Prof. Shankar
Lec. 8 (18/10/12) 1

ENZYME KINETICS

Michaelis Menten Model

$$E + S \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} ES \; ; \quad ES \xrightarrow{k_2} P + E$$

$$r_s = k_2(es) - k_1(e)(s)$$

$$r_{es} = k_1(e)(s) - k_{-1}(es) - k_2(es)$$

$$e_0 = e + es$$

$$r_s = k_{-1}(es) - k_1(e_0 - es)s$$

In Batch Equipment

$$\frac{d}{dt}(\cancel{V} \cdot s) = r_s \cancel{V}$$

$$\cancel{V}\frac{d}{dt}(es) = r_{es}\cancel{V} \underset{QSSA}{=} 0.$$

QSSA: Quasi Steady State Appox

$r_{es} = 0$ so

$$0 = k_1(e_0 - es)s - k_{-1}(es) - k_2(es)$$

$$[es] = \frac{k_1 e_0 s}{k_1 s + k_{-1} + k_2}$$

$$r_p = \frac{k_2 k_1 e_0 s}{k_1 s + k_{-1} + k_2} = \frac{V_M s}{K_M + s}$$

$$\boxed{V_M = k_2 e_0 \quad K_M = (k_{-1} + k_2)/k_1}$$

pH EFFECTS

MODEL

$$E \rightleftharpoons E^{-} + H^{+} \qquad K_1 = \frac{E^{-} H^{+}}{E}$$

$$E^{-} \rightleftharpoons E^{-2} + H^{+}$$

$$E_0 = E^{-} + E^{-2} + E \qquad K_2 = \frac{E^{-}}{H^{+} E^{-2}}$$

$$E_0 = \frac{E^{-} H^{+}}{K_1} + \frac{K_2}{H^{+}} E^{-} + E^{-}$$

$$e_0 = e^- \left[1 + \frac{H^+}{K_1} + \frac{K_2}{H^+} \right]$$

$$y = \frac{e^-}{e_0} = \frac{1}{\left[1 + \frac{H^+}{K_1} + \frac{K_2}{H^+} \right]}$$

$$(v_m) = K_3\, e^- = k_3 \frac{e^-}{e_0} \cdot e_0 .$$

$$= K_3\, y\, e_0$$

$$v_m = k_3 e_0 \left[\frac{1}{1 + \frac{H^+}{K_1} + \frac{K_2}{H^+}} \right]$$

$$v_p = f\left[V_M, K_M, S\right]$$

Highest value of V_M at

$$H^+ = \sqrt{K_1 K_2}$$

SUBTRATE INHIBITION KINETICS

$$E + S \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} ES$$

$$ES + S \underset{k_{-2}}{\overset{k_2}{\rightleftharpoons}} SES$$

$$ES \xrightarrow{3} P + E$$

$$0 = r_{es} = k_1(e)s - k_{-1}(es) + k_{-2}(ses)$$

$$0 = r_{ses} = k_2(es)(s) - k_{-2}(ses)$$

$$(ses) = \frac{k_2(es)s}{k_{-2}}$$

$r_{es} = 0$

$$0 = k_1(e_0 - es - ses)s - k_{-1}(es) - k_3(es)$$

$$es\left\{k_1 s + k_{-1} + k_3\right\} = k_1 e_0 - \frac{k_1 k_2 (es)(s)^2}{k_{-2}}$$

$$es\left\{k_1 s + k_{-1} + k_3 + \frac{k_1 k_2 s^2}{k_{-2}}\right\} = k_1 e_0 s$$

$$r_p = k_3 e s$$

$$= \frac{k_3 \cancel{k_1} e_0 s}{\left(\cancel{k_1} s + \frac{k_{-1} + k_3}{k_1} + \frac{\cancel{k_1} k_2 s^2}{k_{-2}}\right)}$$

