Consider linear perturbation model

\[
\frac{dx}{dt} = \begin{bmatrix}
-\frac{1}{62} & 0 & 1 & 0 \\
0 & -\frac{1}{90} & 0 & \frac{1}{23} \\
0 & 0 & -\frac{1}{23} & 0 \\
0 & 0 & 0 & -\frac{1}{30}
\end{bmatrix} x(t) + \begin{bmatrix}
7 \\
\frac{84}{1} \\
0 \\
\frac{1}{32}
\end{bmatrix} u(t)
\]

\[
y(t) = \begin{bmatrix}
0.5 & 0 & 0 & 0 \\
0 & 0.5 & 0 & 0
\end{bmatrix} x(t)
\]

- **Step 1:** Convert the continuous time model into a discrete time model of the form

\[
x(k + 1) = \Phi x(k) + \Gamma u(k)
\]

\[
y(k) = C x(k)
\]

where \( \Phi = e^{AT} \) and \( \Gamma = [\Phi - I]A^{-1}B \) with sampling interval of \( T = 5 \) units.

**Note:** MATLAB command to compute \( e^M \), where \( M \) represents a matrix, is \( \text{expm}(M) \)

- **Step 2:** Generation of data for system identification

Simulate the plant in open loop for \( k = 1, 2, \ldots, 500 \) samples using the following set of equations

\[
x(k + 1) = \Phi x(k) + \Gamma [u(k) + w(k)]
\]

\[
y(k) = C x(k) + v(k)
\]

Here, \( w(k) \) and \( v(k) \) are zero mean Gaussian white noise sequences with

\[
\text{Cov}[w(k)] = \begin{bmatrix}
(0.06)^2 & 0 \\
0 & (0.08)^2
\end{bmatrix}
\]

\[
\text{Cov}[v(k)] = \begin{bmatrix}
(0.05)^2 & 0 \\
0 & (0.04)^2
\end{bmatrix}
\]

Generate \( u(k) \) PRBS input signal using following sequence of MATLAB commands

\[
uk = \text{zeros}(2,500) ;
\]

\[
Sk = \text{sign} \left( \text{randn}(2,50) \right) ;
\]

\[
jk = 0 ; \ uk1 = 0.5 ; \ uk2 = -0.6 ;
\]
for $k = 1 : 500$

if $(\text{rem}(k,10) == 0)$

$\text{j}\text{k} = \text{j}\text{k} + 1$ ;

$\text{uk}1 = 0.5 \ast \text{Sk}(1,\text{jk})$ ; $\text{uk}2 = 0.6 \ast \text{Sk}(2,\text{jk})$ ;

end

$\text{uk}(1,k) = \text{uk}1$ ; $\text{uk}(2,k) = \text{uk}2$ ;

end

• **Step 3:** Using data generated in Step 2 for output 1, develop a 2' th order ARX model of the form

$$y(k) = -a_1 y(k-1) - a_2 y(k-2) + b_1 u_1(k-1) + b_2 u_1(k-2)$$
$$+ \beta_1 u_2(k-1) + \beta_2 u_2(k-2) + e(k)$$

Parameters of this model can be identified using the following algorithm

$$\text{M} = \begin{bmatrix}
-y(2) & -y(1) & u_1(2) & u_1(1) & u_2(2) & u_2(1) \\
-y(3) & -y(2) & u_1(3) & u_1(2) & u_2(3) & u_2(2) \\
... & ... & ... & ... & ... & ...
\end{bmatrix}$$

$$\text{Y} = \begin{bmatrix} y(3) & y(4) & ... & y(N-1) & y(N) \end{bmatrix}^T$$

$$\theta = \begin{bmatrix} a_1 & a_2 & b_1 & b_2 & \beta_1 & \beta_2 \end{bmatrix}^T$$

$$\hat{\theta} = [\text{M}^T\text{M}]^{-1} \text{M}^T\text{Y}$$

• **Step 4:** Graphical presentation of results

Figure 1: Find predicted output as

$$\hat{\text{Y}} = \text{M}\hat{\theta}$$

and compare vectors $\hat{\text{Y}}$ and $\text{Y}$ in same figure.

Figure 2 and 3: Plot input sequences $u_1(1,:)$ and $u_2(1,:)$ using MATLAB function ‘stairs’

Figure 4: Compute model residual vector

$$\text{E} = \text{Y} - \hat{\text{Y}}$$

and plot it.