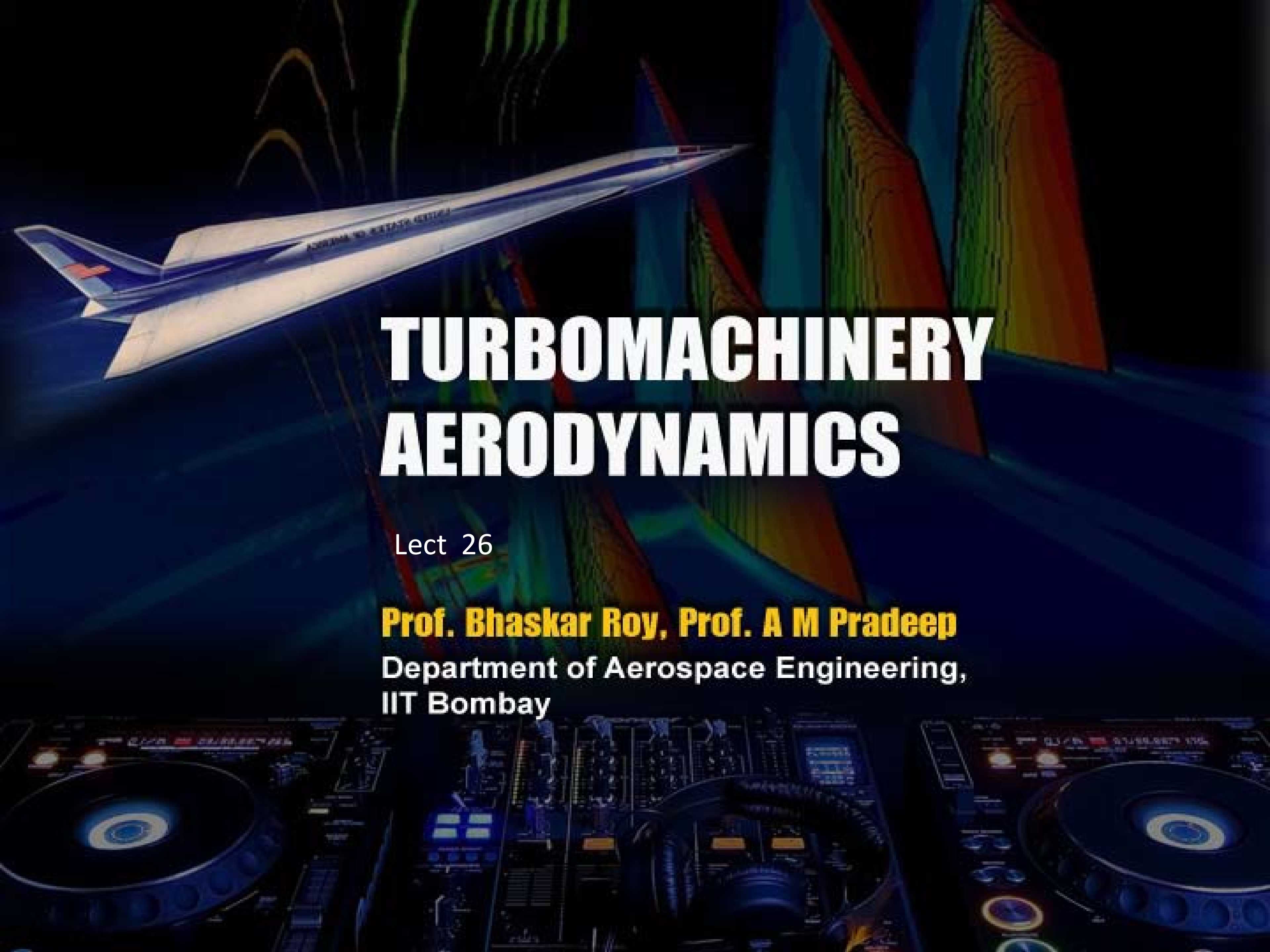

TURBOMACHINERY AERODYNAMICS

Lect 26

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Tutorial - 4

Solved Problems

And

Exercise Problems

On

3-D flows in Axial Flow Turbine

Example 1.