$$r_p = \frac{V_M s}{\left(K_2 + s + \frac{s^2}{K_1}\right)}$$

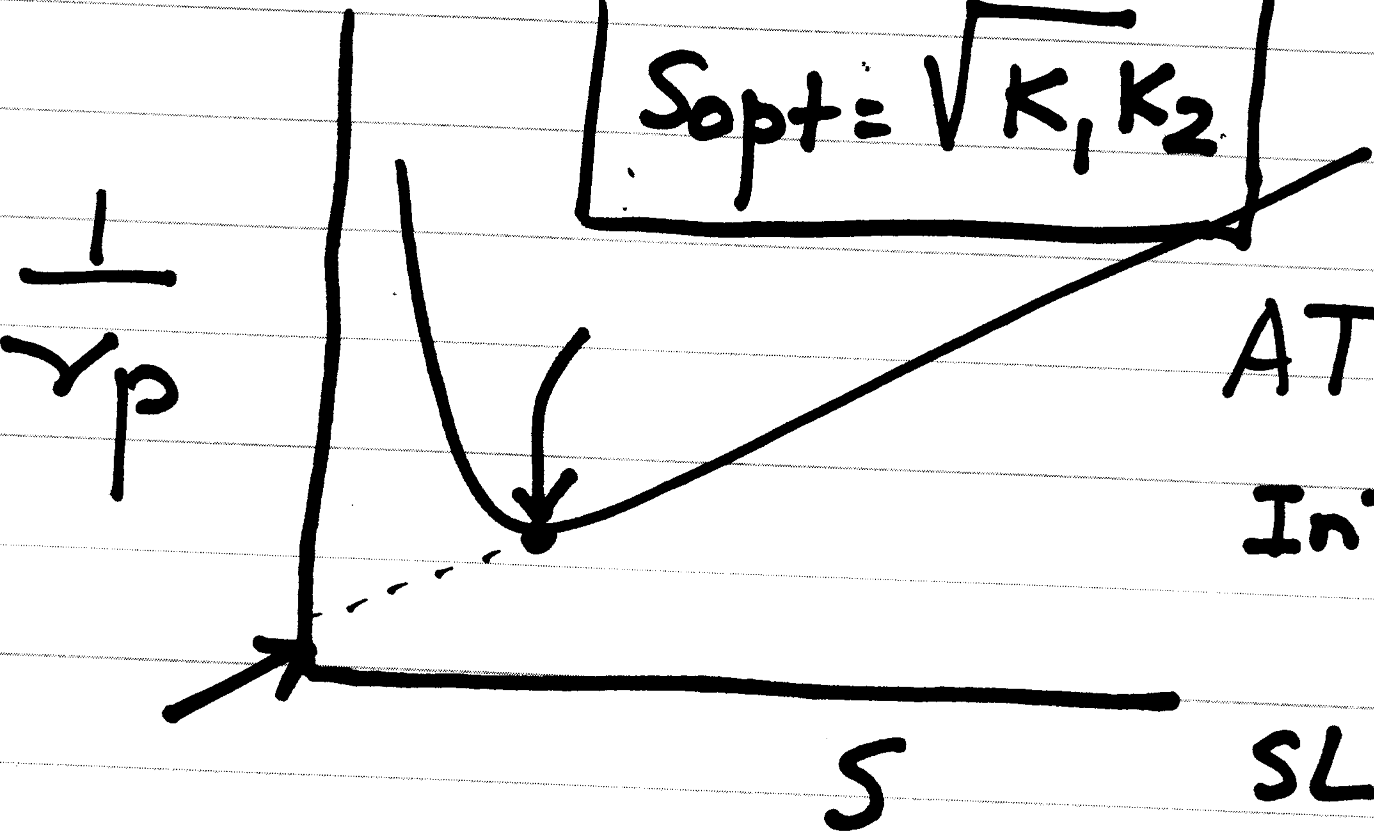
$$K_2 = (k_{-1} + k_3)/k_1$$

$$K_1 = k_{-2} | k_2$$

$$V_M = k_3 e_0$$

$$\frac{1}{v_p} = \frac{K_2}{V_M S} + \frac{1}{V_M} + \frac{S^{\cancel{2}}}{V_M \cancel{S} K_1}$$

