Module-5

Lecture-24

Lateral Stability and Control
**Lateral stability**

An airplane is said to have roll (lateral) stability, if a restoring moment is generated when it is disturbed in bank orientation ($\phi$).

- The restoring moment is function of side slip angle, $\beta$.
- The requirement for roll stability is that $C_{l\beta} < 0$.
- The rolling moment created in airplane due to side slip angle also depends on
  - Wing dihedral
  - Wing sweep
  - Position of wing and fuselage
  - Vertical tail
- The major contributor to $C_{l\beta}$ is the wing dihedral angle, $\Gamma$.
- When an aircraft is disturbed from a wing-level attitude, it will begin to side slip.

![Figure 1: Wing-body dihedral effect](image)

On wing 1,

$$\Delta \alpha = \frac{v_n}{u} \quad V \sin \Gamma$$

$$\beta \approx \frac{v}{u}$$

$$\Delta \alpha = \beta \Gamma$$

On wing 2, angle of attack will decrease. Resulting in negative rolling moment to positive side slip angle.

$$C_{l\beta} < 0$$
Rolling moment due to right wing

\[ L_{w,R} = -\frac{1}{2} \rho V^2 C_{L_{\alpha,w}} \Delta \alpha \int_{0}^{\frac{b}{2}} c(y) \cdot y \, dy \]

Rolling moment due to left wing

\[ L_{w,L} = -\frac{1}{2} \rho V^2 C_{L_{\alpha,w}} \Delta \alpha \int_{-\frac{b}{2}}^{0} c(y) \cdot y \, dy \]

Rolling moment (total)

\[ L_w = -\frac{1}{2} \rho V^2 C_{L_{\alpha,w}} \Delta \alpha \int_{0}^{\frac{b}{2}} c(y) \cdot y \, dy \]

\[ \therefore \Delta \alpha = \beta \Gamma \text{ and } \bar{y} = \frac{2}{S_w} \int_{0}^{\frac{b}{2}} c(y) \, dy \]

\[ L_w = -\frac{1}{2} \rho V^2 C_{L_{\alpha,w}} \Gamma \beta \frac{S}{2} \bar{y} \]

\[ (C_{l,w})_\Gamma = \frac{L_w}{\frac{1}{2} \rho V^2 S b} = -\Gamma \beta C_{L_{\alpha,w}} \frac{\bar{y}}{b} \]

\[ C_{l_\beta} = -\Gamma C_{L_{\alpha}} \frac{\bar{y}}{b} \]

\[ C_{l_{\beta,w}} < 0, \text{ Stabilizing for } \Gamma > 0 : \text{ Dihedral} \]

\[ C_{l_{\beta,w}} > 0, \text{ Destabilizing for } \Gamma < 0 : \text{ Anhedral} \]

- \( C_{l_\beta} \): Due to wing sweep

\[ (L_w)_{R,\Lambda} = -C_L \frac{1}{2} \rho V^2 \bar{y} \cos^2 (\Lambda - \beta) \]

\[ (L_w)_{L,\Lambda} = C_L \frac{1}{2} \rho V^2 \bar{y} \cos^2 (\Lambda + \beta) \]

Hence,

\[ (L_w)_{\Lambda,Total} = -C_L \frac{1}{2} \rho V^2 \bar{y} \left\{ \cos^2 (\Lambda - \beta) - \cos^2 (\Lambda + \beta) \right\} \]

\[ \bar{y} \] is the location of resultant lift on the wing half

Assume \( \beta \) to be small; \( \cos \beta \to 1 \) and \( \sin \beta \to \beta \)

\[ (L_w)_{\Gamma,Total} = -C_L \frac{1}{2} \rho V^2 \bar{y} \beta \sin 2\Gamma \]

\[ (C_{l_w})_{\Gamma} = -C_L \frac{\bar{y}}{b} \beta \sin 2\Gamma \]

\[ C_{l_\beta} = -C_L \frac{\bar{y}}{b} \sin 2\Gamma \]

\[ C_{l_\beta} < 0 \Rightarrow \text{ Stabilizing} \]
- $C_{I\beta}$: Due to vertical tail

  The rolling moment due to vertical tail when aircraft is side slipping can be written as

  $$l = -\frac{1}{2} \rho V^2 \eta_v S_v \left( \frac{dC_L}{d\alpha} \right)_v \beta_v z_v$$

  hence,

  $$(C_{I\beta})_v = -\eta_v \frac{S_v z_v}{S_w} C_{L_{\alpha,v}}$$

**Roll control**

- It is achieved by differential deflection of small flaps called ailerons.

- The basic principle lies on the fact that due to differential deflection of ailerons, the lift distribution over the wing becomes unequal, causing a rolling moment.

- An approximate expression for roll control power can be obtained using simple strip integration method.

$$\Delta(\text{Rolling moment due to } \delta_a) = \Delta l$$
\[ \Delta C_l = \frac{\Delta L}{qSb} = \frac{C_l Q_c y dy}{Q S_b} = \frac{C_{lcy} dy}{S_b} \]

\[ C_l = C_{La} \frac{d\alpha}{d\delta a} \delta a = C_{La} \tau \delta a \]

Integrating over the region containing the aileron yields

\[ C_l = \frac{2C_{La,w} \tau \delta a}{S_b} \int_{y_1}^{y_2} dy \]

\[ C_{ls\alpha} = \frac{2C_{La,w} \tau}{S_b} \int_{y_1}^{y_2} dy \]