Module-2

Lecture-6

Cruise Flight - Power required, Velocity for Minimum Power required
Cruise flight: power required & velocity for minimum power required

\[ P_{\text{req}} = T_{\text{req}}V = DV = \frac{WV}{C_L/C_D} = \sqrt{\frac{2W^3C_D^2}{S\rho C_L^3}} \]  

\[ \Rightarrow P_{\text{req}} \propto \frac{1}{C_L/C_D} \]

Thus, following conclusions can be made from the above expression:

- For power required to be minimum, \( C_L^{3/2}/C_D \) should be maximum.
- The airplane needs to be flown at \( C_L \) such that \( C_L^{3/2}/C_D \) is maximum.
- The airplane needs to have sufficient speed so that the lift produced by the aircraft at \( C_L \) corresponding to \( C_L \) for \( C_L^{3/2}/C_D \) \( \text{max} \) is able to balance the weight of the aircraft.

Now an interesting question arises that how the power required is dependent on the velocity of an aircraft?

- In order to find an answer to this question let us write Equation \[1\] in another way

\[ P_{\text{req}} = DV = \bar{q}SV C_D = \left( C_{D_o} + \frac{C_L^2}{\pi AR e} \right) \bar{q}SV \]

\[ \Rightarrow P_{\text{req}} = \begin{cases} 
C_{D_o} + \frac{\left( \frac{W}{\bar{q}S} \right)^2}{\pi AR e} \end{cases} \bar{q}SV \]

\[ P_{\text{req}} = \frac{1}{2} \rho V^3 SC_{D_o} + \frac{W^2}{2\rho VS} \pi AR e \]  

\[ (2) \]

A typical variation of power required, \( P_{\text{req}} \) with velocity is presented in Figure \[1\]. Referring to the figure it can easily be interpreted that there is a particular speed at which power required to maintain level flight is minimum.

- For minimum power required,

\[ \frac{\partial P_{\text{req}}}{\partial V} = 0 \]

\[ \frac{\partial^2 P_{\text{req}}}{\partial V^2} > 0 \]
So differentiating Equation 1 and equating it to zero, we will get:

\[
\frac{\partial P_{req}}{\partial V} = \frac{3}{2} \rho V^2 S C_{D_o} - \frac{W^2}{\pi A Re} = 0
\]

\[
\Rightarrow \frac{3}{2} \rho V^2 S \left( C_{D_o} - \frac{W^2}{\frac{1}{3} \rho V^2 S^2} \right) = 0
\]

\[
\Rightarrow \frac{3}{2} \rho V^2 S \left( C_{D_o} - \frac{1}{3} C_{D_i} \right) = 0
\]

\[
\Rightarrow C_{D_o} - \frac{1}{3} C_{D_i} = 0
\]

where $C_{D_i}$ is the induced drag.

So the aerodynamic condition for minimum power required is

\[
C_{D_i} = 3C_{D_o} \tag{3}
\]

Now to calculate $C_L$ for minimum power required we know that, induced drag coefficient,

\[
C_{D_i} = KC_{L}^2
\]

Using Equation 2 we can write:

\[
KC_{L}^2 = 3C_{D_o}
\]

\[
C_{L_{\text{min}P_{req}}} = \sqrt{\frac{3C_{D_o}}{K}} \tag{4}
\]
Once we get $C_{L_{\text{minPreq}}}$, then what is the velocity for cruise at which $C_L^{3/2}/C_D$ value is maximum (for minimum power required)?

- As it is well known that, lift needs to balance weight in cruise. So,
  \[ L = \frac{1}{2} \rho V^2 S C_L = W \]

- This could also be written as
  \[ W = \frac{1}{2} \rho V^2 S C_{L_{\text{minPreq}}} \]

\[ \Rightarrow V = \sqrt{\frac{2 \left( \frac{W}{S} \right)}{\rho C_{L_{\text{minPreq}}}}} \]

- So by using the result obtained in Equation 3 the velocity for minimum power required can be shown to be:
  \[ V_{\text{minPreq}} = \sqrt[3]{\frac{2 \left( \frac{W}{S} \right)}{\rho \sqrt[6]{\frac{3 C_{D_c} K}}}} \]  \hspace{1cm} (5)

- Hence once the altitude is decided, the pilot should be instructed to trim the airplane at $V_{\text{minPreq}}$ to satisfy the minimum power required condition.

- This speed at a given altitude could easily be obtained by substituting the values of $W/S$, $C_{D_c}$, $K$ in Equation 5.