$$\boxed{S_{opt} = \sqrt{K_1 K_2}}$$



AT LARGE S

$\text{Int} = \frac{1}{V_M}$

SLOPE $\left(\frac{1}{V_M K_1}\right)$

$$\text{SLOPE}: \frac{19-4}{30} = 0.5 = \frac{1}{V_M K_1}$$

$$\text{Intercept} = 4 = \frac{1}{V_M}$$

$$V_M = 0.25$$

$$\frac{1}{V_M K_1} = 0.5 \Rightarrow K_1 = \frac{1}{(0.5)(0.25)}$$

$$K_1 = 8 \quad \text{MMOL}/\text{L} \cdot \text{MIN}$$

$$S_{opt} = \sqrt{K_1 K_2}$$

$$4 = \sqrt{K_1 K_2}$$

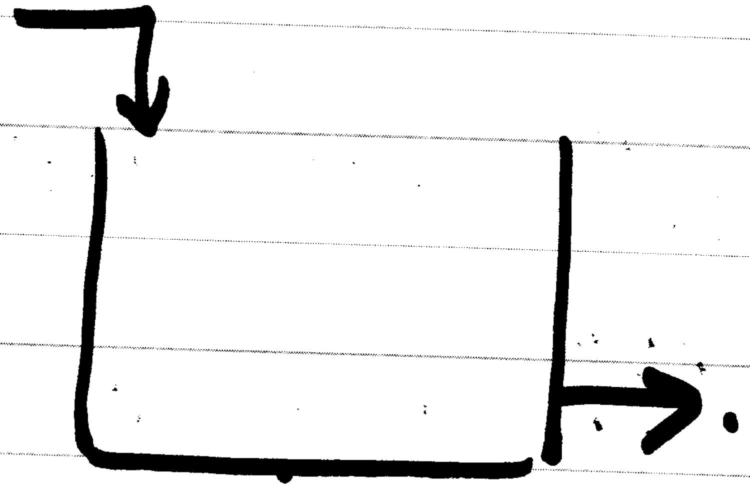
$$K_1 K_2 = 16$$

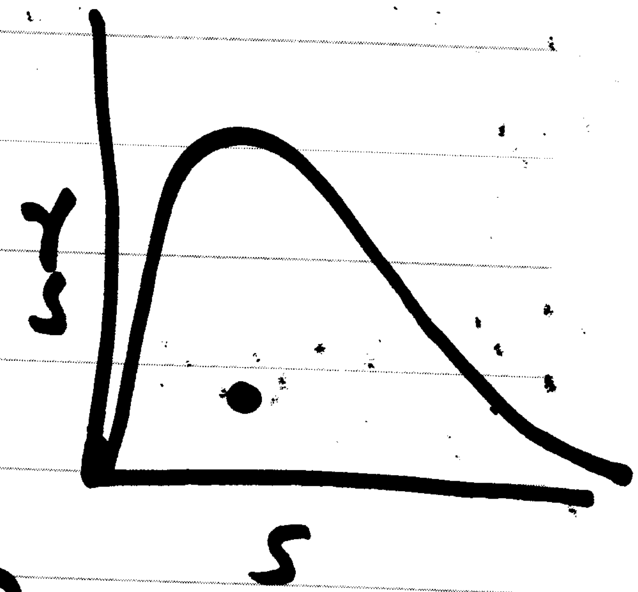
$$K_1 = 8 \; ; \; K_2 = 2 \ \frac{\text{mmol}}{\text{L}}$$

mmol/L

$$V_M = 0.25 \text{ mmol}/\text{L} \cdot \text{min}$$

12/1





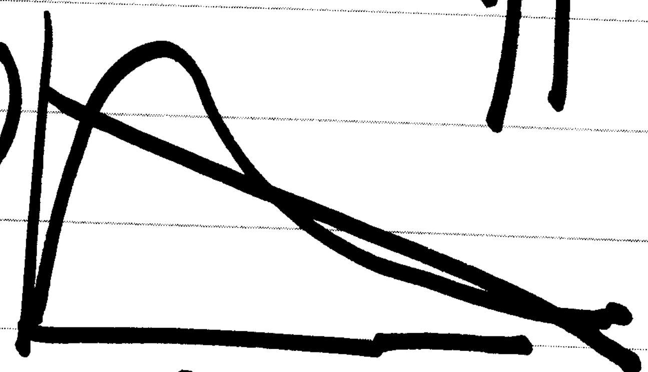
$$F S_0 - F S + r_s V = 0.$$

$$\underbrace{F(S_0 - S)}_{P(s)} = \underbrace{-\,r_s \cdot V}_{Q(s)}$$

12/2

$-r_s V$	170	140	110	55	50	37
$F(S_0-S)$	147	134	121	57	44.8	41 38
S	4	8	12	32	36	48

