Introduction to TFA and Wavelet Transforms

Assignment 5: Lectures 6.1 to 6.8

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February 24, 2015

1. Select the correct statement with regard to Wigner–Ville distributions:
   (a) The WVD may be thought of as the local auto–covariance function of the signal's Fourier transform.
   (b) The WVD may be thought of as the Fourier transform of a local auto–covariance function.
   (c) Both **(a)** and **(b)**.
   (d) Neither **(a)** nor **(b)**.

2. $W(\tau, \xi)$ is the WVD of a continuous time signal $x(t)$. Then:
   (a) Only if $x(t)$ is real valued, then $W(\tau, \xi) = W^{\star}(\tau, \xi)$.
   (b) If $x(t)$ is purely imaginary, then $W(\tau, \xi) = jW^{\star}(\tau, \xi)$.
   (c) $W(\tau, \xi) = jW^{\star}(\tau, \xi)$ for both real and complex valued signals.
   (d) $W(\tau, \xi) = W^{\star}(\tau, \xi)$ for both real and complex valued signals.

3. Select the correct statement(s) with respect to the WVD from the following:
   (a) $\int W(t, \omega)\, d\omega = 2\pi |x(t)|^2$
   (b) $\int W(t, \omega)\, d\omega = |x(t)|^2$
   (c) $\int W(t, \omega)\, dt = 2\pi |X(\omega)|^2$
   (d) $\int W(t, \omega)\, dt = |X(\omega)|^2$

4. A signal $x(t)$ has the Wigner–Ville distribution $W(t, \omega)$. $g_1(t)$ and $g_2(\omega)$ are two different functions. Then, which of the following statements is **incorrect** with regard to averages ?
   (a) $\langle g_1(t) \rangle = \int g_1(t)\, W(t, \omega)\, dt d\omega$
   (b) $\langle g_2(\omega) \rangle = \int g_2(\omega)\, W(t, \omega)\, dt d\omega$
   (c) $\langle g_1(t) + g_2(\omega) \rangle = \int (g_1(t) + g_2(\omega))\, W(t, \omega)\, dt d\omega$
   (d) None of the above.

5. $W_1(\tau, \xi)$ & $W_2(\tau, \xi)$ are the WVDs of $x_1(t) = \sin(\omega_0 t) + \sin(\omega_2 t)$ & $x_2(t) = e^{j\omega_0 t} + e^{j\omega_1 t}$ respectively. Then:
   (a) $W_1(\tau, \xi)$ has interference terms, $W_2(\tau, \xi)$ is free of interference terms.
   (b) $W_2(\tau, \xi)$ has interference terms, $W_1(\tau, \xi)$ is free of interference terms.
   (c) Both $W_1(\tau, \xi)$ and $W_2(\tau, \xi)$ contain interference terms.
   (d) None of the above.

6. Select the correct statement with respect to discrete–WVDs
   (a) For analytic signals, they are always free of interference terms.
   (b) Interpolation of the signal (padding zeros in the Fourier domain) adds new information.
   (c) For analytic signals, they are free from spectral aliasing.
   (d) All of the above.
7. Select the correct **statement(s)** from the following with respect to Pseudo–WVDs:
   (a) Introducing a window function in the WVD enforces positivity.
   (b) Introducing a window function addresses the issue of the non–local nature of the WVD.
   (c) Introducing a window function reduces smearing in the TF plane.
   (d) Introducing a window function increases smearing in the TF plane.
8. Select the correct **statement(s)** from the following :
   (a) Constructing a pseudo–WVD with a Gaussian window leads to the Choi–Williams distribution.
   (b) Convolving the WVD with a Gaussian kernel leads to the Choi–Williams distribution.
   (c) For Cohen's class of distributions, the weighting function is the 2–dimensional Fourier transform of the smoothing kernel.
   (d) To obtain the STFT from the WVD, the smoothing kernel used is the WVD of the window associated with the STFT itself.
9. Select the correct **statement(s)** from the following :
   (a) The auto–covariance of a signal at lag s is the ambiguity function evaluated at delay s and relative velocity 0.
   (b) The energy of the signal is equal to the value of the ambiguity function at the origin.
   (c) The magnitude of the ambiguity function is minimum at the origin.
   (d) The ambiguity function is insensitive to time shifts in the signal up to a modulation.
10. $P(t,\omega)$ is a positive valued energy distribution that satisfies the marginality property. Then:
    (a) It cannot be bi–linear.
    (b) It satisfies the strong finite support property.
    (c) If the marginal is 0 at a fixed point in time, say t_0, then $P(t_0,\omega) = 0 \ \forall\omega$
    (d) All of the above.
11. $x(t) = \left(\frac{\alpha}{\pi}\right)^{1/4} \exp\left(-0.5\alpha t^2 + j0.5t\right)$. The value of $\pi \times W_{xx}(t=0, \omega=0.5)$ is _____
12. Consider the signal $x(t) = \left(\frac{32}{\pi}\right)^{1/4} te^{-t^2+jt^2+j\omega_0 t}$. The WVD of this signal is given by:

$$W(t,\omega) = \frac{\gamma}{\pi}\left[\alpha t^2 + (\omega - \beta t - \omega_0)^2/\alpha - \frac{1}{2}\right] e^{-\alpha t^2 - (\omega-\beta t-\omega_0)^2/\alpha}$$

    The value of $\alpha + \beta + \gamma$ is _____