ADVANCED GEOTECHNICAL ENGINEERING

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Lecture No - 48

Module 3:

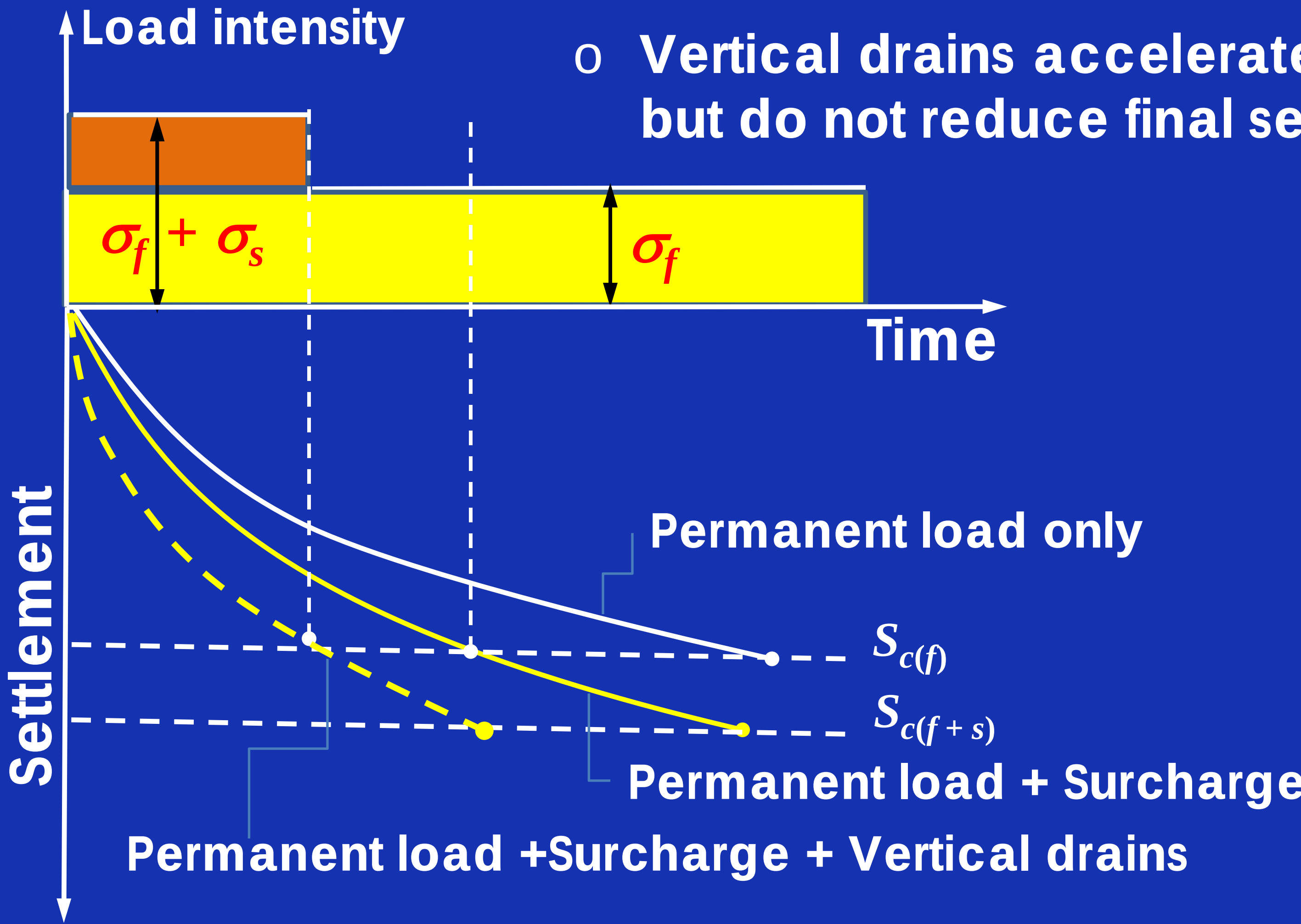
Lecture - 10 on Compressibility and Consolidation

Contents

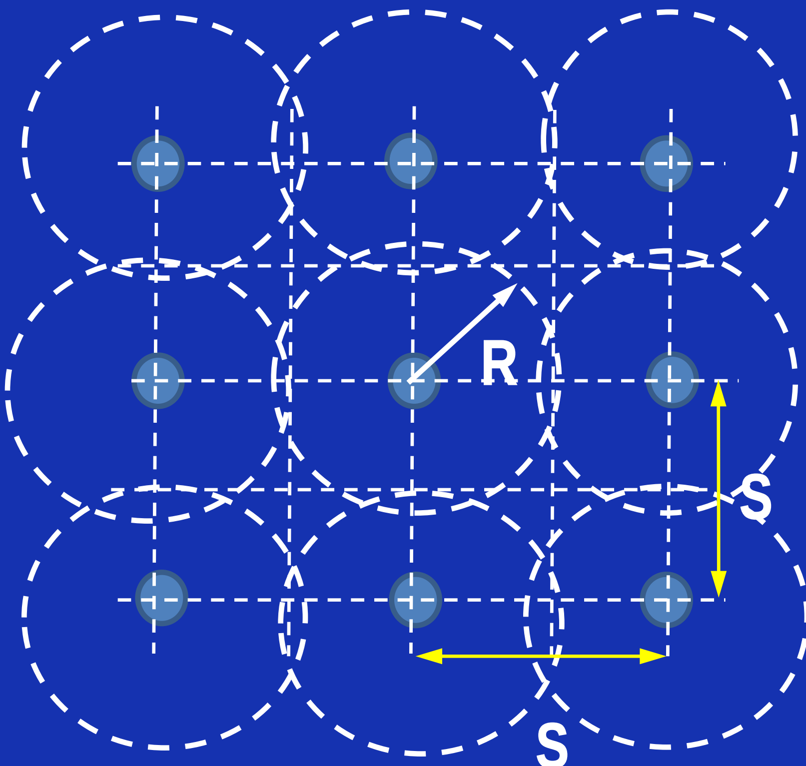
- Stresses in soil from surface loads;
- Terzaghi's 1-D consolidation theory;
- Application in different boundary conditions;
- Ramp loading;
- Determination of Coefficient of consolidation;
- Normally and Over-consolidated soils;
- Compression curves; Secondary consolidation;
- Radial consolidation;
- Settlement of compressible soil layers and
- Methods for accelerating consolidation settlements.

