LEC 23 19/11/12

Advanced Reaction Engg

Residence Time Distribution

17 Nov 12

1030 - 1130

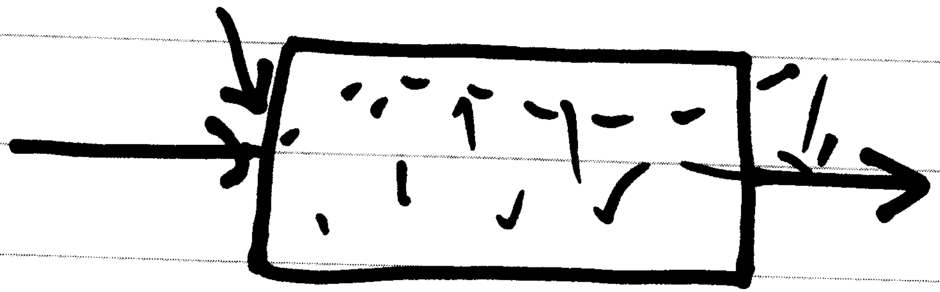
Tracers

Gas: any gas which can be measured

Solid: Radiotracer, Sod. Chloride

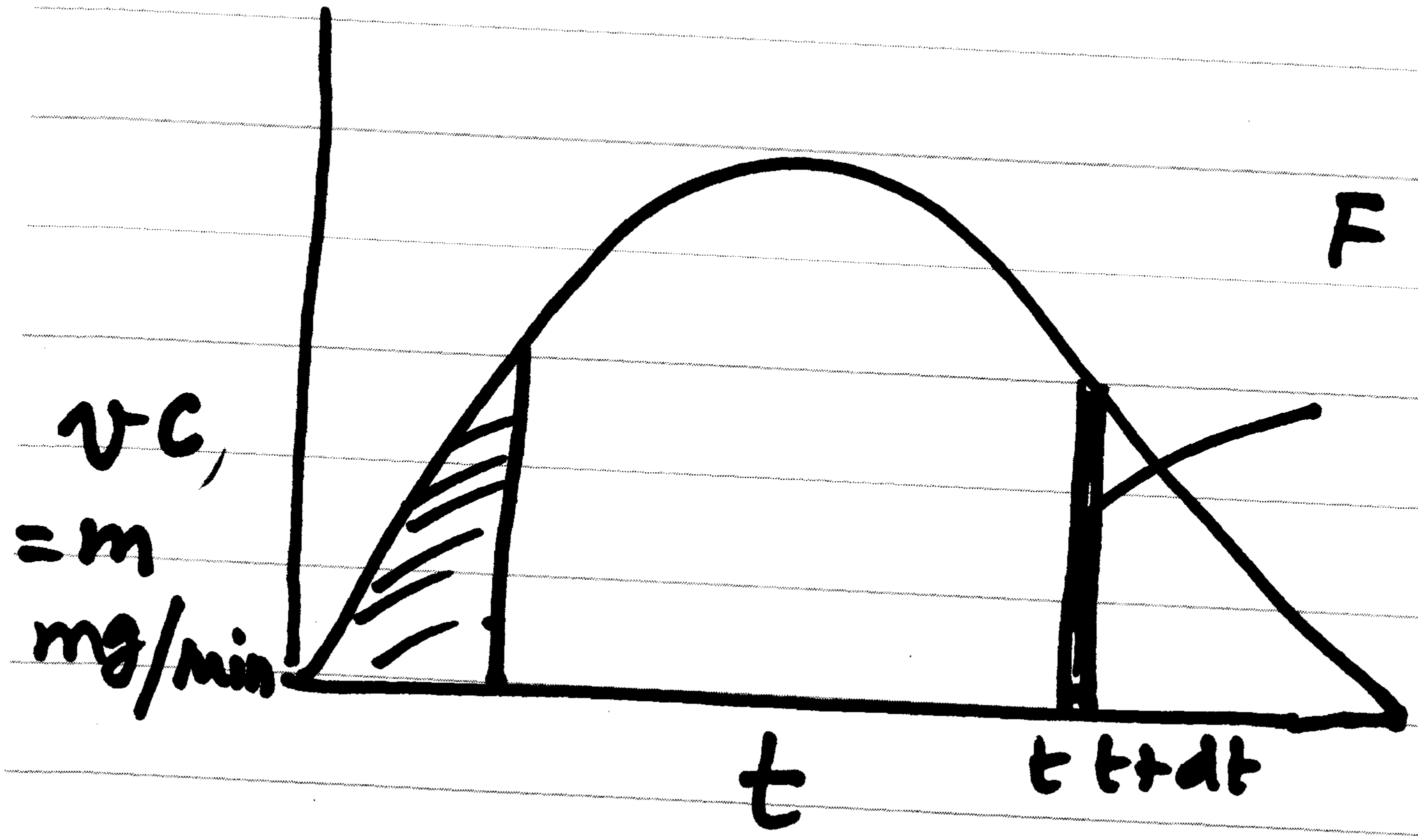
Liquid: Dye

Pulse



t (MIN)	0	1	2	3	4	5	6	7	8	9	10	12	14
