Module 4:
Lecture 8 on Stress-strain relationship
and Shear strength of soils
Contents

- Stress state, Mohr’s circle analysis and Pole, Principal stress space, Stress paths in p-q space;
- Mohr-Coulomb failure criteria and its limitations, correlation with p-q space;
- Stress-strain behavior; Isotropic compression and pressure dependency, confined compression, large stress compression, Definition of failure, Interlocking concept and its interpretations,
- Triaxial behaviour, stress state and analysis of UC, UU, CU, CD, and other special tests, Drainage conditions; Stress paths in triaxial and octahedral plane; Elastic modulus from triaxial tests.
The triaxial test: Introduction

- Most widely used shear strength test and is suitable for all types of soil.
- A cylindrical specimen, generally “L/D = 2” is used for the test, and stresses are applied under conditions of axial symmetry.
- Typical specimen diameters are 38mm and 100mm

Stress system in triaxial test:
- Axial stress
- Equal all round pressure
The triaxial test: Components

- Loading ram
- Perspex cell
- Latex sheet
- Soil sample
- Porous discs
- Pressure supply to cell
- To pore pressure measuring device
The triaxial test: **Mechanism**

- **Intermediate principal stress** $\sigma_2$ must be equal to major $\sigma_1$ or minor $\sigma_3$ stress, so as to facilitate representation of stress state in two dimensional Mohr’s circle.
- A cylindrical specimen is placed inside Perspex cell filled with water.
- The specimen is covered with latex sheet so as to avoid direct contact with water.
- The specimen is loaded initially by surrounding water pressure so as to achieve isotropic loading conditions.
- A deviatoric stress is then applied gradually on the sample with the help of Ram axially.
- A duct at the bottom of the sample allows water to pass through the sample which is further monitored, or conversely, in some cases, no drainage is allowed.
The triaxial test: **Mechanism**

- **Fine grained soil can stand the mould without any support**
- **But the coarse grained soils samples have to kept in some supporting mould until the application of negative pore pressure to the sample through drainage duct.**

So,

\[ u = u_e \text{ (negative)} \]

\[ \sigma_a = \sigma_r = 0 \]

\[ \sigma'_a = \sigma'_r = -u_e \]

**where,** \( \sigma_a \) **is the axial stress,** \( \sigma_r \) **is the radial stress**
The triaxial test: **Mechanism**

- If cell pressure increased to $s_{cp}$, this isotropic pressure is taken entirely by the pore water. Thus pore pressure increases, but no change occurs in effective stresses.

So,

- $u_i = \sigma_{cp} + u_e$ (negative)

- $\sigma_a = \sigma_r = \sigma_{cp}$

- $\sigma'_a = \sigma'_r = -u_e$

thus,

- $u - u_e = \Delta \sigma_{cp}$

i.e. $\Delta u = \Delta \sigma_{cp}$
Drainage conditions: **Combinations in triaxial test**

**Step 1**
- Under all-around cell pressure $\sigma_c$
- Drainage valve condition
  - Open
  - Closed
- Consolidated sample
- Unconsolidated sample

**Step 2**
- Shearing (loading)
- Drainage valve condition
  - Open
  - Closed
- Drained loading
- Undrained loading

Possible combinations:
- CD
- CU
- UU
### Drainage conditions: Combinations in triaxial test

<table>
<thead>
<tr>
<th>Test Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unconfined Compressive test (UC)</td>
<td>Specimen is taken to failure with no confinement</td>
</tr>
<tr>
<td>Unconsolidated Undrained test (UU)</td>
<td>Specimen is taken to failure with no drainage permitted</td>
</tr>
<tr>
<td>Consolidated Undrained test (CU)</td>
<td>Drainage valve initially opened to allow pore pressure $u_i$ to dissipate to zero, and then closed so that specimen is taken to failure without any further drainage</td>
</tr>
<tr>
<td>Consolidated Drained test (CD)</td>
<td>The drainage valve is initially opened to allow the pore pressure $u_i$ to dissipate to zero, and is kept open while the specimen is taken to failure at a sufficiently slow rate.</td>
</tr>
</tbody>
</table>

**Applying back pressure:**
- Decreases cavitation, and
- Reduction of voids.
Stresses and strains on a sample in the Triaxial compression test

**Axisymmetric condition**, \( \sigma'_2 = \sigma'_3 \) or \( \sigma_2 = \sigma_3; \ \varepsilon_2 = \varepsilon_3 \)

\[ p' = (\sigma'_1 + 2\sigma'_3)/3 \quad \text{and} \quad p = (\sigma_1 + 2\sigma_3)/3 \quad p' = p - u \]

\[ q = \sigma_1 - \sigma_3; \]
\[ q' = \sigma'_1 - \sigma'_3 \]
\[ = (\sigma_1 - \Delta u) - (\sigma_3 - \Delta u) = \sigma_1 - \sigma_3 \]

Thus, \( q' = q \); Shear is unaffected by PWP.