$P(S)$ $\Phi(S)$



S

$$K_M \ln \frac{S}{S_0} + (S - S_0) = -V_m t$$

$$\boxed{K_M \ln \frac{S_0}{S} + (S_0 - S) = V_m t}$$

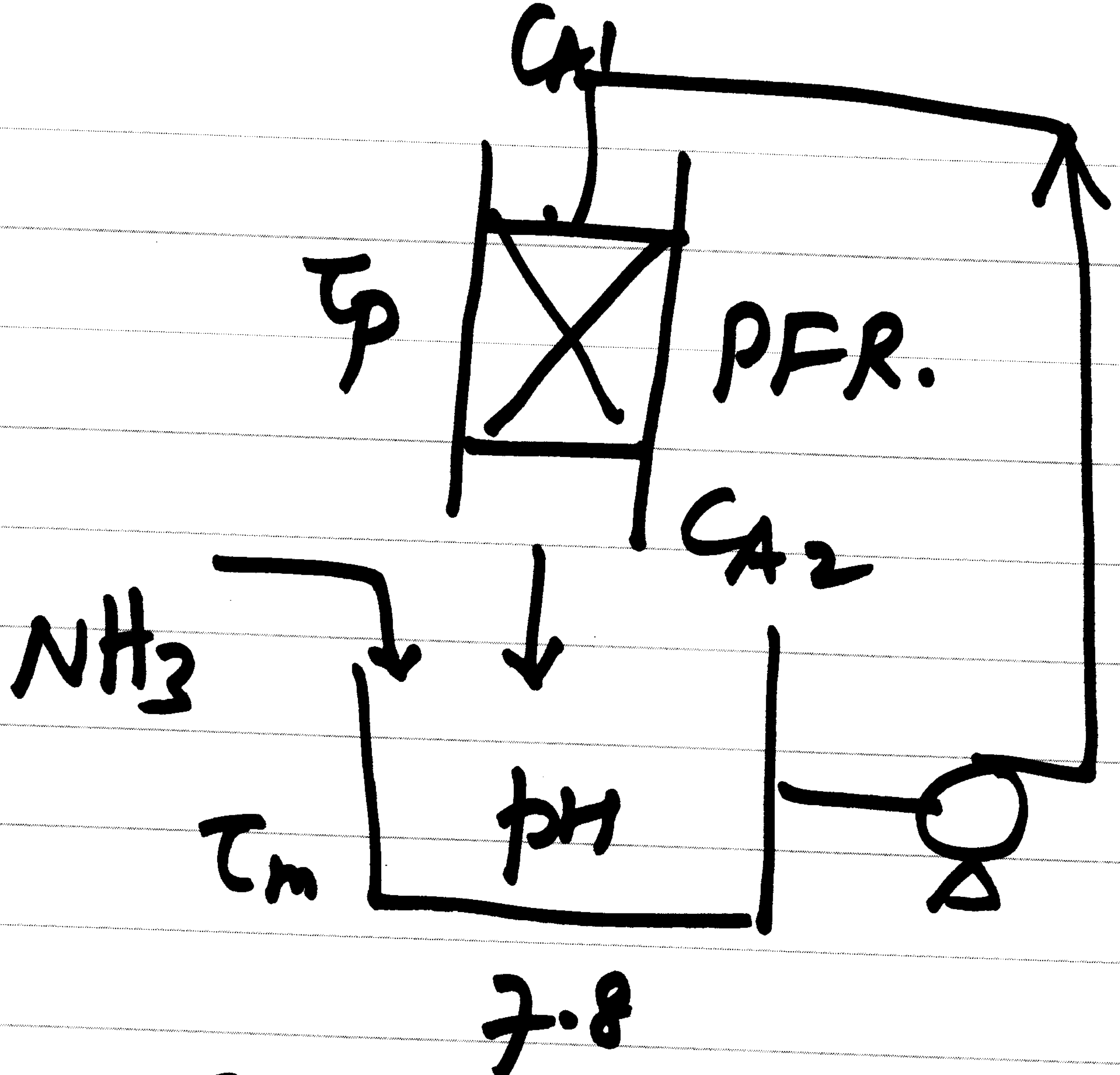
12/4

$$-\frac{ds}{dt} = r_s = \frac{V'_M S}{K_M + s}$$

$$\int \frac{(K_M + s)}{s} \frac{ds}{} = -\int_S V'_M \, dt$$

$$\Big[K_M \ln s + s\Big]_{S_0}^{S} = -V'_m t$$

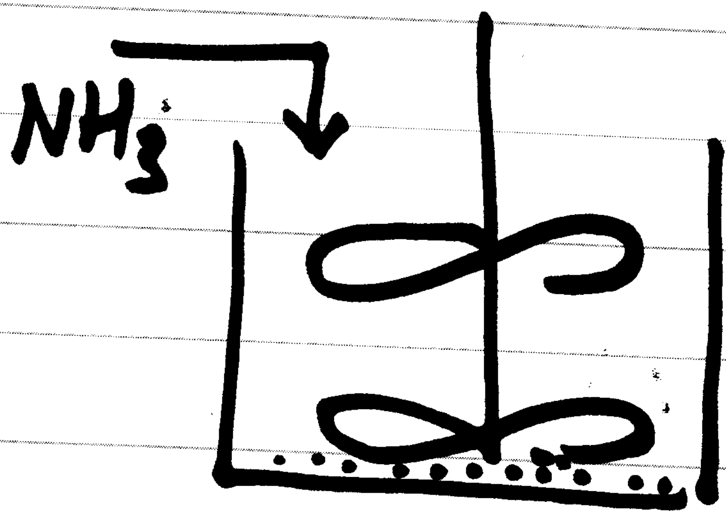
12/5



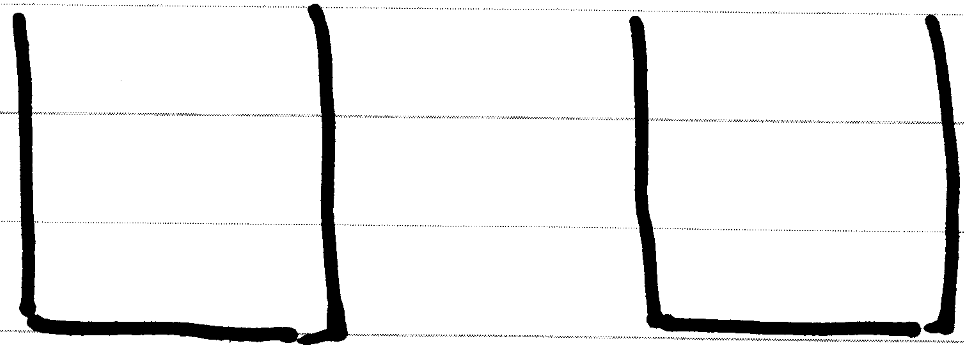
$$\boxed{C_{A2} = C_{A1} e^{-k\tau_p}}$$

