Introduction to TFA and Wavelet Transforms

Practice Questions for DWT

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1. Which of the following statements describes a Discrete Wavelet Transform of a signal ?

   (a) It is the CWT sampled uniformly at finite grid points in the $t - s$ plane.

   (b) It is the CWT evaluated on a uniformly spaced grid of scales and translations.

   (c) It is the CWT evaluated on a specific non–uniform grid of scales and translations.

   (d) None of the above.

2. Select the correct **statement(s)** about the frame bounds A and B

   (a) The frame is linearly independent if $A = 0.5$ and $B = 1.5$.

   (b) $A = 0.5$ and $B = 0.5$ corresponds to an orthonormal basis but not a tight frame.

   (c) $A = 1$ and $B = 1$ corresponds to an orthonormal basis but not a tight frame.

   (d) $A = 1$ and $B = 1$ corresponds to both an orthonormal basis and a tight frame.

3. Select the **incorrect statement(s)** from the following with respect to signal reconstruction from the DWT

   (a) Non-orthonormal bases result in imperfect or approximate reconstruction.

   (b) If the wavelets do not constitute a tight frame, reconstruction is imperfect.

   (c) Non-orthonormal bases need not result in imperfect or approximate reconstruction.

   (d) None of the above.

4. The orthogonal projection of a signal $x(t)$ onto a basis space:

   (a) is the optimal approximation of $x(t)$ in a weighted $1-$norm sense.

   (b) is the optimal approximation of $x(t)$ in an un–weighted $2-$norm sense.

   (c) is sub–optimal with respect to the L_2 norm of the error.

   (d) None of the above.

5. $\mathbf{x} = \begin{bmatrix} 1.5 & 3 & 6 \end{bmatrix}^T$. M is the subspace spanned by the vectors $\phi_{\mathbf{1}} = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}^T$ and $\phi_{\mathbf{2}} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$. Then:

   (a) $\mathbf{x}$ can be perfectly recovered using these bases.

   (b) $\mathbf{x}$ cannot be perfectly recovered using these bases.

   (c) The optimal approximation of $\mathbf{x}$ in M is $1.5\phi_{\mathbf{1}} + 1.5\phi_{\mathbf{2}}$.

(d) The squared $2-$norm error for the orthogonal projection of $\mathbf{x}$ onto M is 3 units.

6. Select the correct statement with respect to MRA:

   (a) Any function in V_{n-1} can be expressed using the basis functions of V_n .

   (b) Any function in V_n can be expressed using the basis functions of V_{n-1}.

   (c) The coefficients $h[n]$ serve as a band pass filter for the scaling function .

   (d) $\sum (h[n])^2 = 2$ for a conjugate mirror filter.

7. Select the correct **statement(s)** in regard to thresholding during signal estimation:

   (a) In Garrote thresholding, large coefficients are shrunk linearly.

   (b) Universal threshold is useful for signals corrupted with pink noise.

   (c) Level-based thresholding is appropriate for signals corrupted by coloured noise.

   (d) Hard thresholding sets all DWT coefficients below a certain threshold to zero, while retaining the remaining coefficients as is.

8. Select the correct **statement(s)** with respect to Wavelet Packet Decomposition or WPD:

   (a) n levels of decomposition produces 2^n different sets of coefficients.

   (b) n levels of decomposition produces 2^{n+1} different sets of coefficients.

   (c) In WPD, detail and approximation coefficients are decomposed to create a full binary tree.

   (d) In WPD, only the approximation coefficients are decomposed to create a full binary tree.

9. Select the correct **statement(s)** from the following:

   (a) The synthesis and analyzing filters are identical in a biorthogonal wavelet transform.

   (b) The analysing scaling and wavelet functions in a biorthogonal wavelet transform are not orthogonal to each other..

   (c) The analysis scaling and the synthesis wavelet functions in a biorthogonal wavelet transform are orthogonal to each other.

   (d) All of the above.

10. Which of the following statements are correct ?

    (a) A `db6` wavelet is narrower and smoother than a `db4` wavelet.

    (b) A `db6` wavelet is wider and sharper than a `db4` wavelet.

    (c) A `db6` wavelet is wider and smoother than a `db4` wavelet.

    (d) A `db6` wavelet is narrower and sharper than a `db4` wavelet.

11. Which of the following statements are correct with respect to the Haar wavelet ?

    (a) $\phi(t) = \phi(2t) + \phi(2t-1)$.

    (b) $\psi(t) = \phi(2t) + \phi(2t-1)$.

(c) $\phi(t) = \phi(2t) - \phi(2t-1)$.

(d) $\psi(t) = \phi(2t) - \phi(2t-1)$.

12. For a signal of length of length 256, the coarsest approximation coefficients are obtained at the level $J =$ _____

13. The vectors $\phi_{\mathbf{1}} = \begin{bmatrix} 1 & 2 & 4 \end{bmatrix}^T$ and $\phi_{\mathbf{2}} = \begin{bmatrix} 4 & 2 & 1 \end{bmatrix}^T$ form the basis for a subspace M. The signal $\mathbf{x} = \begin{bmatrix} 1.4 & 1 & 1.1 \end{bmatrix}^T$ is projected onto M. The squared $2-$norm of the error in approximation is _____

14. $D_1 = \begin{bmatrix} -1.0251 & 0.6331 & 0.1569 & 0.8756 \end{bmatrix}^T$ and $D_2 = \begin{bmatrix} 0.9621 & -1.0024 & 0.0557 & 0.6587 \end{bmatrix}^T$ can correspond to the details reconstructed using Coiflets. True or False ?

15. A continuous–time signal $x(t) = 2t^2 - \exp(t), 0 \leq t < 1$ is sampled at a frequency $f_s = 20$Hz starting at $t = 0$. The approximation of the signal at the coarsest possible scale using a Haar wavelet is _____

Answers

1. (c)
2. (a),(d)
3. (a),(b)
4. (b)
5. (a),(c)
6. (b)
7. (c),(d)
8. (a),(c)
9. (b),(c)
10. (c)
11. (a),(d)
12. The coarsest level is at $J_{max} = \log_2 256 = 8$
13. $\mathbf{x} = 0.2\phi_1 + 0.3\phi_2$. So the error in reconstruction is 0.
14. False. D_1 and D_2 should have a zero inner product since coiflets are orthonormal.
15. The answer can be obtained by getting the discrete–time analogue of the signal and taking its average over the first 20 values.