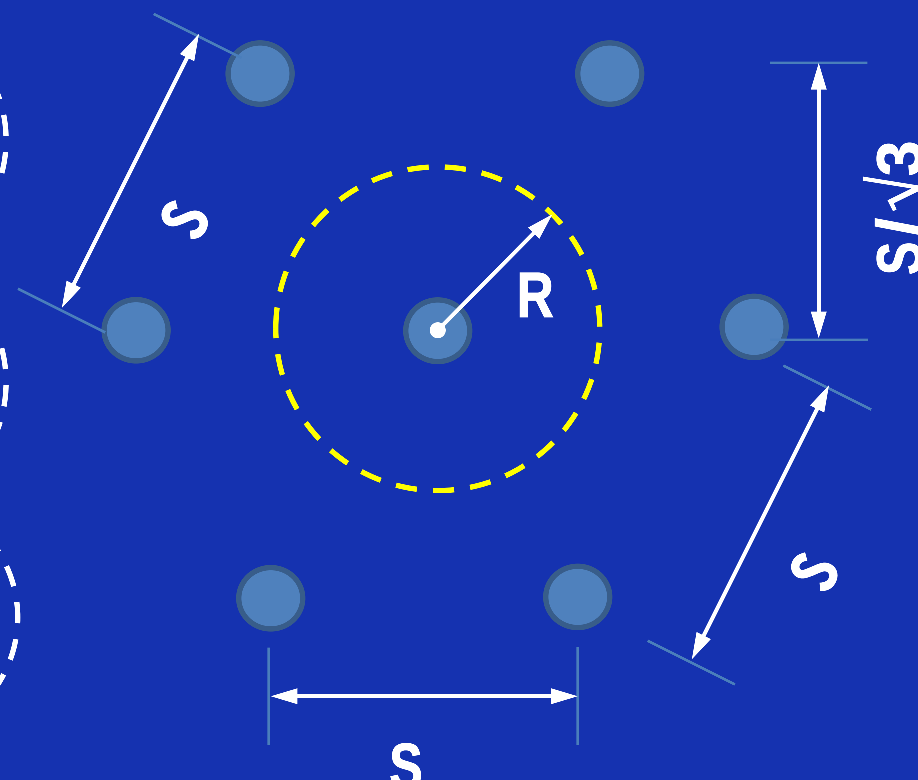
Preloading along with vertical drains

- Vertical drains accelerate settlements; but do not reduce final settlements.



Different layouts of vertical drains





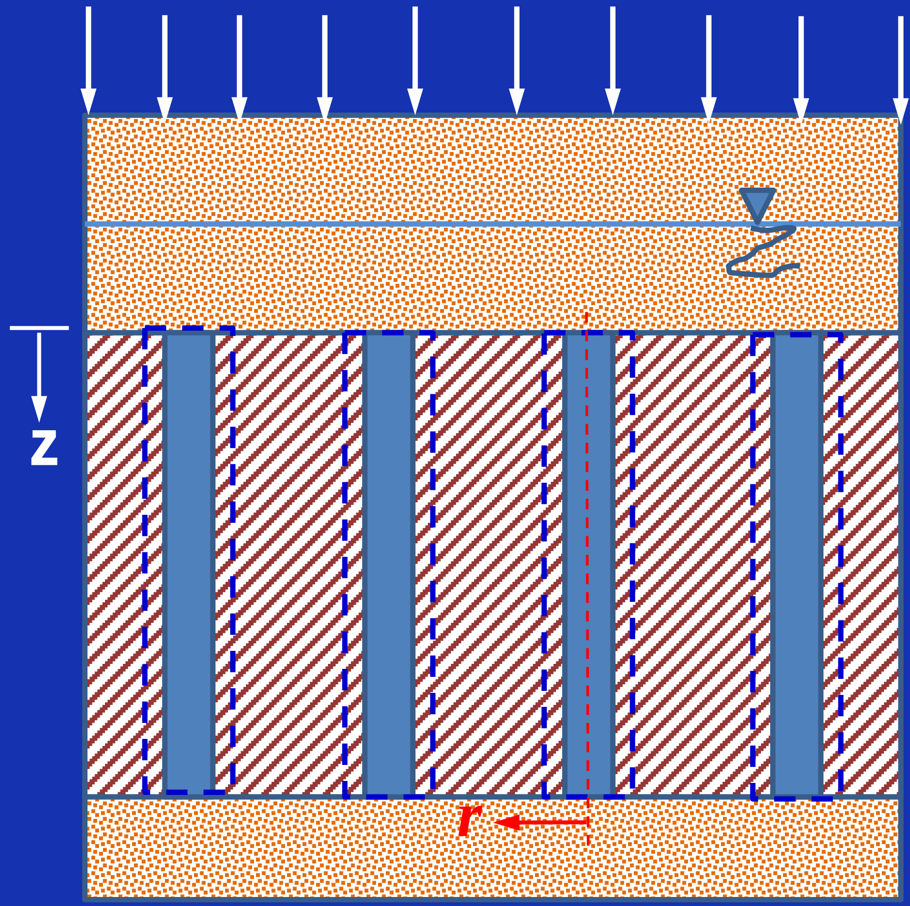
$$S^2 = \pi R^2$$

R = 0.564 S; D = 1.13 S

$$\pi R^2 = 6\left(\frac{\sqrt{3}}{4}\right)\left(\frac{S}{\sqrt{3}}\right)^2$$

R = 0.525S; D = 1.05S

Radial and Vertical consolidation



Governing differential equation for both vertical and radial consolidation

$$\frac{\partial u}{\partial t} = c_r\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) + c_v\frac{\partial^2 u}{\partial z^2}$$

u = excess PWP

r = radial distance measured from centre of the drain well

C_r = Coefficient of Consolidation in radial direction

Radial consolidation

- In order to accelerate the process of consolidation settlement for the construction of some structures, the useful technique of building vertical drains can be used

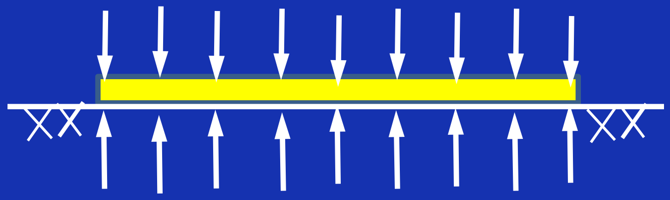
- When a surcharge is applied at ground surface, the pore water pressure in the clay will increase, and there will be drainage in the vertical and horizontal directions

- The horizontal drainage is induced by the vertical drains. Hence the process of dissipation of excess pore water pressure created by the loading (and hence the settlement) is accelerated.

Radial consolidation

The basic theory of vertical drains is presented for two fundamental cases:

Free-strain case: When the surcharge applied at the ground surface is of a **flexible** nature, there will be equal distribution of surface load. This will result in an uneven settlement at the surface.



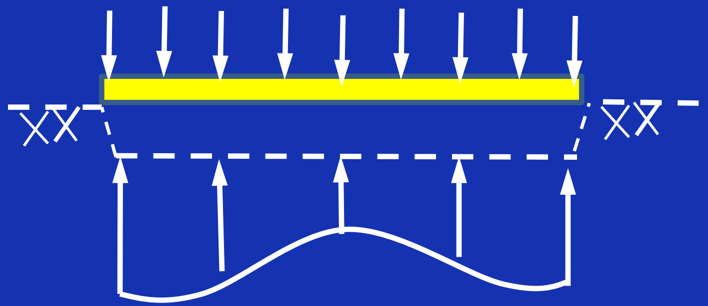
Flexible:

No shear and bending moment because reaction distribution is identical with the distribution loading itself.

- Barron assumes that **no arching** takes place and that shear strains caused by differential settlements **do not redistribute the load induced stresses within the soil at any time during consolidation.**

Radial consolidation

***Equal-strain case*: When the surcharge applied at the ground surface is rigid, the surface settlement will be the same all over. However, this will result in an unequal distribution of stress.**



- **It presumes arching to re-distribute the load so that vertical strains at certain depth become equal.**

Radial consolidation (Smear effect)

- A smear zone in a sand/PV drain is created by the remolding of clay during the drilling operation for building it. This remolding of the clay results in a decrease of the coefficient of permeability in the horizontal direction.

