MODULE – 2

Vibration Theory
**Phase angle vs. Frequency ratio**

![Graph showing phase angle vs. frequency ratio with different values of damping ratio.]

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**Force of Excitation Due to Rotating Imbalance**

\[ (M - m) \frac{d^2 z}{dt^2} + m \frac{d^2}{dt^2} (z + e \sin \omega t) = -kz - c \frac{dz}{dt} \]

After rearranging the terms:

\[ M \ddot{z} + c \dot{z} + kz = me \omega^2 \sin \omega t \]

The solution of this equation:

\[ Z_0 = \frac{me \omega^2}{\sqrt{(k-M \omega^2)^2 + (c \omega)^2}} \quad \text{and} \quad \tan \phi = \frac{c \omega}{k - m \omega^2} \]
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Force Due to Rotating Imbalance (Contd.)

In nondimensional form, these equations may be arranged as,

\[
Z_0 = \frac{\frac{me \omega^2}{M \omega_n^2}}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2 \right]^2 + \left(2 \zeta \omega / \omega_n\right)^2}}
\]

or,

\[
Z_0 = \frac{\frac{r^2}{M}}{\sqrt{\left(1 - r^2\right)^2 + \left(2 \zeta r\right)^2}} = DMF
\]

and \(\tan \phi = \frac{2 \zeta r}{1 - r^2}\)

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DMF (Dynamic Magnification Factor)

[Graph showing DMF values for different values of \(\xi\) and \(\nu / \omega_n\)]

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**DMF (Force Due to Rotating Imbalance)**

Now, \( DMF = \frac{1}{2\xi} \) for \( r = 1 \)

\((DMF)_{\text{max}} \text{ at } r = ?\)

\[
\frac{d}{dr} (DMF) = 0
\]

from this, \( r = \frac{1}{\sqrt{1 - 2\xi^2}} \)

\[
(DMF)_{\text{max}} = \frac{1}{2\xi \sqrt{1 - \xi^2}}
\]

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**Half Power Band Width Method**

(Experimental method to determine \( \xi \))

For small value of \( \xi \)

\((DMF)_{\text{max}} = \frac{1}{(2\xi)}\)

for, \( r = 1 \)

\[
\frac{(DMF)_{\text{max}}}{\sqrt{2}} = \frac{1}{2\sqrt{2}\xi} = \frac{1}{\sqrt{(1-r)^2 + (2\xi r)^2}}
\]

or, \( \frac{1}{8\xi^2} = \frac{1}{(1-r)^2 + (2\xi r)^2} \)

or, \( \xi = \left( \frac{\omega_1 + \omega_2}{2\omega_n} \right) \left( \frac{\omega_1 - \omega_2}{2\omega_n} \right) \)

or, \( \xi = \left( \frac{\omega_1 - \omega_2}{2\omega_n} \right) \) since, \( \left( \frac{\omega_1 + \omega_2}{2\omega_n} \right) \approx 1 \)