$$\text{PenG} + H_2O = \underline{6APA} + \underline{\underline{PAA}}$$

12/6



12/7



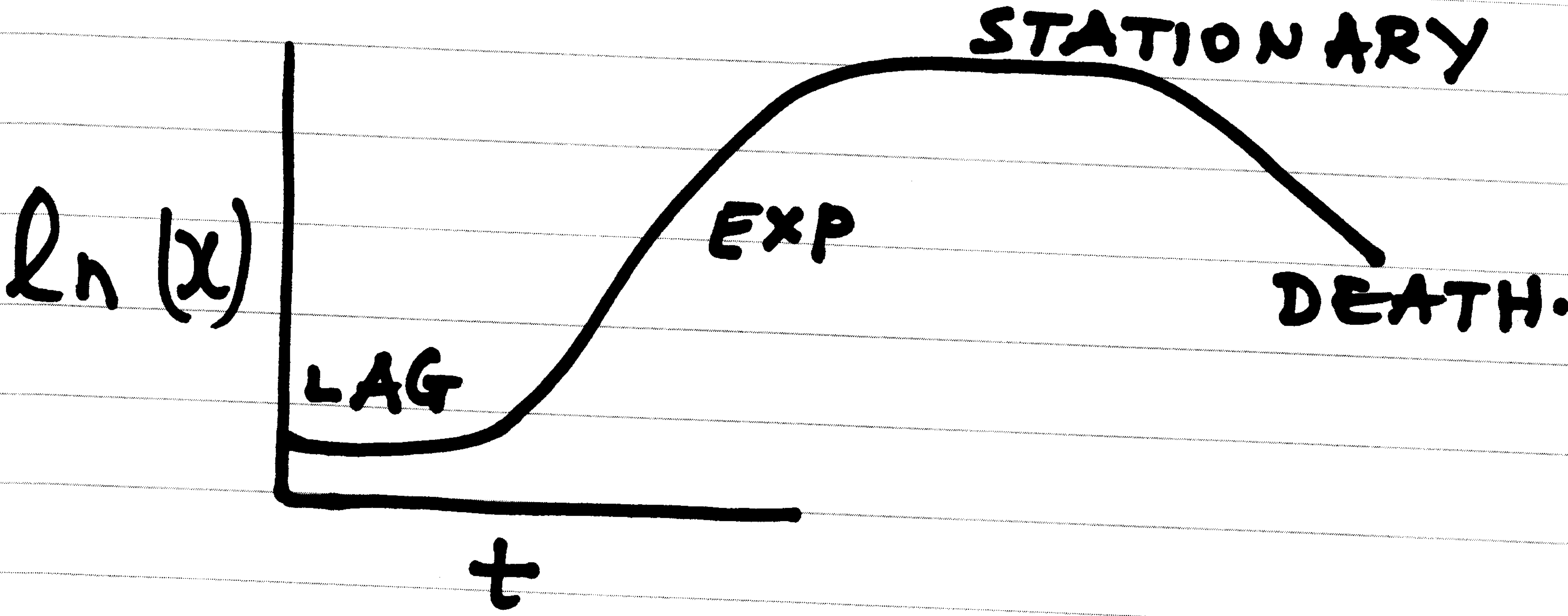
$e_0 = 5 g/L$ $\qquad$ $e_0 = 0.001 \frac{g}{L}$

$V_M = 1.33 \frac{mol}{s \cdot L}$ $\qquad$ $V_M' ? = \frac{1.33}{5} \times 0.001$

$V_M' = 0.266 \times 10^{-3} \frac{mol}{L \cdot s}$

$K_M = 0.266 \frac{mol}{L}$

MICROBIAL KINETICS



BATCH GROWTH

$$V \frac{dx}{dt} = r_x V$$

$$r_x = \mu x$$

$$\mu = \frac{\mu_m S}{K_s + S}$$

MONOD MODEL

$$V \frac{dS}{dt} = r_S V$$

$$dx/dS = \frac{r_x}{r_S} = -Y$$

$$x - x_1 = Y(S_i - S)$$

$$\frac{dx}{dt} = \frac{\mu_m S}{K_s + S}\left[x_i + Y(S_i - S)\right]$$

$$-Y\frac{dS}{dt} = \frac{\mu_m S}{K_s + S}\left[x_i + Y(S_i - S)\right]$$

x_i

Integrated Form BATCH GROWTH

$$(x_i + Y K_s + Y S_i)\ \ln \frac{x_i + Y S_i - Y S}{x_i}$$

$$+ Y K_s \ln \frac{S_i}{S} = (x_i + Y S_i)\,\mu_m t.$$

FOR CONTINUOUS GROWTH PFR

	PFR	IMR
REPLACE	$x_i = x_0$	$x_i + x_0$
	$S_i = S_0$	$S_i = S_0.$
	$t = \tau$	$\tau = (1 - \epsilon_s - \epsilon_g)\tau$

IMMOBILISED CELL REACTOR.