- The smear zone, which is reported to have a hydraulic conductivity lower than that of undisturbed soil away from the PVD installation zone, impedes the horizontal consolidation of soft clays.

Radial consolidation (Smear effect)

The smear zone also alters typically anisotropic initial hydraulic conductivity of clays. The reduction of the rate of consolidation in the radial direction at the smear zone is defined as the smear effect.

The smear effect, which is expected to be dictated by a number of factors such as:

(i) the sensitivity of soil,
(ii) installation process, and
(iii) the size and shape of the mandrel, are not fully comprehended, particularly the extent of the smear zone and its hydraulic conductivity.

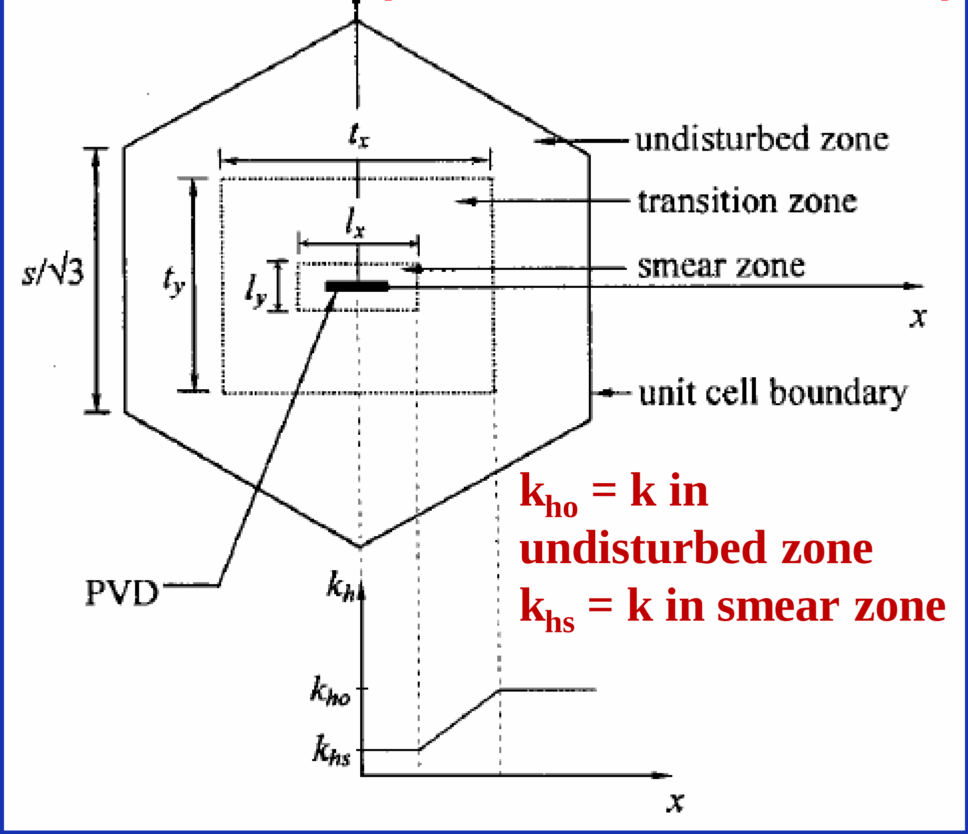
Radial consolidation (Smear and Transition zones)

The disturbed zone around the PVD consists of basically two zones: the smear zone and the transition zone

The smear zone is the completely remoulded zone of soil immediately adjacent to the drain.

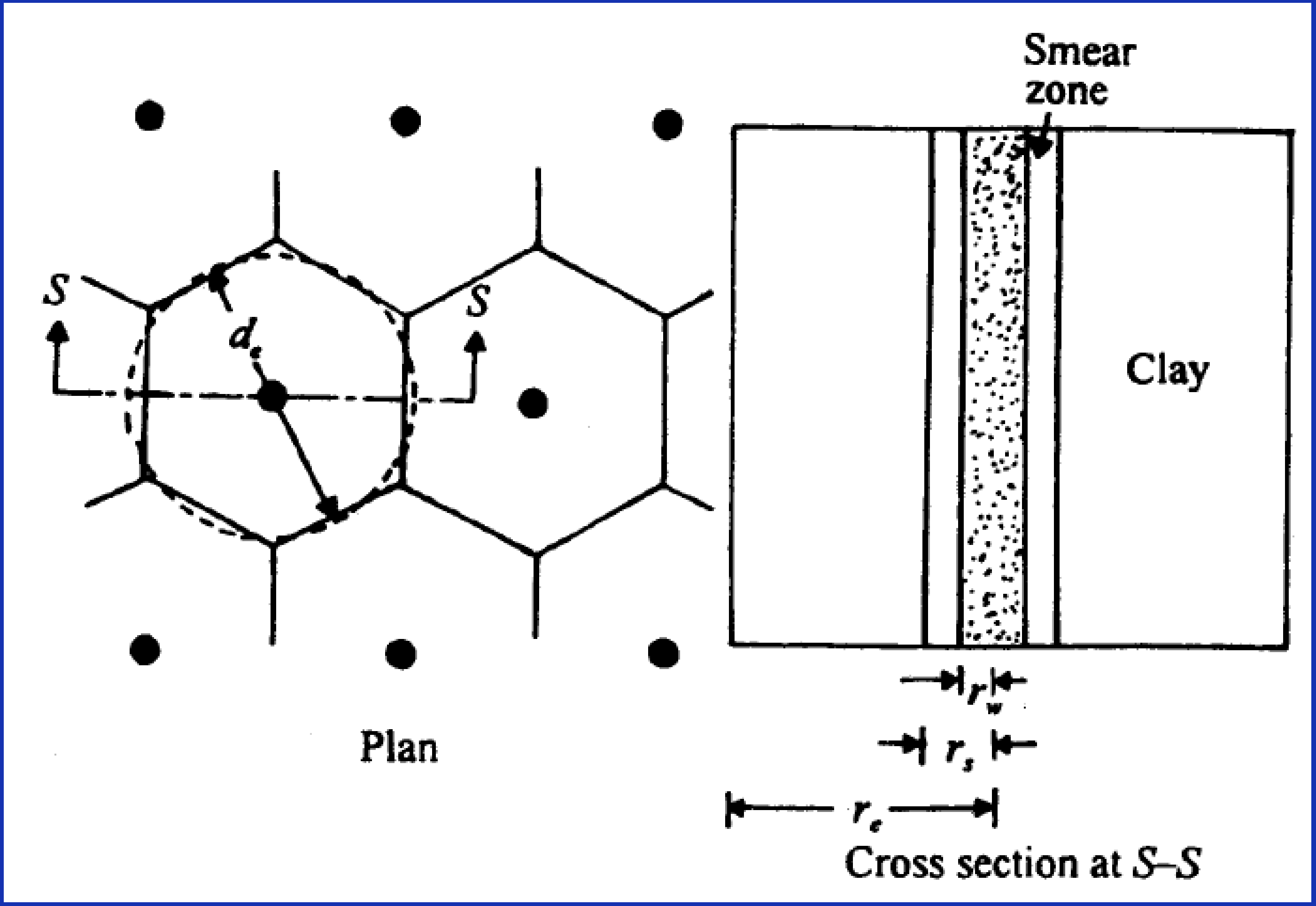
The transition zone is the zone in which there is a gradual transition of soil properties, with the degree of disturbance decreasing with increasing distance from the drain.