C_{exit} mg/L	0	1	5	8	10	8	6	4	3	2.2	1.5	0.6	0
m mg/min													

$v_0 = 10$ Lt/min

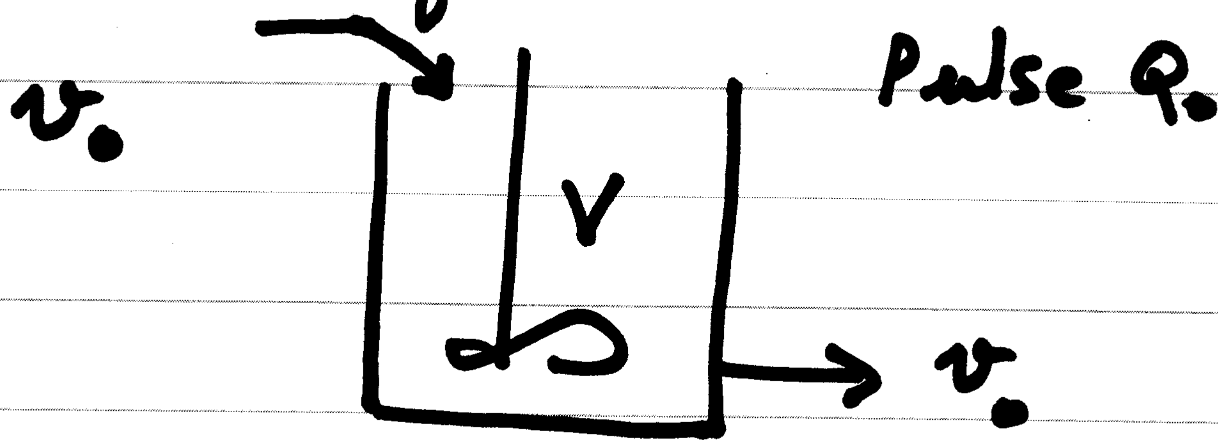


$$F = \int_0^t E(t)\,dt$$

$$1 = \int_0^\infty E(t)\,dt$$

$$\frac{(m\,dt)}{M_0} = \frac{v\,c(t)\,dt}{\int_0^\infty vc(t)\,dt} = E(t)\,dt$$

RTD of Stirred Tank.



