TURBOMACHINERY AERODYNAMICS

Lect- 36

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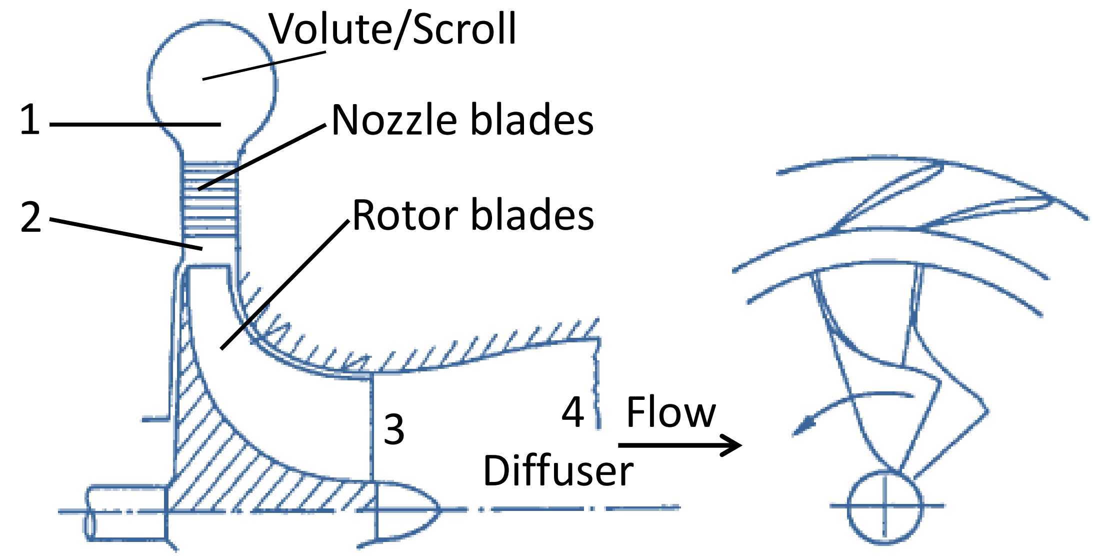
In this lecture...

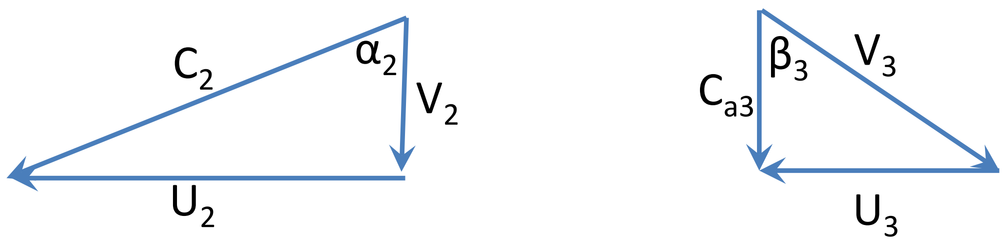
- Tutorial on radial flow turbines

Problem # 1

The rotor of an IFR turbine, which is designed to operate at the nominal condition, is 23.76 cm in diameter and rotates at 38,140 rpm. At the design point the absolute flow angle at rotor entry is 72^o. The rotor mean exit diameter is one half of the rotor diameter and the relative velocity at rotor exit is twice the relative velocity at rotor inlet. Determine the specific work done.