(After Basu and Prezzi, 2007)



Hexagonal unit cell with rectangular disturbed zone

Radial consolidation



Radial consolidation

- The theories for free-strain and equal-strain consolidation are presented herewith by assuming that drainage takes place only in the radial direction, i.e., no dissipation of excess pore water pressure in the vertical direction.

Free-strain consolidation with no smear

For triangular spacing of the sand drains, the zone of influence of each drain is hexagonal in plan. This hexagon can be approximated as an equivalent circle of diameter d_e.

r_e = radius of the equivalent circle = $d_e/2$; r_w = radius of the sand drain well; r_s = radial distance from the center-line of the drain well to the farthest point of the smear zone. For no-smear case, $r_w = r_s$.

Free-strain consolidation with no smear

The basic differential equation for radial drainage, this equation can be written as:

$$\frac{\partial u}{\partial t} = C_{vr}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial t}\right)$$

Where,

u = excess pore water pressure
r = radial distance measured from center of drain well
C_{vr} = coefficient of consolidation in radial direction

For solution, the following boundary conditions are adopted:

1. At time $t = 0$, $u = u_i$
2. At time $t > 0$, $u = 0$ at $r = r_w$
3. At $r = r_e$, $\partial u/\partial r = 0$

Free-strain consolidation with no smear

The solution for excess pore water pressure at any time *t* and radial distance *r* is given by:

$$u = \sum_{\alpha_1, \alpha_2, \ldots}^{\alpha=\infty} \frac{-2U_1(\alpha)U_0(\alpha r/r_w)}{\alpha[n^2 U_0^2(\alpha n) - U_1^2(\alpha)]} exp(-4\alpha^2 n^2 T_r)$$

$$n = \frac{r_e}{r_w}$$

$$U_1(\alpha) = J_1(\alpha)Y_0(\alpha) - Y_1(\alpha)J_0(\alpha)$$

$$U_0(\alpha n) = J_0(\alpha n)Y_0(\alpha) - Y_0(\alpha n)J_0(\alpha)$$

$$U_0\left(\frac{\alpha r}{r_w}\right) = J_0\left(\frac{\alpha r}{r_w}\right)Y_0(\alpha) - Y_0\left(\frac{\alpha r}{r_w}\right)J_0(\alpha)$$

$$T_r = \text{time factor for radial flow} = \frac{C_{vr} t}{d_e^2}$$

$$C_{vr} = \frac{k_h}{m_v \gamma_w}$$

J_0 = Bessel function of first kind of zero order
J_1 = Bessel function of first kind of first order
Y_0 = Bessel function of second kind of zero order
Y_1 = Bessel function of second kind of first order
$\alpha_1, \alpha_2, \ldots$ = roots of Bessel function that satisfy $J_1(\alpha n)Y_0(\alpha) - Y_1(\alpha n)J_0(\alpha) = 0$

Free-strain consolidation with no smear

The average pore water pressure u_{av} throughout the soil mass may now be obtained as:

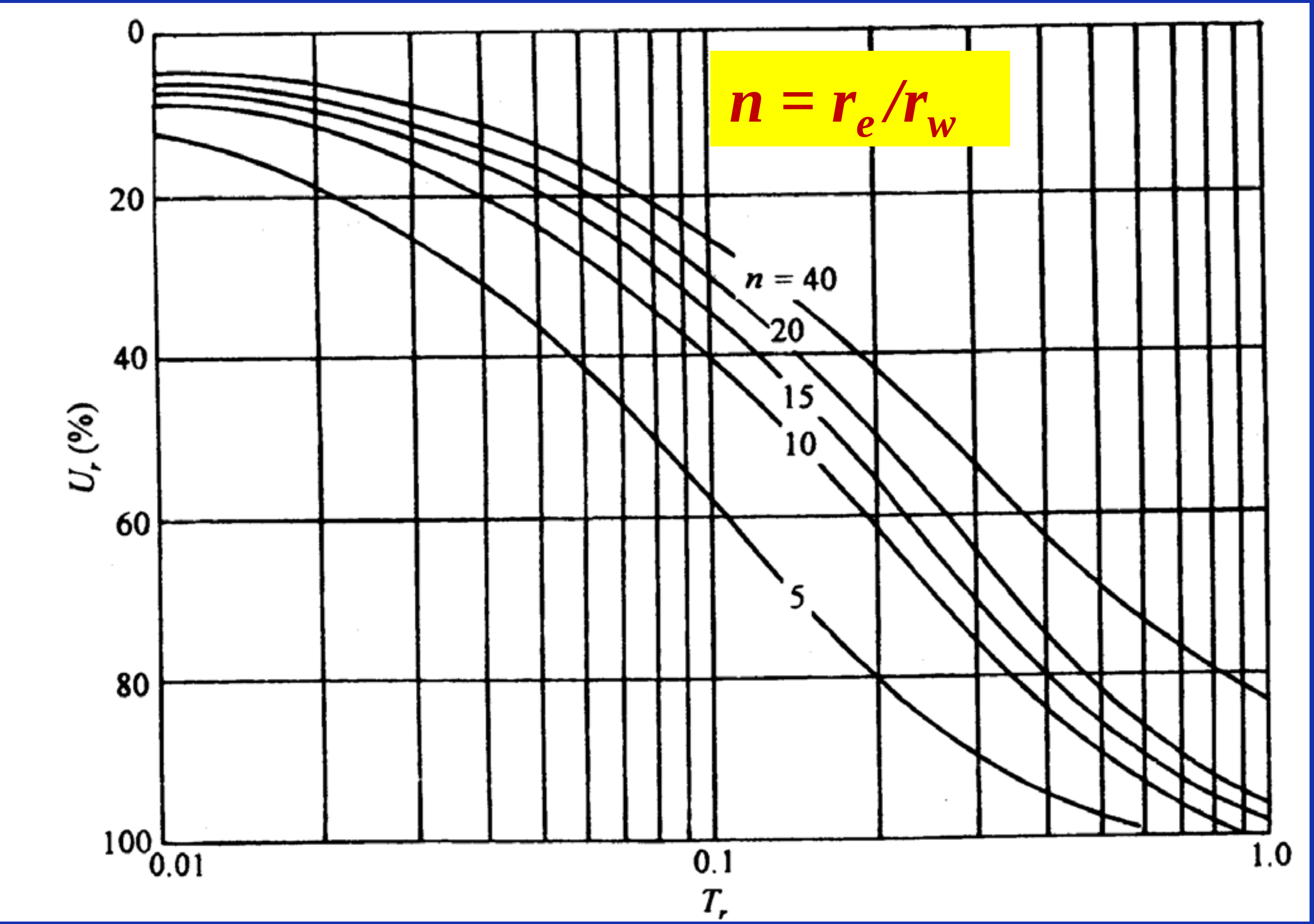
$$u_{\mathrm{av}} = u_i \sum_{\alpha_1,\alpha_2,\ldots}^{\alpha=\infty} \frac{4U_1^2(\alpha)}{\alpha^2(n^2-1)[n^2U_0^2(\alpha n) - U_1^2(\alpha)]} \times \exp(-4\alpha^2 n^2 T_{\mathrm{r}})$$

