Module-2

Lecture-8

Cruise Flight - Range and Endurance of Propeller Driven Aircraft
Range and Endurance

Range is defined as the total distance (measured with respect to ground) traversed by the airplane on a full tank of fuel.

Endurance is defined as the total time that an airplane stays in the air on a full tank of fuel.

For different applications, it may be desirable to maximize one or other, or both characteristics. The parameters that maximize range are different from those that maximizes the endurance. Additionally, these parameters are also different for propeller and jet powered aircrafts.

Range & Endurance : Propeller driven aircraft

For a propeller driven aircraft, the most important factor that influences range and endurance is the specific fuel consumption of the reciprocating engine.

Specific Fuel Consumption (SFC), is defined as the weight of the fuel consumed by the reciprocating engine per unit power per unit time.

\[
SFC = \frac{N(fuel)}{(J/s)(s)} \tag{1}
\]

Endurance

In order to stay airborne for the longest duration, i.e. for maximum endurance the engine must use minimum Newton’s of fuel per unit time. From the Equation 1 we can see that:

\[
\frac{N(fuel)}{(s)} \propto SFC(P_R) \tag{2}
\]
So from Equation 2 depicting the proportionality, we quickly conclude that for maximum
endurance, the power required by the airplane should be minimum. We have already
shown in our previous discussions, that for an aircraft to fly at the minimum power
required, \( C_L^{3/2} / C_D \) should be maximum. Designating \( SFC \) as \( c \) and considering the
product \( c.P.dt \), where \( P \) is engine power and \( dt \) is a small increment in time, we have:

\[
cPdt = \frac{N(fuel)}{(J/s)(s)} \times \frac{J}{s} \times s = N(fuel)
\]

(3)
Thus, \( cPdt \) represents the differential change in the weight of fuel over a small interval of
time, \( dt \). Let,

- \( W_o \) - gross weight of the airplane
- \( W_1 \) - weight of the airplane without fuel
- \( W_f \) - weight of the fuel

Then, we have:

\[
W_1 = W_o - W_f
\]
(4)
and

\[
dW_f = dW = -cPdt
\]
(5)
\[
\Rightarrow dt = -\frac{dW}{cP}
\]
(6)
Denoting endurance as \( E \)

\[
\int_0^E dt = -\int_{W_o}^{W_1} \frac{dW}{cP}
\]
\[
\Rightarrow E = \int_{W_o}^{W_1} \frac{dW}{cP}
\]
(7)

**Range**

Now considering range; in order to cover the longest distance, we must ensure minimum
weight of fuel consumed per unit distance. From the relations discussed above, we can
get the proportionality:

\[
\frac{N(fuel)}{(m)} \propto \frac{SFC(P_R)}{V}
\]

Thus for obtaining maximum range for any flight, the ratio \( P_R/V \) should be minimum.
\([P_R/V]_{min}\) for cruise flight implies that thrust required is minimum and for \( T_R \) to be
minimum, \( C_L/C_D \) should be maximum. Minimum value of \( P_R/V \) precisely corresponds
to the tangent point in Figure 1, which also corresponds to \([L/D]_{max}\) or \([C_L/C_D]_{max}\). Now
to calculate the range, from Equation 7:

$$ds = Vdt = - \int_{W_o}^{W_1} \frac{VdW}{cP}$$

$$\Rightarrow \int_0^R ds = \int_{W_o}^{W_1} \frac{VdW}{cP}$$

$$\Rightarrow R = \int_{W_1}^{W_o} \frac{VdW}{cP} \quad (8)$$

Breguet Formula

For a propeller driven aircraft, we know that:

$$P_A = \eta P$$

thus,

$$P = \frac{P_A}{\eta} = \frac{DV}{\eta} \quad (9)$$

Substitute Equation 9 in Equation 8, we get:

$$R = \int_{W_1}^{W_o} \frac{VdW}{cP} = \int_{W_1}^{W_o} \frac{V\eta dW}{cDV} = \int_{W_1}^{W_o} \frac{\eta dW}{cD} \quad (10)$$

Multiplying Equation 10 by $W/W$ and noting that for steady, level flight, $W = L$, we get:

$$R = \int_{W_1}^{W_o} \frac{\eta W}{cD W} dW = \int_{W_1}^{W_o} \frac{\eta L dW}{cD W}$$

$$\Rightarrow \eta \frac{C_L}{cC_D} \int_{W_1}^{W_o} \frac{dW}{W}$$
thus,
\[ R = \frac{\eta C_L}{c C_D} \ln \frac{W_o}{W_1} \quad (11) \]

Similarly by using Equation 7 and Equation 9 and by applying steady, level flight condition, \( L = W \), we get:

\[ E = \int_{W_1}^{W_o} \frac{dW}{cP} = \int_{W_1}^{W_o} \eta dW = \int_{W_1}^{W_o} \frac{\eta}{c DV} \frac{L}{W} dW \]

Substituting,

\[ L = W = \frac{1}{2} \rho V^2 S C_L \text{ and then } V = \sqrt{\frac{2 (W/W_1)}{\rho C_L}} \]

we get:

\[ E = \int_{W_1}^{W_o} \frac{\eta C_L}{c C_D} \sqrt{\frac{\rho S C_L}{2}} \frac{dW}{W^{\frac{3}{2}}} \quad (12) \]

Assuming \( C_L, C_D, \eta, c \) and \( \rho \) (constant altitude) are all constant, Equation 12 becomes:

\[ E = -\frac{2 \eta C_L^3}{c C_D} \left[ \frac{\rho S}{2} \right]^{\frac{1}{2}} \left[ W^{-\frac{1}{2}} \right]_{W_1}^{W_o} \]

\[ E = \frac{\eta C_L^3}{c C_D} (2\rho S)^{\frac{1}{2}} \left( W_1^{-\frac{1}{2}} - W_o^{-\frac{1}{2}} \right) \quad (13) \]