**Deviator stress**
\[ \sigma_1 - \sigma_3 = \sigma_d = \frac{P}{A} \]

**Deviatoric strain**
\[ \varepsilon_d = \frac{2}{3}(\varepsilon_1 - \varepsilon_3) \]

**Volumetric strain**
\[ \varepsilon_v = \varepsilon_1 + 2\varepsilon_3 \]

**Axial total stress**
\[ \sigma_1 = \sigma_3 + \frac{P}{A} \]

**Axial strain**
\[ \varepsilon_1 = \frac{\Delta z}{H_0} \]

**Radial strain**
\[ \varepsilon_r = \frac{\Delta r}{r_0} \]

Schematic of a Triaxial cell
Consolidated-drained test (CD Test)

Step 1: At the end of consolidation

\[ \sigma = u + \sigma' \]

Step 2: During axial stress increase

\[ \sigma'_{V} = \sigma_{V} \]

\[ \sigma'_{V} = \sigma_{V} + \Delta \sigma = \sigma'_{1} \]

\[ \sigma'_{h} = \sigma_{h} = \sigma'_{3} \]

Step 3: At failure

\[ \sigma'_{Vf} = \sigma_{V} + \Delta \sigma_{f} = \sigma'_{1f} \]

\[ \sigma'_{hf} = \sigma_{h} = \sigma'_{3f} \]
Consolidated- drained test (CD Test)

\[ \sigma_1 = \sigma_{VC} + \Delta \sigma \]

\[ \sigma_3 = \sigma_{hc} \]

**Deviator stress (q or \( \Delta \sigma_d \)) = \sigma_1 - \sigma_3**
Consolidated-drained test (CD Test): Volume change of sample during consolidation
CD Test: Stress-strain relationship during shearing

- Deviator stress, $\Delta \sigma_d$
- Axial strain

For Dense sand or OC clay:
- $(\Delta \sigma_d)_f$

For Loose sand or NC Clay:
- $(\Delta \sigma_d)_f$

Volume change of the sample:
- Loose sand / NC Clay
- Dense sand or OC clay

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CD tests: How to determine strength parameters $c$ and $\phi$

Deviator stress, $\Delta \sigma_d$

Confining stress $= \sigma_{3c}$

Confining stress $= \sigma_{3b}$

Confining stress $= \sigma_{3a}$

Mohr – Coulomb failure envelope

$\sigma_1 = \sigma_3 + (\Delta \sigma_d)_f$

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**CD tests**

**Strength parameters $c$ and $\phi$ obtained from CD tests**

Since $u = 0$ in CD tests, $\sigma = \sigma'$

Therefore, $c = c'$ and $\phi = \phi'$

Parameters are denoted as $c_d$ and $\phi_d$
CD tests: Failure envelopes

For sand and NC Clay, $c_d = 0$

Mohr – Coulomb failure envelope

Therefore, one CD test would be sufficient to determine $\phi_d$ of sand or NC clay
CD tests: Failure envelopes

For OC Clay, $c_d \neq 0$
Stress paths during CD Test

**Stage 1:**
Isotropic consolidation phase

\[ \Delta \sigma_1 = \Delta \sigma'_1 = \Delta \sigma_3 = \Delta \sigma'_3; \Delta \sigma_1 > 0; \]
\[ \Delta u = 0 \text{ (end of consolidation)} \]

\[ \Delta p' = \Delta p = (\Delta \sigma_1 + 2 \Delta \sigma_1)/3 = \Delta \sigma_1; \]
\[ \Delta q = \Delta \sigma_1 - \Delta \sigma_3 = 0 \]
\[ \Delta q/\Delta p' = \Delta q/\Delta p = 0 \]