- **The average degree of consolidation U_{r} can be determined as**

$$U_{\mathrm{r}} = 1 - \frac{u_{\mathrm{av}}}{u_i}$$

Free-strain consolidation with no smear

Variation of U_r with the time factor T_r



Equal-strain consolidation with no smear

The excess pore water pressure at any time *t* and radial distance *r* is given by:

$$u = \frac{4u_{av}}{d_e^2 F(n)}\left[r_e^2 \ln\left(\frac{r}{r_w}\right) - \frac{r^2 - r_w^2}{2}\right]$$

$$F(n) = \frac{n^2}{n^2-1}\ln(n) - \frac{3n^2-1}{4n^2}$$

u_{av} = average value of pore water pressure throughout clay layer

$$= u_i e^{\lambda} \qquad \lambda = \frac{-8T_r}{F(n)}$$

The average degree of consolidation due to radial drainage is:

$$U_r = 1 - \exp\left[\frac{-8T_r}{F(n)}\right]$$

⇒ For $r_e / r_w > 5$ the free-strain and equal-strain solutions give approximately the same results for the average degree of consolidation.

Effect of smear zone on radial consolidation

- Barron (1948) also extended the analysis of equal-strain consolidation by sand drains to account for the smear zone.

- The analysis is based on the assumption that the **clay in the smear zone will have one boundary with zero excess pore water pressure and the other boundary with an excess pore water pressure that will be time dependent**.

Using the above assumption, we obtain:

$$u = \frac{1}{m'} u_{\mathrm{av}} \left[\ln\left(\frac{r}{r_{\mathrm{e}}}\right) - \frac{r^2 - r_{\mathrm{s}}^2}{2r_{\mathrm{e}}^2} + \frac{k_{\mathrm{h}}}{k_{\mathrm{s}}}\left(\frac{n^2 - S^2}{n^2}\right) \ln S \right]$$

Effect of smear zone on radial consolidation

Where k_s = Coefficient of permeability in the smeared zone

$$S = \frac{r_s}{r_w}$$

Note: S = 1 → No smear

$$m' = \frac{n^2}{n^2 - S^2} \ln\left(\frac{n}{S}\right) - \frac{3}{4} + \frac{S^2}{4n^2} + \frac{k_h}{k_s}\left(\frac{n^2 - S^2}{n^2}\right) \ln S$$

$$u_{av} = u_i \exp\left(\frac{-8T_r}{m'}\right)$$

The average degree of consolidation is given by:

$$U_r = 1 - \frac{u_{av}}{u_i} = 1 - \exp\left(\frac{-8T_r}{m'}\right)$$

Radial and Vertical consolidation

- In reality, the drainage for the dissipation of excess pore water pressure takes place in both directions simultaneously.

For such a case, Carrillo (1942) has proposed:

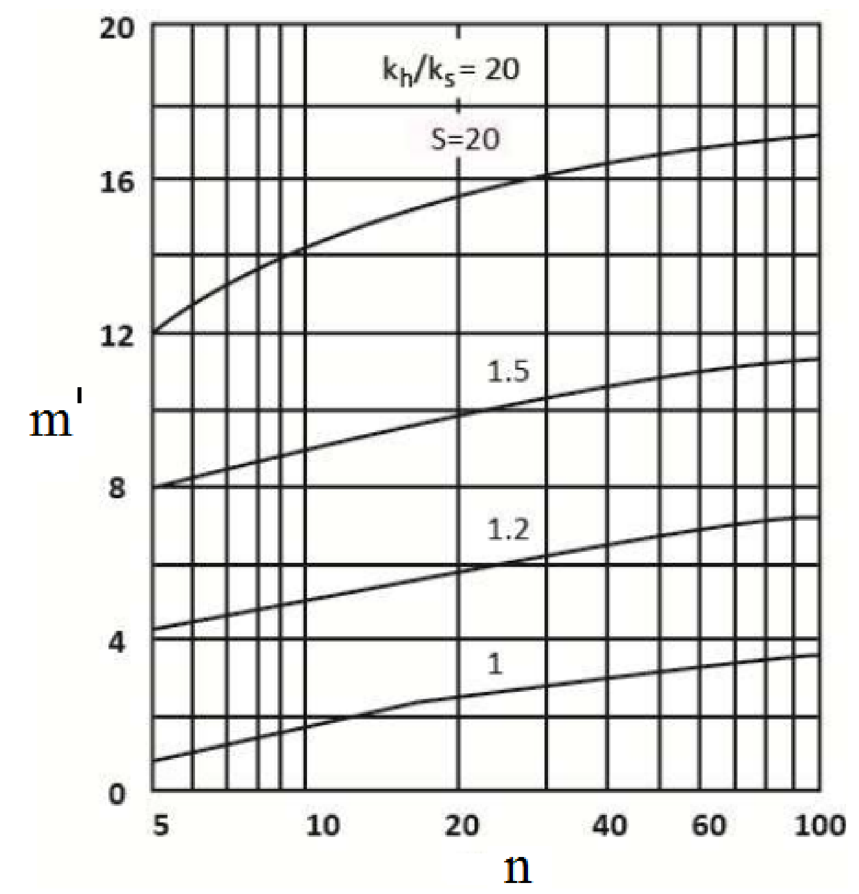
$$U = 1 - (1 - U_v)(1 - U_r)$$

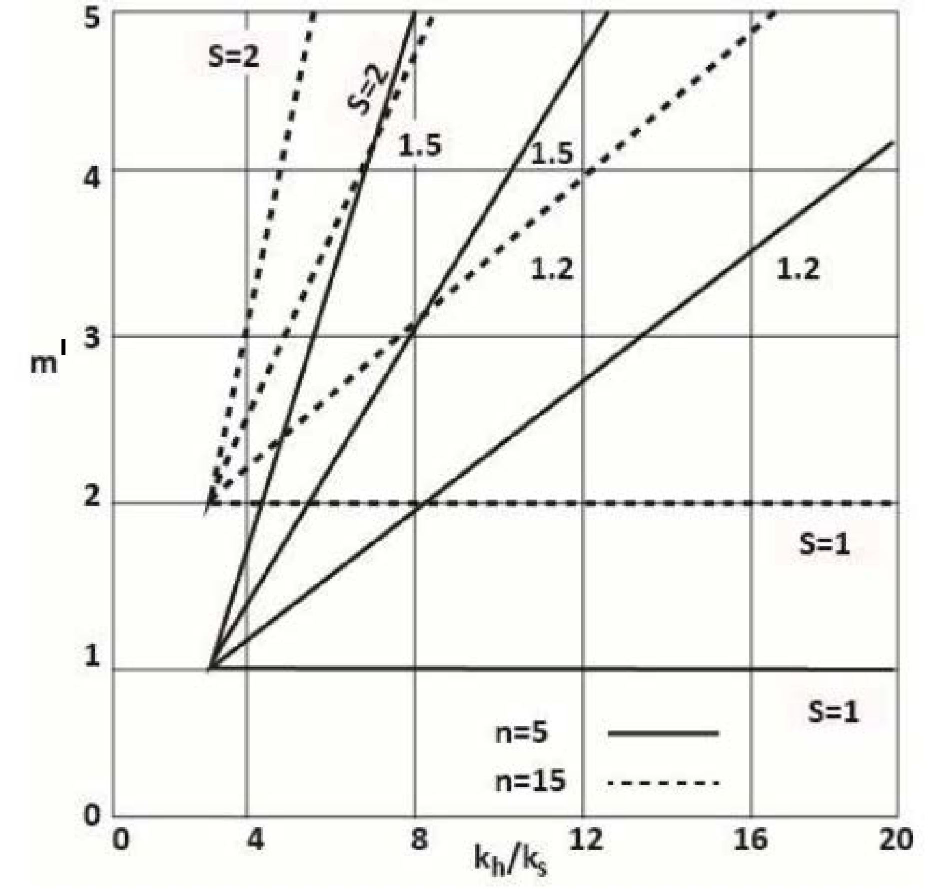
U = average degree of consolidation for simultaneous vertical and radial drainage