Solution: Problem # 1





90° IFR turbine arrangement and velocity triangles

Solution: Problem # 1

The blade tip speed is

$U_2 = \pi N D_2/60 = \pi \times 38{,}140 \times 0.2376/60$

$= 474.5$ m/s

Since $V_2 = U_2 \cot\alpha_2 = 154.17$ m/s

and $C_2 = U_2 \sin\alpha_2 = 498.9$ m/s

$C_3^2 = V_3^2 - U_3^2 = (2 \times 154.17)^2 - (0.5 \times 474.5)^2$

$= 38{,}786\ m^2/s^2$

Hence, $(U_2^2 - U_3^2) = U_2^2(1 - 1/4) = 168{,}863\ m^2/s^2$

$(V_3^2 - V_2^2) = 3 \times V_2^2 = 71{,}305\ m^2/s^2$ and

$(C_2^2 - C_3^2) = 210{,}115\ m^2/s^2$

Solution: Problem # 1

We can sum up the three terms and divide by 2 to get the specific work as

$$\Delta W = 225,142\ m^2/s^2$$

The fractional contributions of each of the three terms to the work output is 0.375 for U^2, 0.158 for V^2 and 0.467 for C^2.

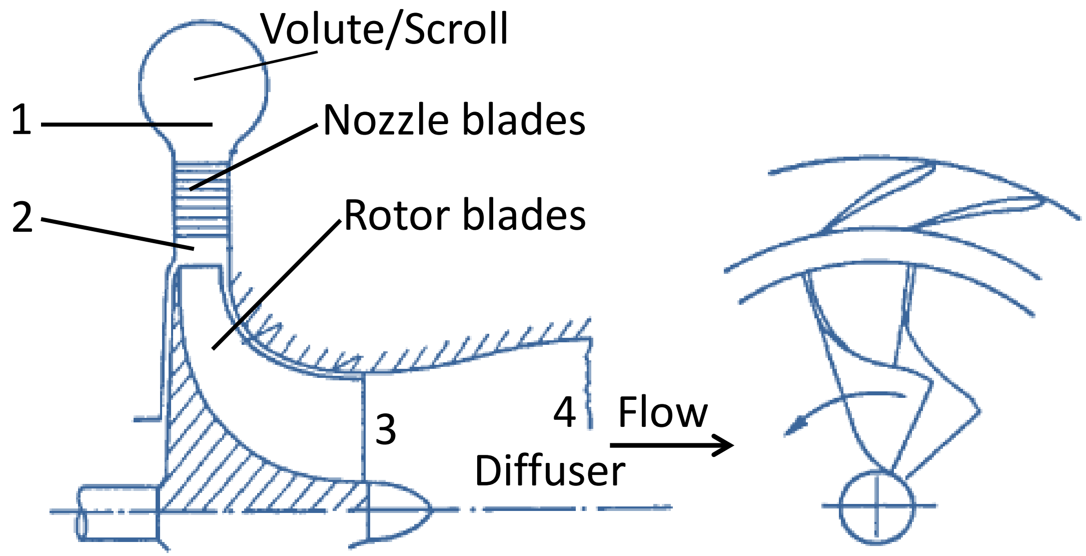
We can also calculate the specific work by

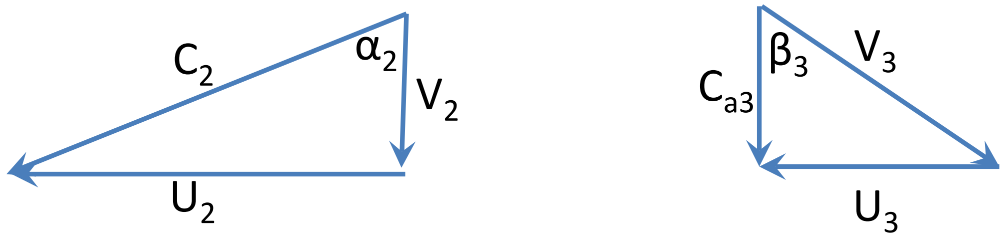
$$\Delta W = U_2^2 = 474.5^2 = 225,150\ m^2/s^2$$

Problem # 2

- A radial inflow turbine develops 60 kW power when running at 60,000 rpm. The pressure ratio (P_{01}/P_3)of the turbine is 2.0. The inlet total temperature is 1200 K. The rotor has an inlet tip diameter of 12 cm and an exit tip diameter of 7.5 cm. The hub-tip ratio at exit is 0.3. The mass flow rate is 0.35 kg/s. The nozzle angle is 70^o and the rotor exit blade angle is 40^o. If the nozzle loss coefficient is 0.07, determine the total-to-static efficiency of the turbine and the rotor loss coefficient.