$$\frac{dFx}{dV} = r_x$$

$$\frac{d F_{\cancel{s}} S}{dV} = r_s$$

$$\frac{dX}{dV_l} = r_x$$

$$\frac{F\,dx}{dV_l} = r_x$$

$$\frac{F\,dx}{(1-\epsilon_s-\epsilon_g)\,dV} = r_x$$

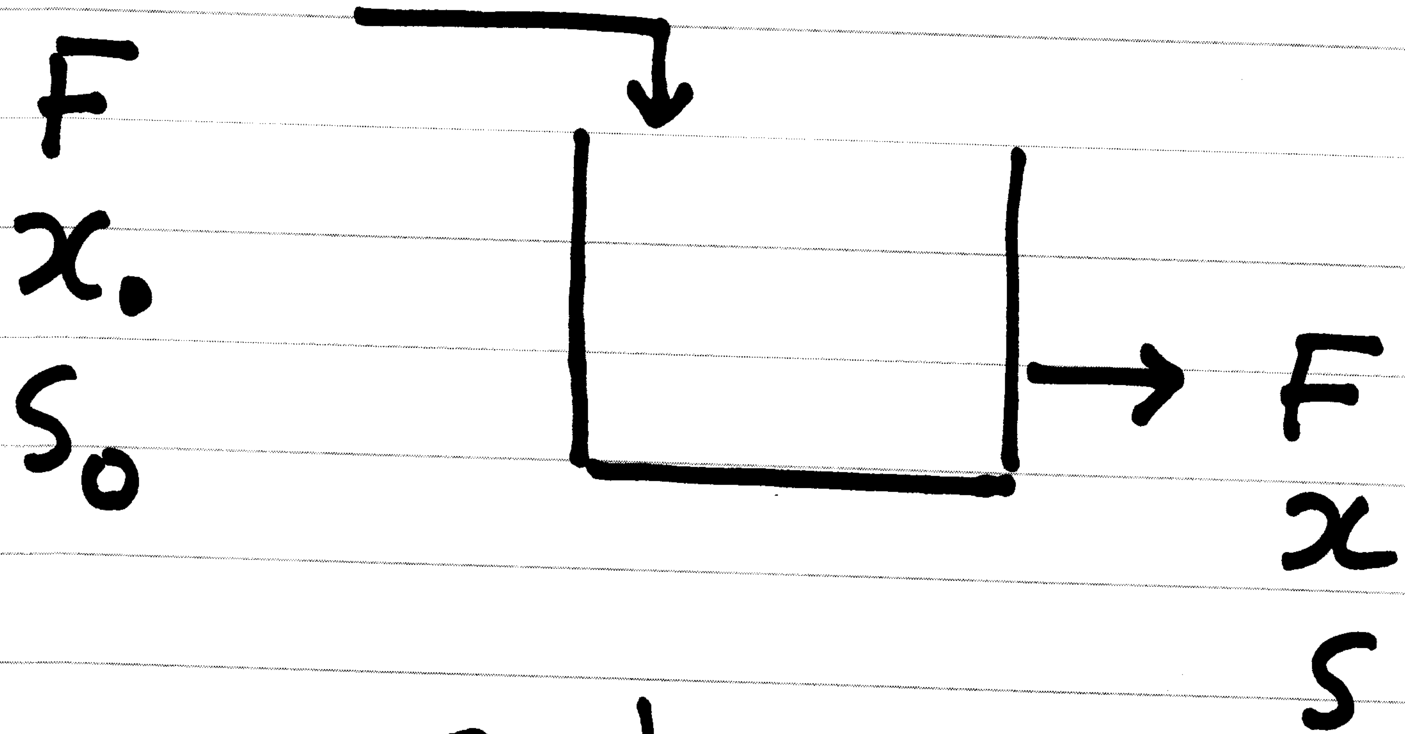
$$\frac{1}{(1-\epsilon_s-\epsilon_g)}\frac{dx}{d\tau} = r_x = \mu x$$

$$x = x_i + x_0 + Y(S_0 - S)$$

Integrated form IMR

$$(x_0 + x_i + yK_s + yS_0) \ln \frac{x_0 + x_i + yS_0 - yS}{x_i + x_0} + yK_s \ln \frac{S_0}{S} = \frac{(x_i + x_0 + yS_0)\mu_m}{(1 - \epsilon_s - \epsilon_s)}$$

CHEMOSTAT (CSTR)



Cell Balance

$$F x_0 - F x + r_x V = \frac{d}{dt}(Vx)$$

AT SS

$$F(\cancel{x_0} - x) + \mu x V = 0.$$

Let $F/V = D$ ($1/\text{TIME}$)