U_v = average degree of consolidation calculated on the assumption that only vertical drainage exists

U_r = average degree of consolidation calculated on the assumption that only radial drainage exists

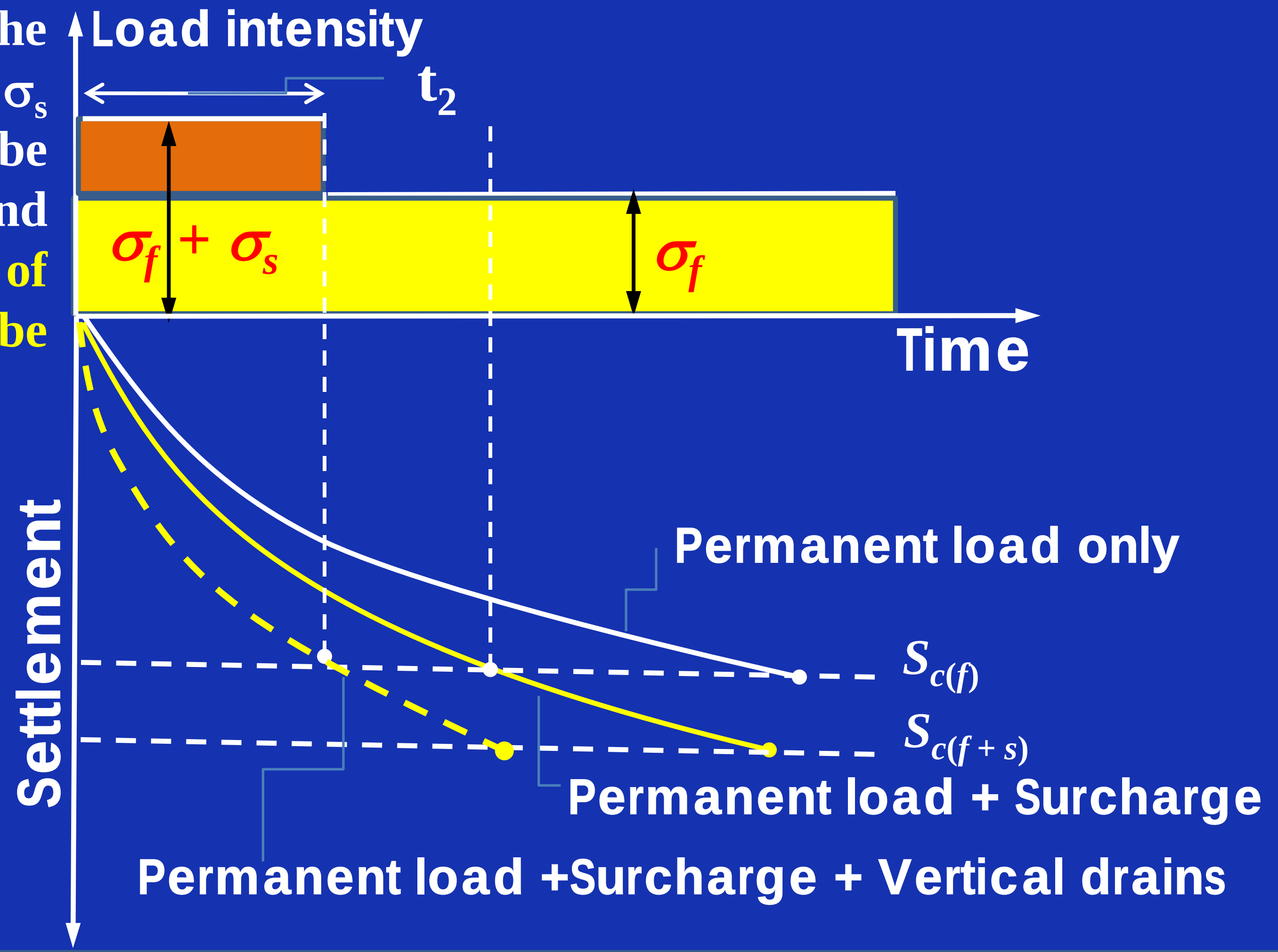
Effect of smear zone on radial consolidation





Design of preloading with sand drains (or PV drains)

- To determine the surcharge intensity σ_s that needs to be applied at the ground surface and length of time that it has to be maintained.



Design of preloading with sand drains (or PV drains)

Average degree of consolidation both in vertical and radial directions

$$U_{v,r} = \frac{\log\left[1+\frac{\sigma_p}{\sigma_0}\right]}{\log\left[1+\frac{\sigma_p}{\sigma_0}\left(1+\frac{\sigma_s}{\sigma_p}\right)\right]}$$

Note: In the case of preloading only, it is the degree of consolidation at mid-plane

We need to determine $U_{v,r}$

First U_r:

$$F(n) = \frac{n^2}{n^2-1}\ln(n) - \frac{3n^2-1}{4n^2}$$

$$T_r = \text{time factor for radial flow} = \frac{C_{vr}t}{d_e^2}$$

$$U_r = 1 - \exp\left[\frac{-8T_r}{F(n)}\right]$$

$n = d_e/(2r_w) = d_e/d_w$

$d_e = 1.13S$ (square grid)

$= 1.05S$ (triangular grid)

Design of preloading with sand drains (or PV drains)

In the case of PVDs, dw can be assumed as a f (drain geometry & configuration)

After Hansbo (1979)

$$d_w = \left[2 \frac{(a+b)}{\pi} \right]$$

a = Width of a band-shaped drain cross-section

b = Thickness of a band-shaped drain cross-section

The above equation was found to be generally valid when the portion of the perimeter area of the band-shaped drain (not obstructed by the drain core) exceeds approximately 10 – 20% of total perimeter. → For most PV drains, this condition is easily met.