Solution: Problem # 2





90º IFR turbine arrangement and velocity triangles

Solution: Problem # 2

- The rotor tip rotational speed is

  $U_2 = \pi D_2 N/60 = 377$ m/s

- From the velocity triangle at the rotor inlet, $\beta_2 = 0$, therefore,

  $\sin \alpha_2 = U_2/C_2$

  $C_2 = U_2 \operatorname{cosec} \alpha_2 = 401.185$ m/s

  $T_2 = T_{02} - (C_2^2/2c_p) = 1130$ K

- To calculate the stagnation temperature drop (isentropic) across the turbine, we shall use the pressure ratio.

Solution: Problem # 2

- The stagnation temperature drop

$$T_{01} - T_{3s} = T_{01}\left[1 - \frac{T_{3s}}{T_{01}}\right]$$

$$= T_{01}\left[1 - \left(\frac{P_3}{P_{01}}\right)^{(\gamma-1)/\gamma}\right] = 190.92 \text{ K}$$

The turbine power is

$$P = \dot{m}\, c_p(T_{01} - T_{03})$$

Hence, $(T_{01} - T_{03})$=60000/(0.35x1.148)

=149.33 K

Solution: Problem # 2

- The total-to-static efficiency is,

$$\eta_{ts} = (T_{01} - T_{03})/(T_{01} - T_{3s})$$

$$=149.33/190.92= 0.782$$

- The radius ratio r_3/r_2 is

$\frac{r_3}{r_2} = \frac{d_{3h}+d_{3s}}{2d_2} = \frac{\xi d_{3s}+d_{3s}}{2d_2}$, where, ξ is the hub-tip ratio.

- Substituting the values, $r_3/r_2=0.406$

Solution: Problem # 2

- We have seen than the total-to-static efficiency can be derived as

$$\eta_{ts} = \left[1 + \tfrac{1}{2}\left\{\zeta_N \frac{T_3}{T_2} \mathrm{cosec}^2\alpha_2 + \left(\frac{r_3}{r_2}\right)^2 (\zeta_R \mathrm{cosec}^2\beta_3 + \cot^2\beta_3)\right\}\right]^{-1}$$

- Here, T_3/T_2 can be defined in terms of

$$\frac{T_3}{T_2} = 1 - \frac{U_2^2}{2c_pT_2}\left[1 + \left(\frac{r_3}{r_2}\right)^2\{(1+\zeta_R)cosec^2\beta_3 - 1\}cot^2\alpha_2\right]$$

- Therefore, $\frac{T_3}{T_2} = 0.9396 - 0.02187\,\zeta_R$
- Substituting this in the above equation for efficiency, we get the rotor loss coefficient as $\zeta_R = 0.62$

Problem # 3

An IFR turbine with 12 vanes is required to develop 230 kW at an inlet stagnation temperature of 1050 K and a flow rate of 1 kg/s. Using the optimum efficiency design method and assuming a total-to-static efficiency of 0.81, determine (i) the absolute and relative flow angles at rotor inlet; (ii) the overall pressure ratio, P_{01}/P_3; (iii) the rotor tip speed and the inlet absolute Mach number.

Solution: Problem # 3

For optimum design, we use the Whitfield's equation, $\cos^2\alpha_2 = 1/N$, where, N is the number of vanes.

For 12 vanes, $\alpha_2 = 73.22^o$

As a consequence of the Whitfield's equation,

$$\beta_2 = 2(90 - \alpha_2) = 33.56^o$$

Solution: Problem # 3

The relation between the pressure ratio and the total-to-static efficiency is,

$$\frac{P_3}{P_{01}} = \left(1 - \frac{\Delta W}{c_p T_{01} \eta_{ts}}\right)$$

$$=0.32165$$

Or, $P_{01}/P_3=3.109$

To determine the absolute Mach number at the inlet, let us first determine the Mach number corresponding the stagnation conditions.