DILUTION RATE

$$D(X_0 - X) + \mu x = 0$$

Sterile Feed

$$X(\mu - D) = 0.$$

$$x(\mu - D) = 0.$$

$$x \neq 0.$$

$$\mu = D$$

$$\boxed{\frac{\mu_m S}{K_s + S} = D.}$$

$$S = \frac{D K_s}{\mu_m - D}.$$

$$x - x_0 = y\,(S_0 - S)$$

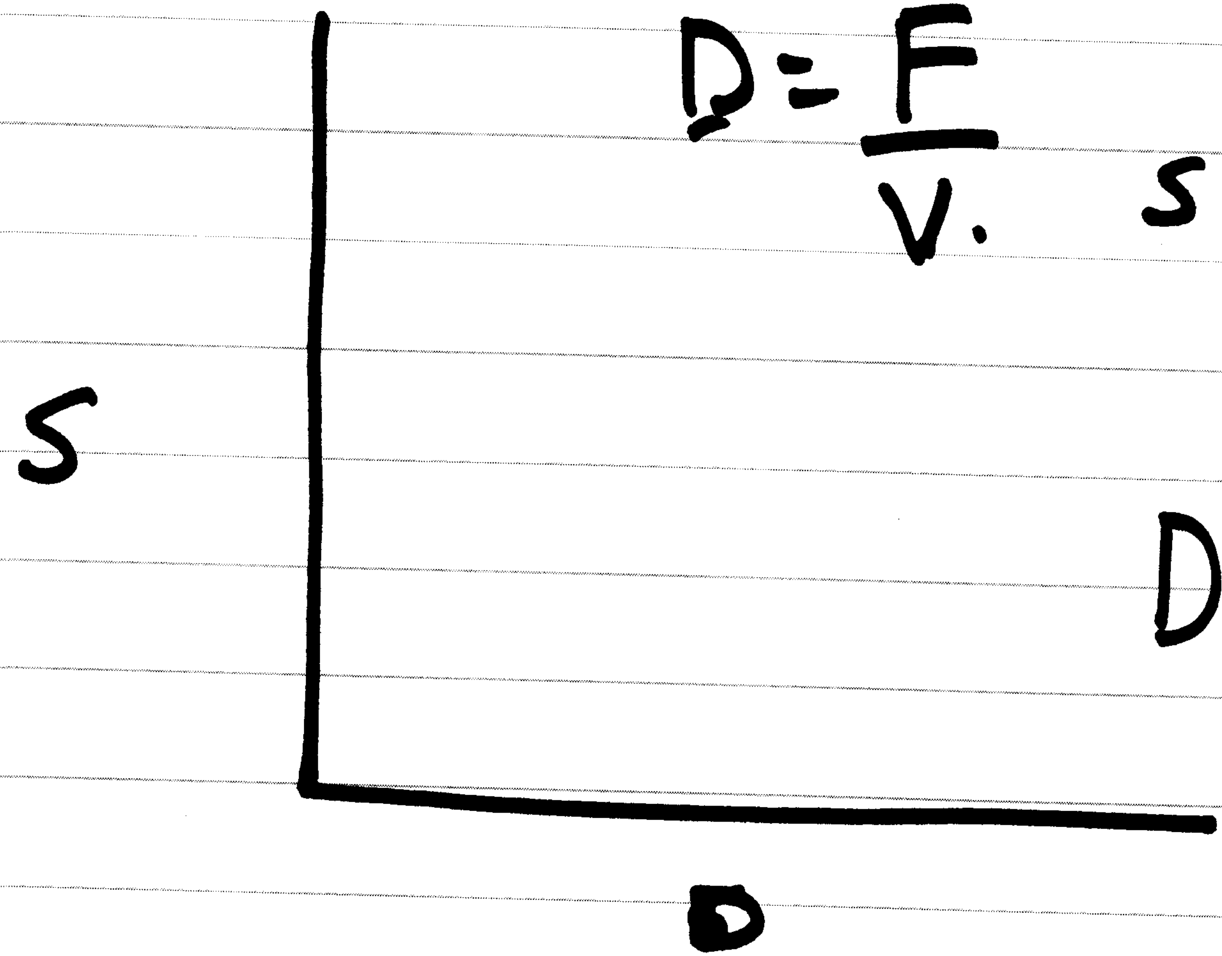
$$x = x_0 + y\,(S_0 - S)$$

Cell Production.