Design of preloading with sand drains (or PV drains)

Now, average degree of consolidation due to vertical direction only,

For t_2, determine T_v using $T_v = t_2 c_v/(H_{dr}^2)$

And then determine U_v using $T_v = \pi/4\ (U_v/100)^2$ for U < 60%

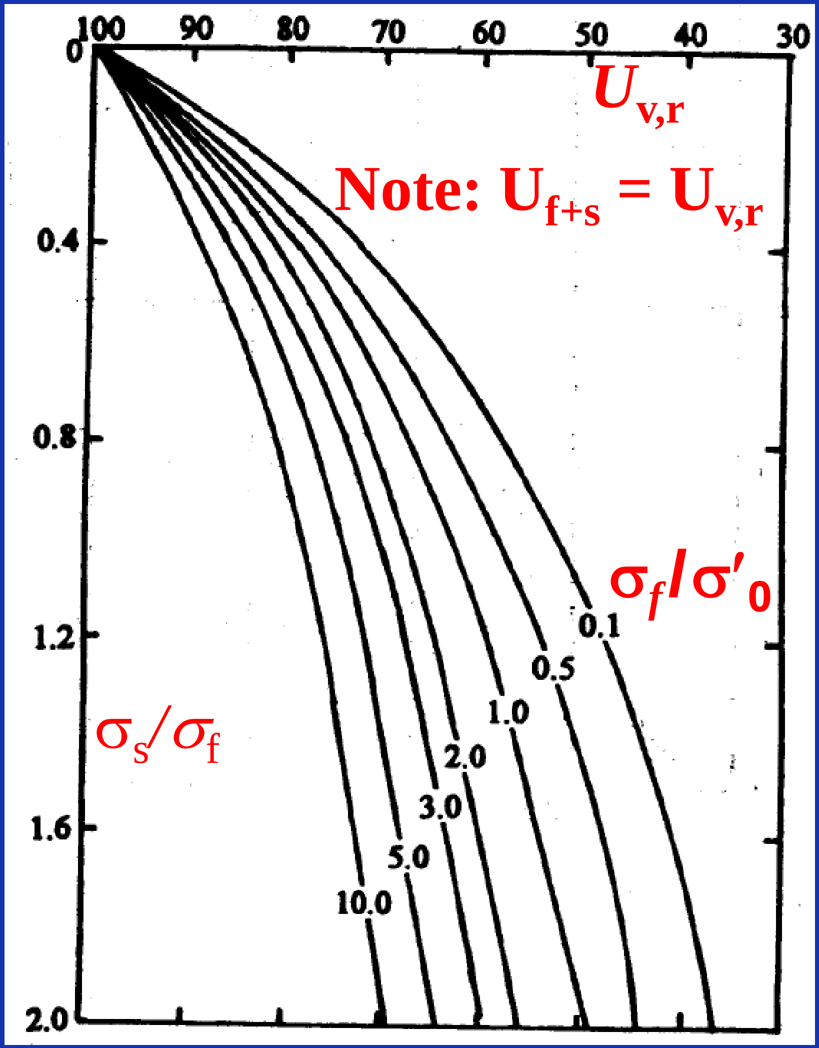
T_v = 1.781 -0.933 log(100-U%) for U > 60%

Degree of consolidation both in vertical and radial drainage is given by:

$$U = 1 - (1 - U_v)(1 - U_r)$$

Preloading with Vertical draiins

- The values of $U_{v,r}$ for several combinations of σ_f/σ'_0 and σ_s/σ_f are given by $U_{v,r}$ vs σ_s/σ_f



Example problem

This is in continuation of design problem solved with preloading only (time = 9 months)

Assume $r_w = 0.1$ m; $d_e = 3$ m; $c_v = c_h = c_{v,r} = 0.36$ m^2/year (No smear)

From the given data, average degree of consolidation in the vertical direction $U_v = 67\%$ for $T_v = 0.36$

$n = d_e/(2r_w) = 3/(2 \times 0.1) = 15$

Using the following, we get

$$F(n) = \frac{n^2}{n^2-1}\ln(n) - \frac{3n^2-1}{4n^2}$$

$$T_r = \text{time factor for radial flow} = \frac{C_{vr}t}{d_e^2}$$

$$U_r = 1 - \exp\left[\frac{-8T_r}{F(n)}\right]$$

$U_r = 77\%$

Example problem

Degree of consolidation both in vertical and radial drainage is given by:

$$U = 1 - (1 - U_v)(1 - U_r)$$

U = 92.4%

With $(\Delta\sigma)_p / \sigma_0$ = 115/210 = 0.548 and U = 92.4%

$(\Delta\sigma)_f / (\Delta\sigma)_p$ = 0.12

Hence $(\Delta\sigma)_f$ = 0.125 x 115 = 14 kPa (very nominal preloading is required with sand drains; In contrast, only preloading requires a surcharge of 207 kPa.

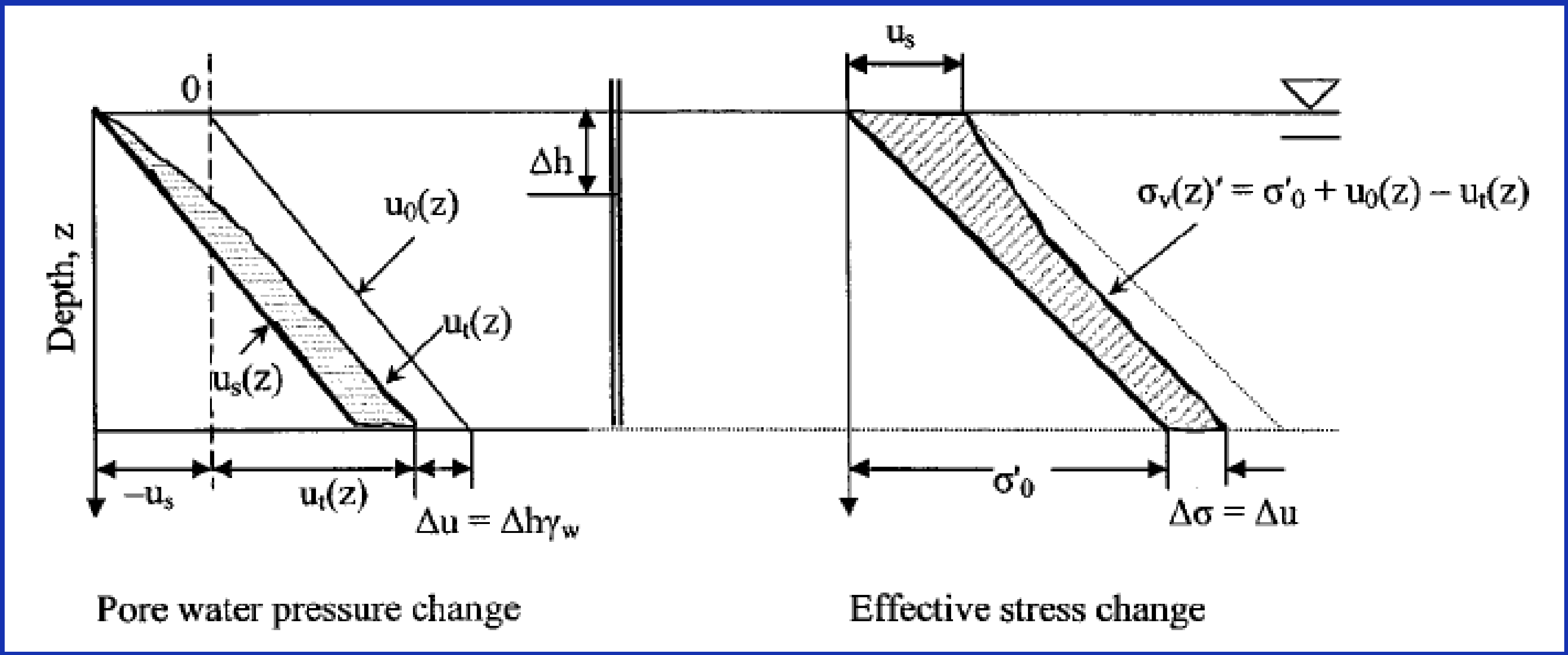
Example problem

An oil tank is to be sited on a soft alluvial deposit of clay. Below the soft clay is a thick layer of stiff clay. It was decided that a circular embankment with sand drains inserted into the clay would be constructed to pre-consolidate the soil. The height of the embankment is 6 m and the saturated unit weight of the soil comprising embankment is 18 kN/m^3.