Solution: Problem # 3

$$M_{02}^2 = \left(\frac{\Delta W}{\gamma - 1}\right)\frac{2cos\beta_2}{1 + cos\beta_2}$$

Substituting, M_{02}=0.7389

The absolute Mach number based on static conditions, M_2 is related to M_{02} by

$$M_2^2 = \frac{M_{02}^2}{1 - \frac{1}{2}(\gamma - 1)M_{02}^2}$$

Therefore, M_2=0.775

Solution: Problem # 3

We have seen that

$$\frac{\Delta W}{c_p T_{01}} = (\gamma - 1)cos\beta_2 \left(\frac{U_2^2}{a_{01}^2}\right)$$

Here, $a_{01} = \sqrt{\gamma R T_{01}}$=633.8 m/s assuming T_{01}=T_{02}

Therefore, substituting the values, the tip speed is

$$U_2 = 538.1 \text{ m/s}$$

Problem # 4

Compare the specific power output of axial and radial turbines in the following caseL

Axial turbine : $\alpha_2 = \beta_3 = 60^o$ and $\alpha_3 = \beta_2 = 0^o$

Radial turbine: $\alpha_2 = 60^o$ and $\beta_3 = \alpha_3 = \beta_2 = 0^o$

The rotational speed is the same in both the cases.

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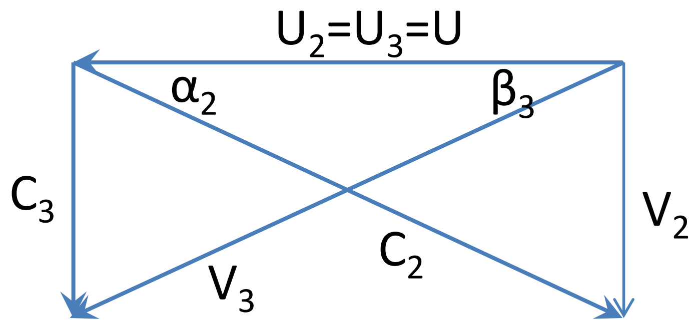
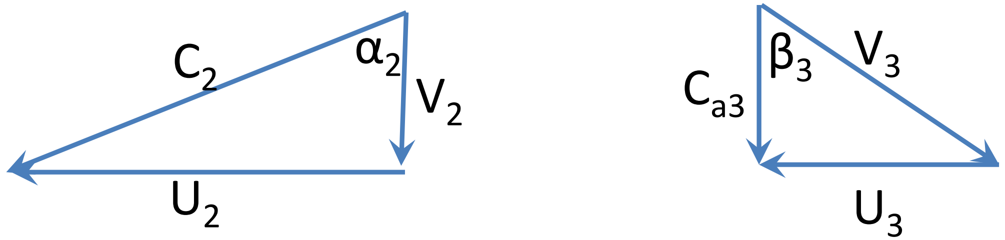
Solution: Problem # 4



Axial turbine



Radial turbine

Solution: Problem # 4

Axial flow turbine:

Since $\alpha_2=\beta_3=60^o$ and $\alpha_3=\beta_2=0^o$

The specific work is

$$\Delta W_{axial}= U(C_{w2}+C_{w3})=U^2$$

Radial flow turbine:

$$\alpha_2=60^o \text{ and } \beta_3=\alpha_3=\beta_2=0^o$$

The specific work is

$$\Delta W_{radial}= U_2C_{w2}-U_3C_{w3})= U_2U_2-U_3x0$$
$$=U_2^2$$

Therefore, the specific work done in both the turbine configurations are the same, given the conditions of operation.