$$V/v = \tau$$

$$VC_0 = Q_0$$

$$VC = Q$$

$$\cancel{IN} - OUT + \cancel{Gen} = Acc$$

$$0 - vC = \frac{d}{dt}(VC)$$

$$-v\frac{Q}{V} = \frac{dQ}{dt}$$

$$-\frac{Q}{\tau} = \frac{dQ}{dt}$$

$$Q = K e^{-t/\tau}$$

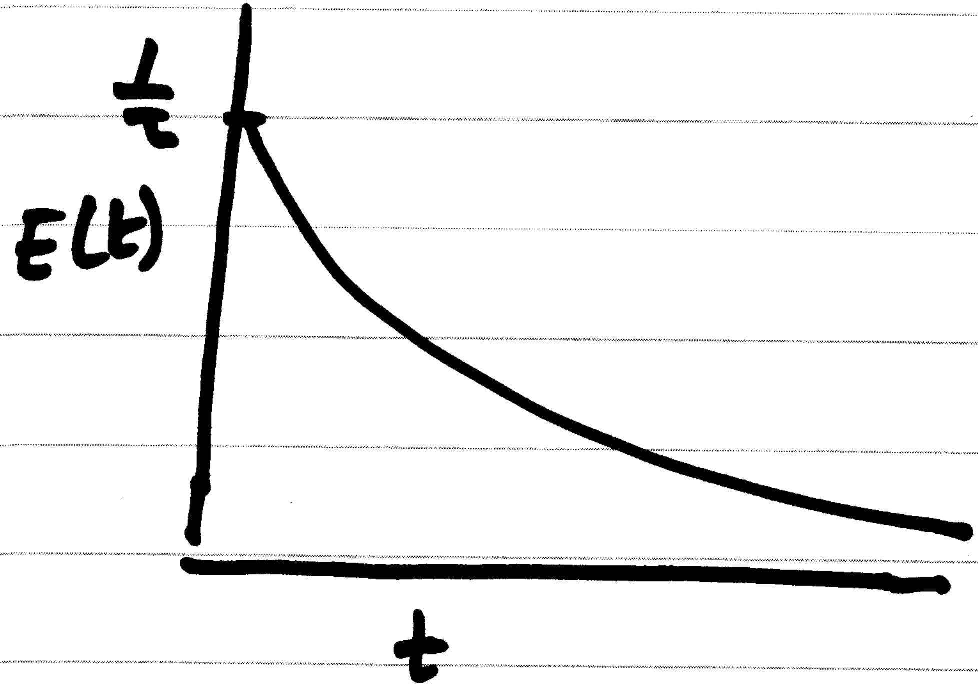
$$Q = Q_0 e^{-t/\tau}.$$

$$RTD = \frac{v\, C(t)\, dt}{Q_0} = E(t)\, dt$$

$$= \frac{v\, C(t)\, dt}{Q_0} = E(t)\, dt.$$

$$= \frac{v}{V} \frac{Q(t)}{Q_0} dt = \frac{1}{\tau} e^{-t/\tau} \cdot dt$$

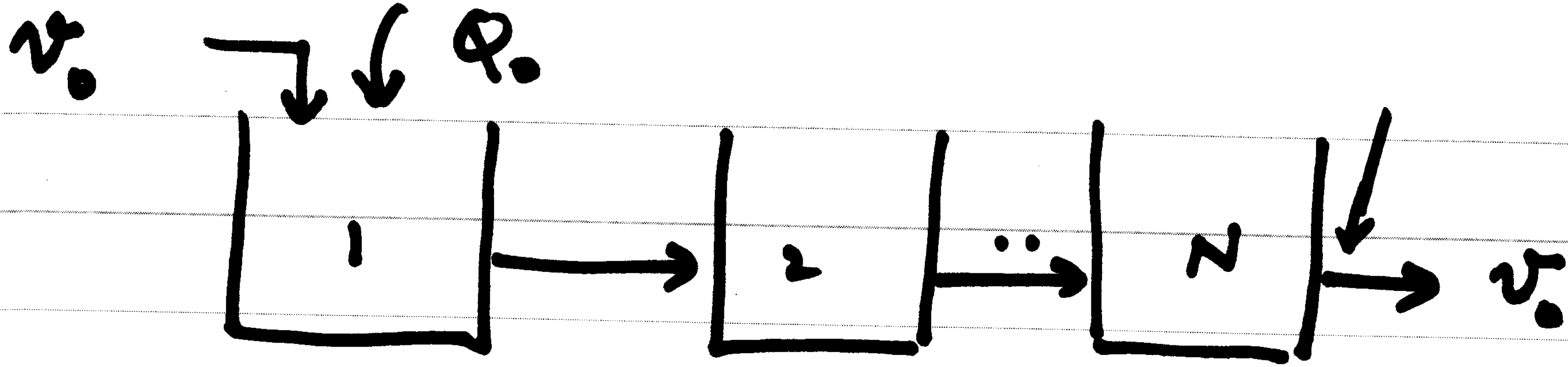
$$\boxed{E(t)\, \cancel{dt} = \frac{1}{\tau} e^{-t/\tau} \cdot \cancel{dt}}$$



