Introduction to Time-Frequency Analysis and Wavelet Transforms

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Assignment 3: Lectures 4.1 to 4.5

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1. Given two discrete–time, finite duration signals $x_1[k]$ and $x_2[k]$ $(x_1 \neq x_2)$ then
   a. $x_1[k]$ and $x_2[k]$ necessarily have different duration and bandwidths.
   b. $x_1[k]$ and $x_2[k]$ may be of same duration but necessarily have different bandwidths.
   c. $x_1[k]$ and $x_1[k]$ may have same duration and bandwidth values.
   d. None of the above.

2. Select the correct **statement(s)** with reference to the given signals:

$$x_1[k] = \begin{cases} \sin(0.6\pi k) & \forall k \in \{0, 1, \ldots, 59\} \\ \sin(0.3\pi k) & \forall k \in \{60, 61, \ldots, 119\} \end{cases} \quad x_2[k] = \begin{cases} \sin(0.3\pi k) & \forall k \in \{0, 1, \ldots, 59\} \\ \sin(0.6\pi k) & \forall k \in \{60, 61, \ldots, 119\} \end{cases}$$

a. They are stationary signals.
b. They are non–stationary.
c. It is possible to distinguish the energy spectra of the two signals.
d. The energy spectra of both the signals are identical.

3. Which of the following **statement(s)** is/are true ?

   a. Fourier transforms are ill–suited to analyze quadratic chirps.
   b. Fourier transforms are ill–suited to analyze linear chirps.
   c. Fourier transforms are well suited to analyze quadratic chirps without amplitude modulation.
   d. Fourier transforms are well suited to analyze linear chirps without amplitude modulation.

4. Given a signal $x(t) = \left(\frac{\alpha}{\pi}\right)^{\frac{1}{4}} e^{-\alpha t^2} e^{j\omega_0 t}$, then

   a. Its bandwidth contains non–zero contributions from amplitude & frequency modulation.
   b. Its bandwidth contains non–zero contributions only from frequency modulation.
   c. Its bandwidth contains non–zero contributions only from amplitude modulation.
   d. None of the above.

5. Select the correct **statement(s)** regarding the instantaneous frequency $\left(\dot{\phi}\right)$ of signals

  a. For a linear chirp, it is constant.

  b. For a quadratic chirp, it is time–varying.

  c. For the sum of two sinusoids, it is time–varying.

  d. For the sum of two sinusoids, it is constant.

6. A signal $x(t)$ has the following Fourier transform: $X(\omega) = \sigma\sqrt{2\pi}e^{-\frac{\sigma^2\omega^2}{2}}$.
Select the correct **statement(s)** from the following:

a. Average time spent by any frequency ω is zero.
b. The duration has non–zero contribution only from phase variation of $X(\omega)$.
c. The duration has non–zero contribution only from amplitude variation of $X(\omega)$.
d. The duration has non–zero contributions from both amplitude and phase variation of $X(\omega)$.

7. Select the correct **statement(s)** regarding the signal $x(t) = e^{-j\omega_0 t}$ where $\omega_0 > 0$:

a. The Hilbert transform of the signal i.e. $\mathcal{H}[x(t)]$, is 0.

b. The analytic representation of the signal i.e. $\mathcal{A}[x(t)]$, is 0.

c. The Hilbert transform of the signal i.e. $\mathcal{H}[x(t)]$, is $e^{-j(\omega_0 t + \frac{\pi}{2})}$.

d. The analytic representation of the signal i.e. $\mathcal{A}[x(t)]$, is $e^{-j(\omega_0 t + \frac{\pi}{2})}$.

8. For a continuous–time signal $x(t)$ which is a Gaussian amplitude modulated sine wave of frequency ω_0:
   a. The product of $\langle t \rangle$ and $\langle \omega \rangle$ is 0.
   b. The value of $\left\langle t\dot{\phi}(t) \right\rangle$ is 0.
   c. The value of $\left\langle t\dot{\phi}(t) \right\rangle$ is equal to $\langle t \rangle \langle \omega \rangle$.
   d. None of the above.

9. Select the correct **statement(s)** from the following
   a. It is not possible to precisely determine the $t - f$ energy density of a given waveform due to Gabor's uncertainty principle.
   b. The energy densities both in time and frequency of a signal cannot be arbitrarily narrow.
   c. The uncertainty principle applies only to pairs of variables whose operators commute.
   d. The uncertainty principle applies only to pairs of variables whose operators do not commute.

Note: Two operators $\hat{A}$ and $\hat{B}$ are said to commute if $\hat{A}\left(\hat{B}\left(f\left(x\right)\right)\right) = \hat{B}\left(\hat{A}\left(f\left(x\right)\right)\right)$

10. Select the correct **statement(s)** from the following

a. For the signal $x(t) = \left(\frac{\alpha}{\pi}\right)^{\frac{1}{4}} e^{-\frac{\alpha t^2}{2}}$, the strict inequality holds for the weak version of the uncertainty principle.

b. For the signal $x(t) = \left(\frac{\alpha}{\pi}\right)^{\frac{1}{4}} e^{-\frac{\alpha t^2}{2}}$, equality holds for the weak version of the uncertainty principle.

c. For the signal $x(t) = \left(\frac{\alpha}{\pi}\right)^{\frac{1}{4}} e^{-\frac{\alpha t^2}{2}} e^{\frac{\beta t^2}{2}+\omega_0 t}$, the strict inequality holds for the strong version of the uncertainty principle.

d. For the signal $x(t) = \left(\frac{\alpha}{\pi}\right)^{\frac{1}{4}} e^{-\frac{\alpha t^2}{2}} e^{\frac{\beta t^2}{2}+\omega_0 t}$, equality holds for the strong version of the uncertainty principle.

Questions **11** to **15** are based on the following signal:

$$x(t) = \sqrt{\frac{9}{8}}t^2 e^{-\frac{3t}{2}+j2t}, t \geq 0$$

The Fourier transform of this signal is given by:

$$X(\omega) = \frac{9}{2\sqrt{\pi}\left[-1.5 + j(\omega - 2)\right]^3}$$

11. Select the correct option from the following
    a. The signal is an energy signal.
    b. The signal is a power signal.
    c. The signal is neither an energy nor a power signal.
    d. The signal is a Gaussian amplitude modulated signal.

12. The instantaneous frequency of the signal is _____ rad/sec.

13. The value of $3\langle t\rangle - 9\sigma_t^2$ is _____

14. The expected value of ω^2 is _____

15. The value of $\dfrac{5}{\sigma_t^2\sigma_\omega^2}$ is equal to _____