 0$$
$$\sum F_y = -V - 2kN - 3kN + 3.5kN = 0$$
$$\sum M_z(about\,C) = -M - 2kN(2m) - 3kN(6m) + 3.5kN(8m) = 0$$
$$P = -2kN$$
$$V = -1.5kN$$
$$M = 6kNm$$

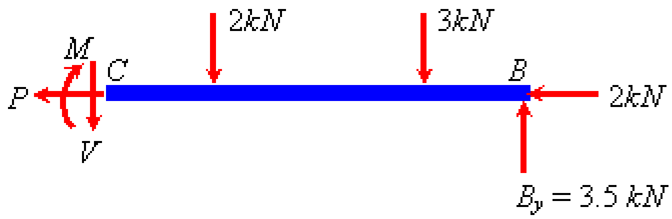

Figure 2.10 Free body diagram of part *CB*

Note that these values match exactly with the values obtained previously by considering the equilibrium of segment *AC* . This is true for any system because there is always a unique set of internal forces on an internal surface for a given loading condition.

Recap

In this course you have learnt the following

- How to obtain internal forces using equilibrium conditions.