$$\frac{1}{\tau} e^{-t/\tau}.$$

Mean value of $E(t)$

$$\mu = \int_0^\infty t \cdot \frac{e^{-t/\tau}}{\tau} \cdot dt = \tau$$



$V_1 = V_2 = \cdots V_N$

$V_1/v_0 = \tau_1 = \tau_i$

$V_1 = V_2 = \#$

Tank-1 $q_1 = q_0 \, e^{-t/\tau_i}$

Tank2

IN − OUT $+ \cancel{G}$ = Acc

$$v_0 C_1 - v_0 C_2 + 0 = \frac{d}{dt}(V_2 C_2)$$

$$v_0 \frac{Q_1}{V_1} - \frac{Q_2}{V_2} = \frac{d}{dt} Q_2$$

$$\frac{Q_1}{\tau_i} - \frac{Q_2}{\tau_i} = \frac{d}{dt} Q_2$$

$$\frac{Q_0 e^{-t/\tau_i}}{\tau_i} - \frac{Q_2}{\tau_i} = \frac{dQ_2}{dt}$$

$$\frac{dQ_2}{dt} + \frac{Q_2}{\tau_i} = \frac{Q_0}{\tau_i} e^{-t/\tau_i}$$

$$Q_2(t=0) = 0$$

$$\frac{dQ_2}{dt} + \frac{Q_2}{\tau_i} = \frac{Q_0}{\tau_i} e^{-t/\tau_i}$$

$$\frac{Q_2}{Q_0} = \frac{t}{\tau_i} e^{-t/\tau_i}$$

Tank 3

IN − OUT + G = Acc

$$v_0 C_2 - v_0 C_3 + 0 = \frac{d}{dt} V C_3$$

$$v_0 \frac{q_2}{V_2} - v_0 \frac{Q_3}{V_3} = \frac{d}{dt} V C_3$$

$$\frac{Q_2}{\tau_2} - \frac{Q_3}{\tau_3} = \frac{d}{dt} Q_3 \qquad \tau_3 = \tau_i$$

$$\frac{Q_2}{\tau_i} - \frac{Q_3}{\tau_i} = \frac{d}{dt} Q_3$$

Tak 3

$$\frac{Q_2}{\tau_i} - \frac{Q_3}{\tau_i} = \frac{d}{dt} Q_3$$

$$Q_3 (t=0) = 0$$

$$Q_3 = Q_0 \frac{t^2}{2\tau_i^2} e^{-t/\tau_i}$$

$$E_3\, dt = \frac{v\, c_3(t)\, dt}{Q_0} = \frac{v\, Q_3}{V\, Q_0} dt$$

$$E_3\, dt = \frac{Q_3\, dt}{Q_0 \tau_i} = \frac{t^2}{\tau_i\, 2\tau_i^2} e^{-t/\tau_i}$$

Tank 3

$$E_3(t)\,dt = \frac{t^2}{2!\,\tau_i^3} e^{-t/\tau_i}$$

$$E_n(t) = \frac{t^{n-1}}{(n-1)!\,\tau_i^n} e^{-t/\tau_i}$$

nV_i

$$\tau_i = \left(\tau/N\right) = \left(V/v_0\right)/N$$

$$\boxed{\tau = N\tau_i}$$

$$E_n(t) = \frac{t^{n-1}}{(n-1)!} \frac{e^{-t/\tau_i}}{\tau_i^{n}}$$

$$\tau = \frac{V}{v_0}$$

$$\boxed{\int_0^\infty t^n e^{-t/\tau} dt = n! \, \tau^{n+1}}$$

$$\mu_n(t) = \int_0^\infty t \, E(t) \, dt = \int_0^\infty \frac{t \cdot t^{n-1}}{(n-1)!} \frac{e^{-t/\tau_i}}{\tau_i^{n}} dt.$$

$$\mu_n(t) = \int_0^{\infty} t \cdot E(t)\,dt$$

$$\int_0^{\infty} \frac{t \cdot t^{n-1}}{(n-1)!\ \tau_i^{\,n}} e^{-t/\tau_i}\,dt.$$

$$= \frac{(n!)\ \tau_i^{\,n+1}}{(n-1)!\ \tau_i^{\,n}} = n\,\tau_i = \tau$$

Variance of $E_n(t)$ function.

