Introduction to TFA and Wavelet Transforms

Assignment 4: Lectures 5.1 to 5.4

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1. Which of the following **statement(s)** describe the STFT of a continuous–time signal $x(t)$?
   (a) It is the inner product of the windowed signal with the time frequency atom $g_{\tau,\xi}(t) = w(t-\tau)\, e^{j\xi t}$ where w is the window function.
   (b) It is the Fourier transform of the sliced signal $x(t)\, w(t-\tau)$.
   (c) It is the inner product of $x(t)$ with frequency modulated sinusoids.
   (d) It is the inner product of $x(t)$ with amplitude modulated sinusoids.

2. Which of the following are acceptable window functions to construct an STFT ?
   (a) $w(t,\tau) = \begin{cases} \dfrac{1}{\alpha^2}\left(\alpha - |t-\tau|\right), \ \forall \ t \in [\tau-\alpha, \tau+\alpha], \ \alpha > 0 \\ 0, \ \text{elsewhere} \end{cases}$
   (b) $w(t,\tau) = \begin{cases} \dfrac{3}{4} - \dfrac{3(t-\tau)^2}{16}, \ \text{when } |t-\tau| \leq 2 \\ 0, \ \text{elsewhere} \end{cases}$
   (c) Both (a) and (b).
   (d) Neither (a) nor (b).

3. Select the correct **statement(s)** regarding the STFT:
   (a) Choosing a window with narrow duration provides poor time resolution.
   (b) Choosing a window with narrow duration provides good time resolution.
   (c) Choosing a window with narrow bandwidth provides poor frequency localization.
   (d) Choosing a window with narrow bandwidth provides good frequency localization.

4. Which of these is a consequence of the spectrogram not satisfying marginality requirements ?
   (a) The values of $\langle t \rangle$ and $\langle \omega \rangle$ are different when calculated from the spectrogram and the signal.
   (b) The energy density could take on negative values.
   (c) The lower bound for the duration–bandwidth product (for the spectrogram) is increased.
   (d) Interference terms arise, leading to spurious frequencies having non–zero energy density.

5. Select the incorrect **statement(s)** from the following:

   (a) The STFT is sensitive to time shifts up to a modulation.

   (b) The STFT is sensitive to frequency shifts without modulation.

   (c) The STFT preserves both time and frequency shifts without modulation.

   (d) The STFT is insensitive to scaling of the signal, because it lacks finite support.

6. σ_t^2 & σ_ω^2 are the respective duration and bandwidth of the spectrogram and τ & ξ are the center in time and frequency of the window function respectively. Then,

   (a) $\frac{d\sigma_t^2}{d\tau} \neq 0, \ \frac{d\sigma_t^2}{d\xi} = 0$

   (b) $\frac{d\sigma_t^2}{d\tau} = 0, \ \frac{d\sigma_t^2}{d\xi} = 0$

   (c) $\frac{d\sigma_\omega^2}{d\tau} = 0, \ \frac{d\sigma_\omega^2}{d\xi} \neq 0$

   (d) $\frac{d\sigma_\omega^2}{d\tau} = 0, \ \frac{d\sigma_\omega^2}{d\xi} = 0$

7. Given the discrete STFT of a signal $X[m, l]$, then:

   (a) The signal can be recovered only if the representation is orthogonal.

   (b) The signal can be recovered only if a $X[m, l]$ is a redundant representation.

   (c) The signal can be recovered from both redundant and orthogonal representations.

   (d) The signal can be recovered from both redundant and orthogonal representations only if rectangular windows are used.

Questions **8** and **9** are based on the following signal:

$$x(t) = \begin{cases} e^{j\omega_0 t}, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

8. When the signal is subjected to a short time Fourier transform, the spectrogram

   (a) Will be zero for all values of $t < 0$ irrespective of the window function.

   (b) Will be non–zero for some times $t < 0$ only if a Gaussian window is used.

   (c) Will be non–zero for some times $t < 0$ only if a rectangular window is used.

   (d) Will be non–zero for some times $t < 0$ for all finite length windows.

9. A rectangular window of width 4 seconds, height of 0.5 units, centered at time $t = 0$ is used to get a windowed Fourier transform of $x(t)$. The value of the spectrogram at time $t = -2$ seconds is _____

10. The signal $x\left(t\right)=\exp\left(j2.5t^2\right)$ is subjected to a STFT with the following window:

$$w\left(t\right)=\left(\frac{1}{9\pi}\right)^{\frac{1}{4}}\exp\left(-\frac{t^2}{9}\right)$$

When $\tau=2$, the angular frequency at which the spectrogram reaches a maximum is ξ_0 rad/sec. The instantaneous frequency at this time is ξ_1 rad/sec. The value of $\xi_0\times\xi_1$ is _____