**Consolidation phase**

\[ \Delta \sigma_3 = \Delta \sigma'_1 \]
\[ \Delta u = 0 \]

**Shearing phase**

\[ \Delta \sigma_3 = \Delta \sigma'_3 \]
\[ ESP = TSP \]
\[ \Delta p = \Delta \sigma_1/3; \]
\[ \Delta q = \Delta \sigma_1 \]
\[ \Rightarrow \Delta q/\Delta p = 3 \]

\[ p = (\sigma_1 + 2\sigma_3)/3 \]
\[ p, p' \]

**Consolidation**

\[ \sigma'_1 = \sigma'_3 + P/A \]
Stress paths during CD Test

Stage 2: Shearing phase

\[ \Delta \sigma_1 = \Delta \sigma'_1 > 0 ; \]

\[ \Delta \sigma_3 = \Delta \sigma'_3 = 0 ; \Delta u = 0 ; \]

\[ \Delta p' = \Delta p = (\Delta \sigma_1)/3 = \Delta \sigma_1/3 ; \]

\[ \Delta q = \Delta \sigma_1 - \Delta \sigma_3 = 0; = \Delta \sigma_1 ; \Delta q/\Delta p' = \Delta q/\Delta p = 3 \]
Consolidated- Undrained test (CU Test)

**Step 1:** At the end of consolidation

\[ \sigma_{VC} + \sigma_{hC} = \sigma_{VC} = \sigma_{VC} + \Delta \sigma + \sigma_{hC} \]

**Drainage**

\[ \sigma_{VC} + \Delta \sigma + \sigma_{hC} = \sigma_{VC} + \Delta \sigma + \sigma_{hC} \]

**No drainage**

\[ \sigma_{VC} + \Delta \sigma + \sigma_{hC} = \sigma_{VC} + \Delta \sigma + \sigma_{hC} \]

**Step 2:** During axial stress increase

\[ \sigma_{VC} + \Delta \sigma + \pm \Delta u = \sigma_{1} \]

\[ \sigma_{hC} + \pm \Delta u = \sigma_{3} \]

**Step 3:** At failure

\[ \sigma_{VC} + \Delta \sigma + \pm \Delta u_f = \sigma_{1f} \]

**No drainage**

\[ \sigma_{hC} + \pm \Delta u_f = \sigma_{3f} \]
Consolidated- Undrained test (CU Test)

Volume change of sample during consolidation

Time

Expansion

Compression

Volume change of the sample
CU Test: Stress-strain relationship during shearing

Devitaor stress, $\Delta\sigma_d$

- Dense sand or OC clay
- Loose sand or NC Clay

Pore water pressure varies with axial strain

$\Delta u$

Axial strain

$\Delta\sigma_d f$
CU tests: - How to determine strength parameters $c$ and $\phi$

Deviator stress, $\Delta\sigma_d$

Confining stress $= \sigma_{3b}$

Confining stress $= \sigma_{3a}$

Total stresses at failure

Mohr – Coulomb failure envelope in terms of total stresses

$\sigma_1 = \sigma_3 + (\Delta\sigma_d)_f$

$\sigma_{1a} \rightarrow \sigma_{1b}$

$\sigma_{3a} \rightarrow \sigma_{3b}$

$\phi_{cu}$
CU tests: Strength parameters $c$ and $\phi$

Mohr – Coulomb failure envelope in terms of effective stresses

Mohr – Coulomb failure envelope in terms of total stresses

Effective stresses at failure

$$\sigma_1' = \sigma_3 + (\Delta \sigma_d)_f - u_f$$

$$\sigma_3' = \sigma_3 - u_f$$

$u_f$

$\phi'$

$\phi_{cu}$

$u_{fa}$

$u_{fb}$

$\sigma_1$, $\sigma_3$, $\sigma_3'$, $\sigma_1'$

$(\Delta \sigma_d)_{fa}$
CU tests

Strength parameters $c_d$ and $\phi_d$ obtained from CD tests

Shear strength parameters in terms of total stresses are $c_{cu}$ and $\phi_{cu}$