$$\sigma_n^2 = \int_0^{\infty} (t - \mu_n)^2 E_n(t)\,dt$$

$$= \int_0^{\infty} \left(t^2 - 2\mu_n t \cancel{E_n(t)\,dt} + \mu_n^2\right) E_n\,dt$$

$$= \int_0^{\infty} t^2 E_n\,dt - 2\mu_n^2 + \mu_n^2$$

$$\sigma_n^2 = \int_0^{\infty} t^2 E_n\,dt - \mu_n^2$$

$$\sigma_n^2 = \int_0^{\alpha} \frac{t^2 \cdot t^{n-1}}{(n-1)!} \frac{e^{-t/\tau_i}}{\tau_i^n} \, dt - \mu_n^2$$

$$= \int_0^{\alpha} \frac{t^{n+1} e^{-t/\tau_i}}{(n-1)! \, \tau_i^n} \, dt - \mu_n^2$$

$$\sigma_n^2 = \frac{1}{\tau_i^n (n-1)!} (n+1)! \, \tau_i^{n+2} - \mu_n^2$$

$$\sigma_n^2 = n(n+1)\tau_1^2 - \mu_n^2$$

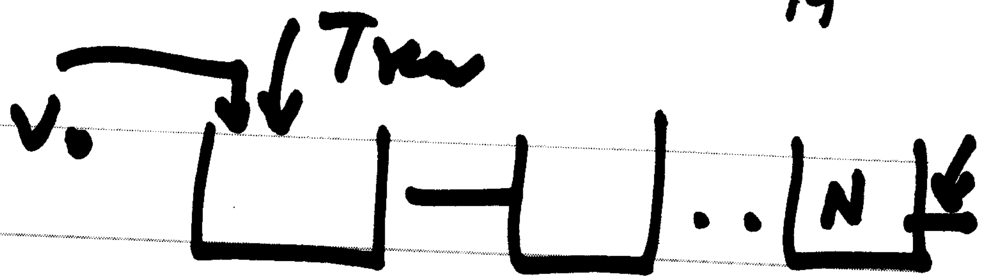
$$\boxed{\mu_n = n\tau_1}$$

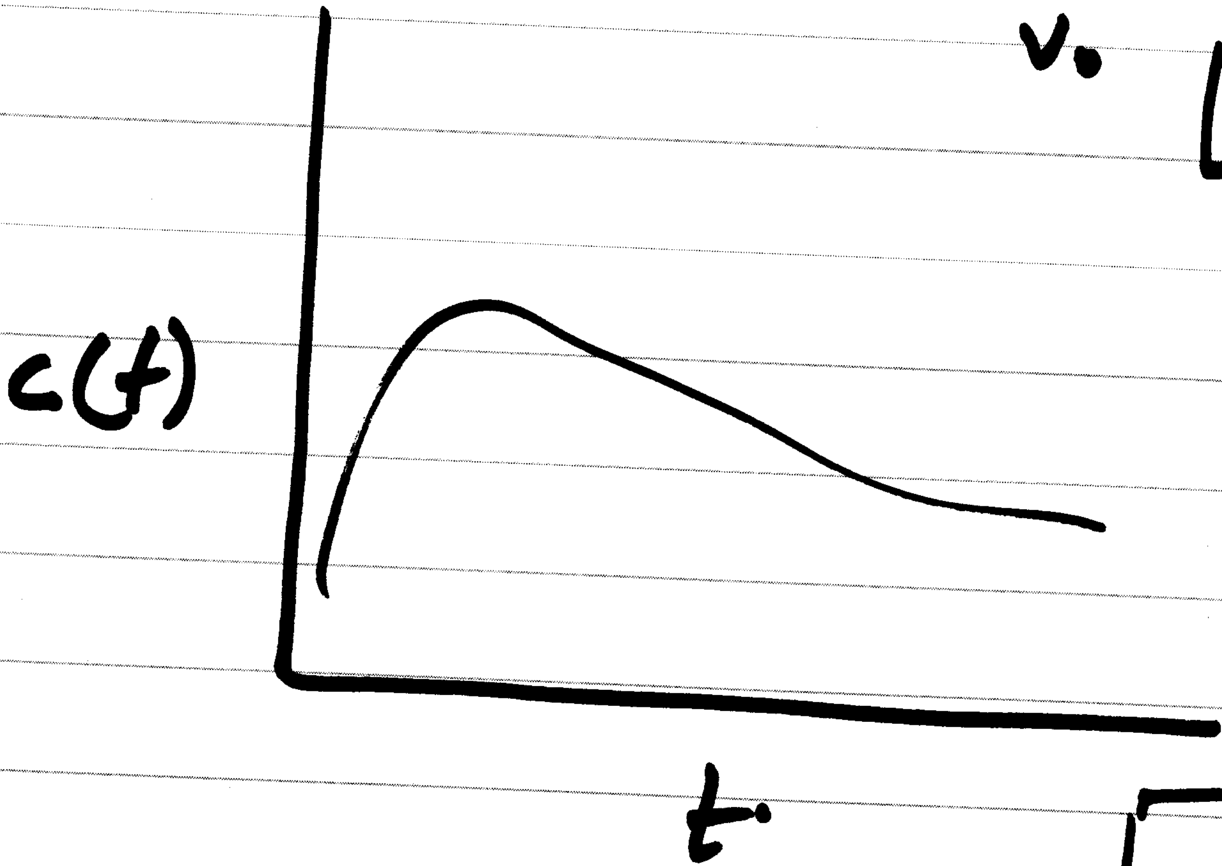
$$\tau_1^2\, n(n+1) - (n\tau_1)^2$$

$$\tau_1^2\left[n^2 + n - n^2\right] = n\tau_1^2$$

$$\sigma_n^2 = n\left(\frac{\tau}{n}\right)^2 = \frac{\tau^2}{n}$$