The following data are available: Thickness of clay = 7 m; m_v = 0.2 m^2/MN; c_v = 3.5 m^2/year; c_h = 6.2 m^2/year; d_w = diameter of sand drain = 0.3 m; The desired degree of consolidation is 90% in 6 months.

Determine the spacing of a square grid of the sand drains such that when the tank is constructed the maximum primary consolidation settlement should not exceed 20 mm.

Vacuum surcharging (equivalent to 4.5 m fill – yields Carbon credits



$u_0(z)$ = hydrostatic pore water pressure profile

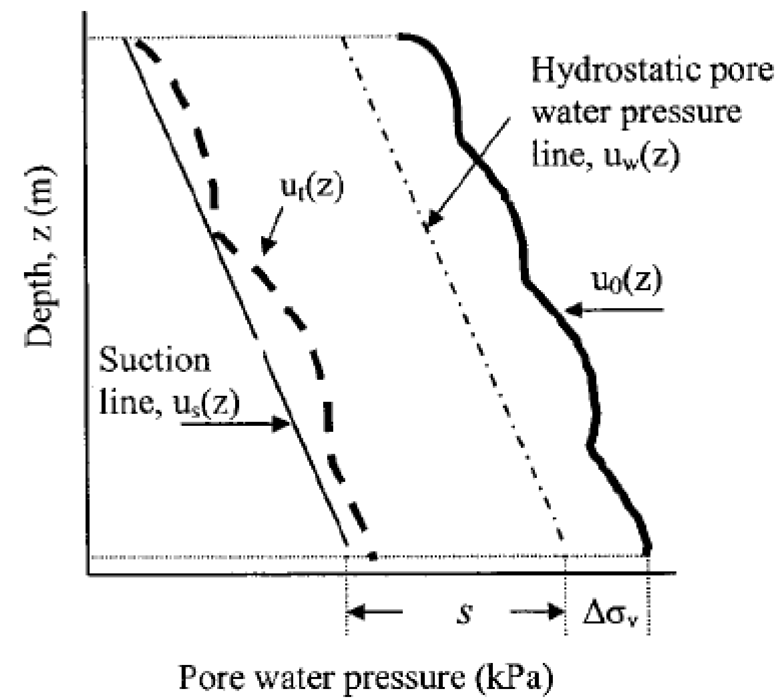
$u_t(z)$ = excess pore water pressure at time t

σ_0' = initial effective overburden stress

$\sigma_v(z)'$ = effective stress at time t

$u_s(z)$ = suction line

Vacuum consolidation



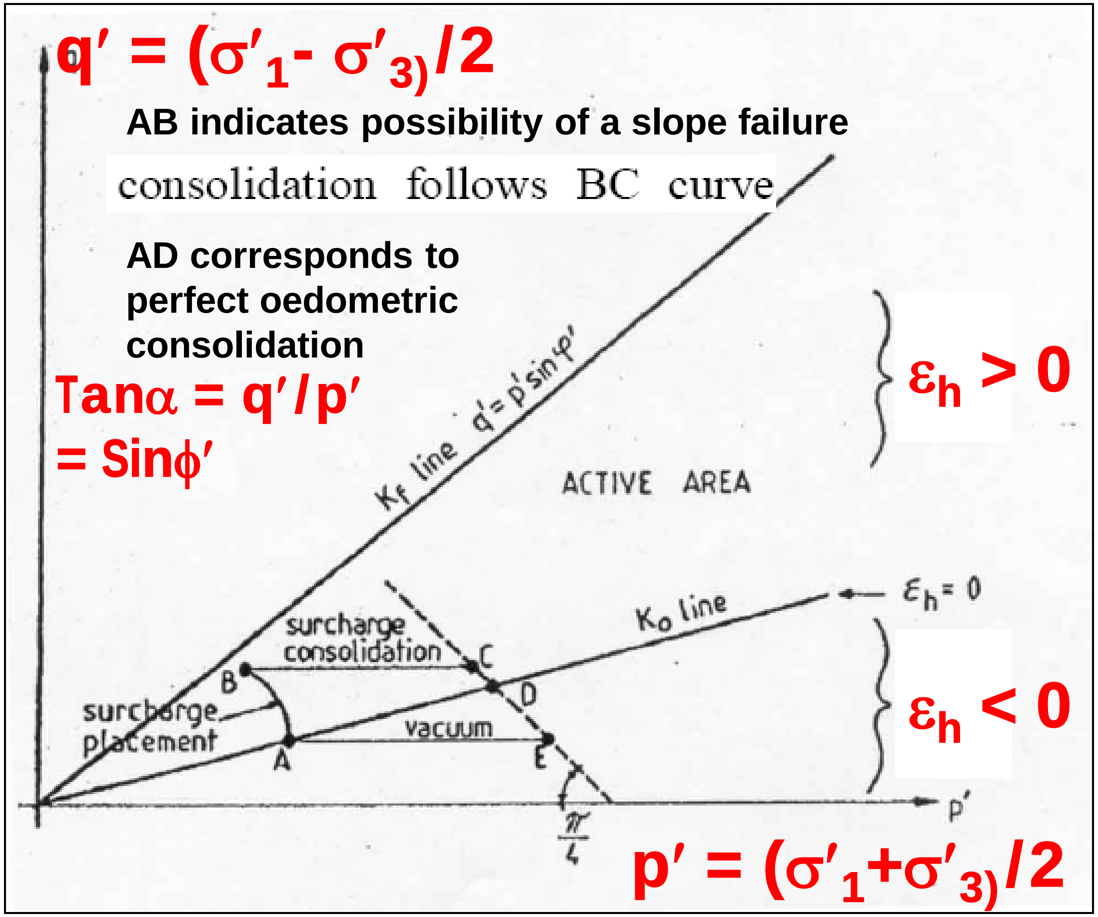
$$U_{avg} = 1 - \frac{\int [u_t(z) - u_s(z)]dz}{\int [u_0(z) - u_s(z)]dz}$$

where

$$u_s(z) = \gamma_w z - s \text{ (kPa)}$$

Schematic illustration of PWP distributions with depth under both combined surcharge and vacuum load

Comparison of classical preloading/vacuum in (p′, q′) diagram



- In the case of vacuum, the stress path simply follows the AE line (lies below the k_0 line)
- As a result of isotropic ($\Delta\sigma_v' = \Delta\sigma_h'$) increase of effective stress, there is no risk of slope failure with vacuum preloading

Pore water pressure dissipations at different durations (After Chu and Yan, 2005)

- A major difference between preloading and vacuum preloading is in the PWP change. Under fill surcharge, the excess PWP will first build-up from its initial state by the same amount as the surcharge and then dissipate gradually.

- Under vacuum pressure, the PWP in the soil will reduce from its initially hydrostatic state by the same amount as the applied vacuum pressure.