Shear strength parameters in terms of effective stresses are $c'$ and $\phi'$

$c' = c_d$ and $\phi' = \phi_d$
Stress paths during CU Test

Stage 1: Isotropic consolidation phase

\[ \Delta\sigma_1 = \Delta\sigma'_1 = \Delta\sigma_3 = \Delta\sigma'_3; \Delta\sigma_1 > 0; \]
\[ \Delta u = 0 \text{ (end of consolidation)} \]

\[ \Delta p' = \Delta p = (\Delta\sigma_1 + 2\Delta\sigma_1)/3 = \Delta\sigma_1; \]
\[ \Delta q = \Delta\sigma_1 - \Delta\sigma_3 = 0 \]
\[ \Delta q/\Delta p' = \Delta q/\Delta p = 0 \]

\[ \Delta\sigma_1 = \Delta\sigma'_1 = \Delta\sigma_3 = \Delta\sigma'_3 \]

\[ \Delta\sigma_3 = \Delta\sigma'_1 \]
\[ \Delta u = 0 \]

\[ \Delta\sigma_3 = 0 \]
\[ \Delta u \neq 0 \]

\[ \Delta\sigma_1 = \Delta\sigma_3 + P/A \]
\[ \Delta\sigma'_1 = \Delta\sigma_1 - \Delta u \]

Shearing phase \( \rightarrow \)

\[ \Delta q = 3 \Delta p \]
\[ \Delta q/\Delta p = 3 \]

\[ \Delta q/\Delta p' = \Delta q/\Delta p' = 3 \]

\[ p = (\sigma_1 + 2\sigma_3)/3 \]

\[ p, p' \]
Stress paths during CU Test

Stage 2: Shearing phase

\[ \Delta \sigma_1 > 0; \Delta \sigma_3 = 0; \Delta \sigma'_1 = \Delta \sigma_1 - \Delta u = 0; \Delta \sigma'_3 = -\Delta u \]

\[ \Delta p = (\Delta \sigma_1)/3; \Delta q = \Delta \sigma_1, \Delta q/\Delta p = 3 \text{ [For TSP]} \]

\[ \Delta p' = \Delta p - \Delta u = (\Delta \sigma_1)/3 - \Delta u; \quad \text{[For ESP]} \]

\[ \Delta q = \Delta \sigma_1; \Delta q/\Delta p' = \Delta \sigma_1/(\Delta \sigma_1/3 - \Delta u) = 3/[1-3(\Delta u/\Delta \sigma_1)] \]
Stress conditions for the UU test

- The purpose of UU test is to determine the undrained shear strength of a saturated soil.

- **Quick test** (Neither during consolidation and shearing stages, excess PWP is allowed to drain).
Mohr failure envelopes for UU tests

For 100% saturated clay →

For partially saturated clay →
Total stress path during UU Test

**Initial stage**

\[
\Delta \sigma_1 = \Delta \sigma_3; \; \Delta u \neq 0
\]
\[
\Delta p = \Delta \sigma_1, \; \Delta q = 0;
\]
\[
\frac{\Delta q}{\Delta p} = 0
\]

**Shearing phase**

\[
\Delta \sigma_1 > 0; \; \Delta \sigma_3 = 0
\]
\[
\Delta p = \frac{\Delta \sigma_1}{3}; \; \Delta q = \Delta \sigma_1
\]
\[
\frac{\Delta q}{\Delta p} = 3
\]

\[
\sigma_1 = \sigma_3 + \frac{P}{A}
\]

\[
\Delta p = \frac{\Delta \sigma_1}{3}; \; \frac{\Delta q}{\Delta p} = 3
\]
Unconfined compressive (UC) test

➢ To determine the un-drained shear strength of saturated clays quickly.

➢ No radial stress \( (\sigma_3 = 0) \)

➢ Deviator load is increased rapidly until the soil sample fails; Pore water can not drain from the soil; the soil sample is sheared at constant volume.