$$\boxed{\underset{\uparrow}{n} = \tau^2/\sigma_n^2} \qquad \boxed{\mu_n = n\tau_1}$$

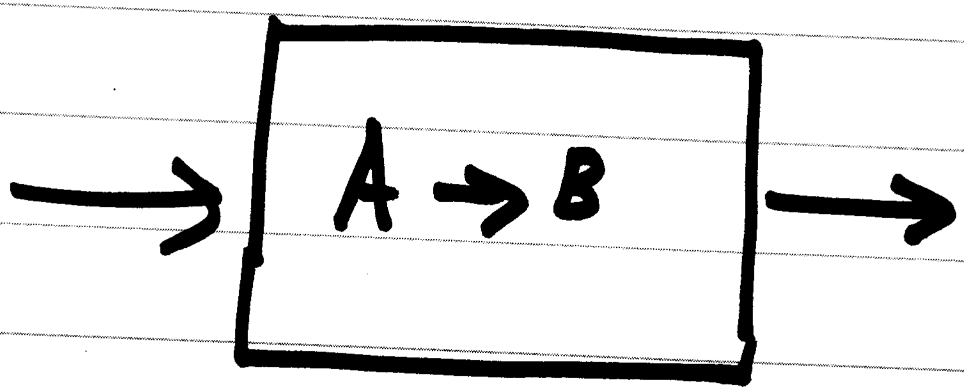




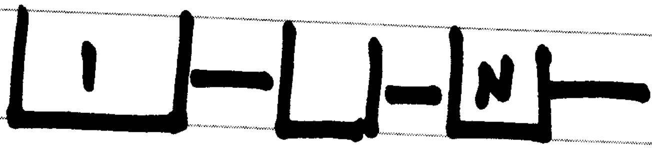
μ_n

σ_n^2

$$\boxed{n = \tau_n^2 / \sigma_n^2}$$



σ_n^2, μ_n^2



$$\boxed{n = \frac{\mu_n^2}{\sigma_n^2}}$$

$$C_1 = \frac{C_0}{(1 + k\tau_i)}$$

$$C_N = \frac{C_0}{(1 + k\tau_i)^N}$$

$$\boxed{C_N = \frac{C_0}{\left[1 + k \frac{\sigma_n^2}{\mu_n}\right]^{\mu_n^2/\sigma_n^2}}}$$

$$X = \left(1 - \frac{C_N}{C_0}\right)$$

$$C_N = \frac{C_0}{(1 + k\tau_i)^N}$$

$$\frac{C_0}{\left[1 + \left(k \frac{\tau}{N}\right)\right]^N} = \frac{C_0}{\left(1 + k \frac{\mu_n \sigma_n^2}{\mu_n^2}\right)^{\frac{\mu_n^2}{\sigma_n^2}}}$$

$$C_N = \frac{C_0}{\left(1 + k \frac{\mu_n \sigma_n^2}{\mu_n^2}\right)^{\mu_n^2/\sigma_n^2}} = \frac{C_0}{\left(1 + k \frac{\sigma_n^2}{\mu_n}\right)^{\frac{\mu_n^2}{\sigma_n^2}}}$$