$$(Fx - Fx_0)/V$$

$$D\,(x - \cancel{x_0}) =$$

$$D\left[x_0 + y\left\{S_0 - \frac{DK_s}{\mu_m - D}\right\}\right]$$



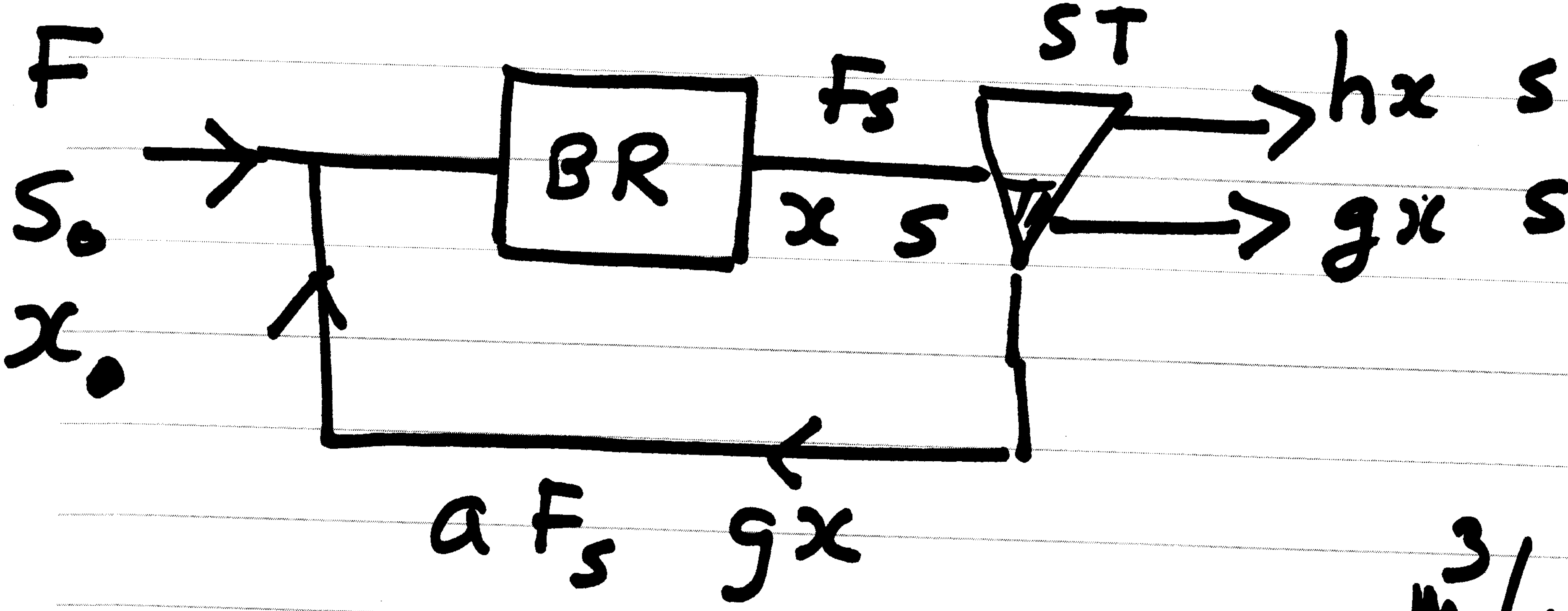
$$D = \frac{F}{V}$$

$$S = \frac{DK_s}{\mu_m - D}$$

$$D_W = \frac{\mu_m S_0}{K_s + S_0}$$

$$Dx = DY\left[S_0 - \frac{DK_s}{\mu_m - D}\right]$$

$$x = Y\left[S_0 - \frac{DK_s}{\mu_m - D}\right]$$



m^3/Hr

ST: Settling Tank

$$aF_s + F = F_s$$

$$F_s = F/(1-a)$$

Cell Balance

$$\cancel{Fx_0} + aF_s gx - F_s x + \mu x V$$

$$\frac{agFx}{(1-a)} - \frac{Fx}{1-a} + \mu x V = 0$$

$$\left[\mu - \left(\frac{1-ag}{1-a}\right)D\right]x = 0$$

$$B = \frac{1-ag}{1-a}$$

$$(\mu - BD)x = 0.$$

$$x \neq 0 \qquad \mu = BD$$

$$\boxed{BD = \frac{\mu_m S}{K_s + S}}$$

$$S = BDK_s \,/\, (\mu_m - BD)$$

$D_W = ?$ $\quad S = S_0$

$$B D_W = \frac{\mu_m S_0}{K_S + S_0}$$

$$D_W = \frac{1}{B}\left[\frac{\mu_m S_0}{K_S + S_0}\right]$$

Cell Productivity

$$P = \left[\cancel{F x_0} - a F_s g x + F_s x\right] / V$$

$$P = \left[-\frac{agFx}{1-a} + \frac{Fx}{1-a}\right] / V$$

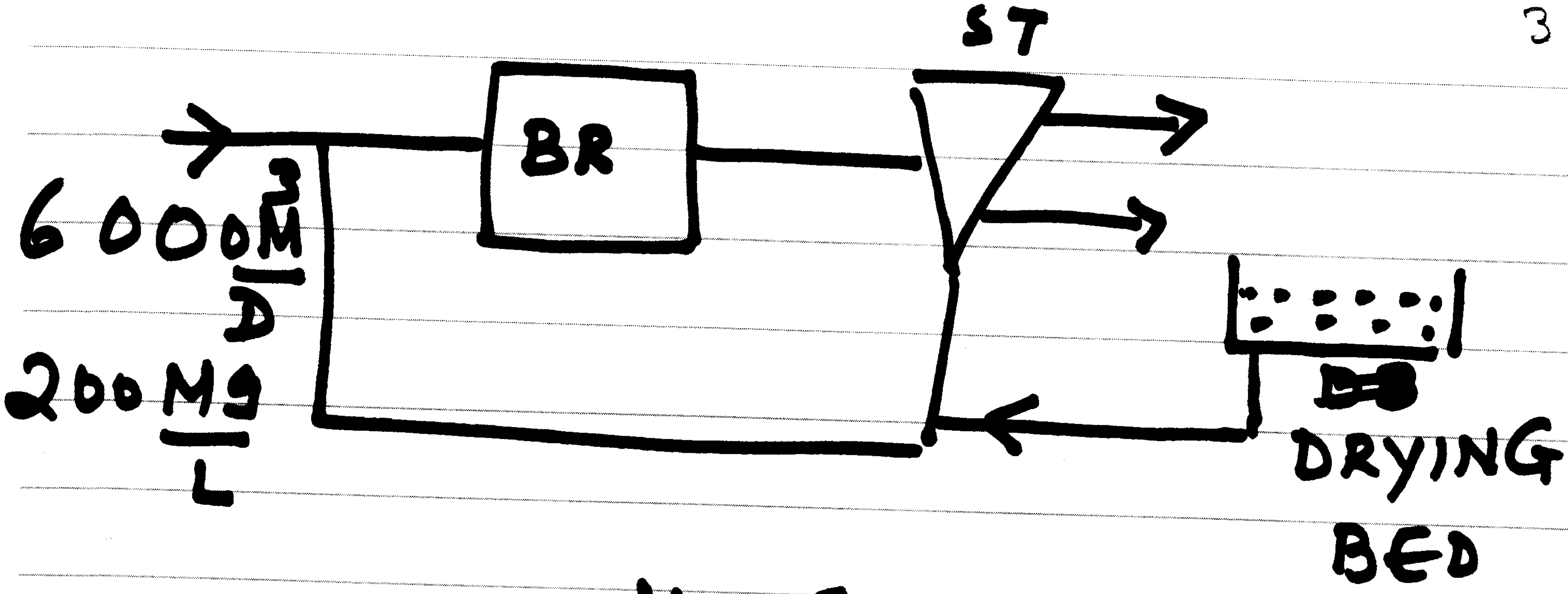
$$Dx\left[\frac{1-ag}{1-a}\right] = \underline{BDx}$$

Cell Productivity

$$P = \left(F_s x - F \cancel{x_0} - a F_s g x\right) / V$$

$$\left(\frac{F}{1-a} x - \frac{aF}{1-a} g x\right) / V$$

$$Dx\left[\frac{1-ag}{1-a}\right] = BDx$$



$$BD: \quad \frac{\mu_m S}{K_s + s} = \frac{(0.3)(10)}{10 + 10.}$$

$$D = 0.3/\text{day}$$

Ft: F

$$\frac{F}{V} = D$$

$$\frac{6000}{0.3} = V$$

V: $12000 M^3$