(After www.geocomp.com)
Stress conditions for the UC test

\[
\text{TOTAL, } \sigma = \text{NEUTRAL, } u + \text{EFFECTIVE, } \sigma'
\]

After sampling and specimen setup; before application of axial load:

- \(\sigma_{vo} = u_r\)
- \(\sigma_{ho} = u_r\)

During application of axial load:

- \(\sigma'_v = \Delta \sigma + u_r \mp \Delta u\)
- \(\sigma'_h = +u_r \mp \Delta u\)

At failure:

- \(\sigma'_{vf} = \Delta \sigma + u_r \mp \Delta u_f\)
- \(\sigma'_{hf} = +u_r \mp \Delta u_f\)

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Total stress path during UC Test

- The effective stress path is unknown since PWP changes are not normally measured.
- If $\Delta u$ is measured, it would be negative.
- Since $\sigma_3 = 0$,
  \[ \sigma'_3 = \sigma_3 - \Delta u = - \Delta u \]
- $\Delta u$ must be -ve because as $\sigma'_3$ can not be -ve (soils can not sustain tension). So $\sigma'_3$ must be +ve.

\[ q = \sigma_1 - \sigma_3 \]

\[ \Delta p = \Delta \sigma_1 / 3; \Delta q / \Delta p = 3 \]

\[ p = (\sigma_1 + 2\sigma_3) / 3 \]
Mohr Circles for UCS

The results of from UC tests can lead to:

- Estimate the short-term bearing capacity of fine-grained soils for foundations.
- Estimate the short-term stability of slopes.
- Determine the stress-strain characteristics under fast (un-drained loading conditions.)
Typical variation of $\sigma_1$ with $\varepsilon_1$ (UCS Test)

$C_u = \frac{136}{2} = 68 \text{ kPa}$
## Consolidated undrained triaxial tests on Silty sand

<table>
<thead>
<tr>
<th>Property</th>
<th>Unit</th>
<th>Silty sand</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific gravity ((G_s))</td>
<td>(-^a)</td>
<td>2.64</td>
</tr>
<tr>
<td><strong>Particle size distribution</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sand ((S))</td>
<td>(%)</td>
<td>80</td>
</tr>
<tr>
<td>Silt ((M))</td>
<td>(%)</td>
<td>10</td>
</tr>
<tr>
<td>Clay ((C))</td>
<td>(%)</td>
<td>10</td>
</tr>
<tr>
<td>Classification (Unified soil classification system)</td>
<td>(-^a)</td>
<td>SM</td>
</tr>
<tr>
<td><strong>Compaction characteristics (standard Proctor)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximum dry unit weight ((MDD))</td>
<td>kN/m(^3)</td>
<td>19.75</td>
</tr>
<tr>
<td>Optimum moisture content ((OMC))</td>
<td>(%)</td>
<td>10.5</td>
</tr>
<tr>
<td>Co-efficient of permeability ((k))</td>
<td>m/sec</td>
<td>(4.0 \times 10^{-7})</td>
</tr>
</tbody>
</table>
Variation in deviator stress with axial strain

- $\sigma' = 50$ kPa
- $\sigma' = 100$ kPa
- $\sigma' = 150$ kPa

Axial strain (%) vs. Deviator stress (kPa)
Variation in excess pore water pressure with axial strain
Status of silty sand sample after CU test
Variation in stress path at various effective stress

Cambridge stress path is plotted between \( p \) or \( p' \) and \( q \). Where,

\[
p = \frac{\sigma_1 + 2\sigma_3}{3}
\]

\[
p' = \frac{\sigma_1' + 2\sigma_3'}{3}
\]

\[
q = q' = (\sigma_1 - \sigma_3)
\]
Mohr circles for consolidated un-drained tests on silty sand

- Effective parameter ($\sigma' = 50$ kPa)
- Effective parameter ($\sigma' = 150$ kPa)
- Effective parameter ($\sigma' = 100$ kPa)
- Total parameter ($\sigma' = 50$ kPa)
- Total parameter ($\sigma' = 100$ kPa)
- Total parameter ($\sigma' = 150$ kPa)

- $c' = 2$ kPa
- $\phi' = 35^\circ$
- $c = 7$ kPa
- $\phi = 32^\circ$
Results of CU triaxial tests on Fine Sand

Max void ratio = 0.778
Min void ratio = 0.542

Void ratio after consolidation stage

i) \( e(\sigma' = 50 \text{ kPa}) = 0.723 \)

ii) \( e(\sigma' = 100 \text{ kPa}) = 0.741 \)

Variation in deviator stress with axial strain

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Variation in excess pore water pressure with axial strain

Effective stress = 50 kPa
Effective stress = 100 kPa

Variation in excess pore water pressure with axial strain
Variation of TSP at various effective stresses

Variation of ESP at various effective stresses

At failure
Status of sample after termination of CU test

\[ \frac{\pi}{4} + \frac{36^\circ}{2} \]