Following data apply to a constant nozzle exit angle (α_2) axial turbine design :
Temp. drop, $\Delta T = 150$ K; at hub $U_{2h} = 300$ m/s ; at tip $U_{2t} = 400$ m/s ; $\alpha_2 = 60$; $\alpha_3 = 0$; and Radius ratio given is, $r_h / r_t = 0.75$

(a) Complete the design velocity diagrams at hub, mean and tip of the stage
(b) Calculate the velocity components if the design is free vortex for the turbine and compare the values with (a)

Solution 1 :

At the rotor inlet station we know,

$$\frac{C_{w2}}{C_{w2m}} = \frac{C_{a2}}{C_{a2m}} = \frac{C_2}{C_{2m}} = \left(\frac{r}{r_m}\right)^{\sin^2\alpha_2}$$

And, at the rotor exit

$$C_{a3}^2 = C_{a3m}^2 + 2U_m C_{w2m}\left[1 - \left(\frac{r}{r_m}\right)^{\cos^2\alpha_2}\right]$$

and

$r_m / r_t = 0.875$, and $r_m / r_h = 1.166$

Work done by the rotor is given by (for $\alpha_3 = 0$)

$$U\,(C_{w2} + C_{w3}) = \Delta H_0 = c_p.\Delta T = U_m.C_{w2m}$$

From which we can write $C_{w2m} = 492$ m/s

$C_{a2m} = C_{w2m} \cot\alpha_2 = 284$ m/s $= C_{a3m}$

At the rotor hub inlet

$$C_{a2h} = C_{a2m}\left(\frac{r_m}{r}\right)^{\sin^2\alpha_2} = 318.8 \text{ m/s}$$

$$C_{w2h} = C_{w2m}\left(\frac{r_m}{r}\right)^{\sin^2\alpha_2} = 552.2 \text{ m/s}$$

At the rotor tip inlet

$$c_{a2t} = c_{a2m}\left(\frac{r_m}{r}\right)^{\sin^2 \alpha_2} = 257 \text{ m/s}$$

$$c_{w2t} = c_{w2m}\left(\frac{r_m}{r}\right)^{\sin^2 \alpha_2} = 447 \text{ m/s}$$

At the rotor tip outlet

$$C_{a3} = \sqrt{C_{a3m}^{2} + 2U_m C_{w2m}\left[1 - \left(\frac{r}{r_m}\right)^{\cos^2\alpha_2}\right]}$$

From which we can calculate the axial velocities,

C_{a3t} = 262 m/s

C_{a3h} = 306 m/s; C_{w3} = is constant radially

(b) Free vortex stage design and comparison

For Free vortex design we have established in the last lecture

$$C_{w3}.r = \text{constant},\ C_{a3} = \text{const} = C_{a2}$$

	Constant Nozzle	Free Vortex
C_{a2h}	318.8	284
C_{a2m}	284	284
C_{a2t}	257	284
C_{w2h}	552	574
C_{w2m}	492	492
C_{w2m}	447	430
C_{a3h}	306	284
C_{a3m}	284	284
C_{a3t}	262.6	284

Example 2

It is proposed that for design of an axial flow turbine two design methods are to be explored :

A) $C_{w2m} = C_{w2h} = C_{w2t}$

B) $C_{a2t} = C_{a2h}\left(\frac{r_h}{r_t}\right)^{\sin^2 \alpha_2}$ and,

c) $C_{w2t} / C_{w2h} = r_h / r_t$

Common design data prescribed are: $C_{am} = 200$ m/s ; $\alpha_2 = 60$; $\alpha_3 = 0$; $R_x = 0.5$; and $r_h / r_t = 0.8$

Complete the velocity diagrams for all the cases.

Solution 2 :

From the prescribed data :
One can calculate that: $r_m / r_t = 0.889$; $r_t / r_m = 1.11$

$C_{w2m} = C_{a2m} \times \tan\alpha_2 = 346.5$ m/s ; and $C_{w3m} = 0$

For all the cases, $R_x = 0.5$ is prescribed at mean
Hence, from symmetrical blading concept
$\alpha_{2m} = \beta_{3m} = 60^0$; $\alpha_{3m} = \beta_{2m} = 0^0$

Also, $U_m = C_{w2m} = 346.5$ m/s and hence at any radius, $U_h = 308$ m/s ; $U_t = 385$ m/s

For Case (A)

This is a fluid behaving like a 'solid body' case for which n = 0 in the equation **$C_w = r^n$**

The axial speed is calculated from the axial velocity expression derived from the energy equation for the case n=0

$$C_{a2} = C_{a2m}\sqrt{1-2\tan^2\alpha_{2m}\ln\left(\frac{r}{r_m}\right)}$$

All the angles across the rotor may be also calculated from above

Tabulated results of Case A

	C_{a2}	C_{a3}	C_{w2}	C_{w3}	α_2	α_3	β_2	β_3
Hub	261.3	200	346.5	0	53	0	8.4	57
Mean	200	200	346.5	0	60	0	0	60
Tip	121.3	200	346.5	0	70.7	0	-17	62.5

Case (B)

Prescribed condition is $C_{a2t} = C_{a2h}\left(\dfrac{r_h}{r_t}\right)^{\sin^2 \alpha_2}$

Which essentially means : $\dfrac{C_{a2t}}{C_{a2h}} = \dfrac{C_{a2t}}{C_{a2m}} = \dfrac{C_{a2m}}{C_{2h}}$

For constant nozzle angle: $C_{a2} = C_{a2m}\left(\dfrac{r_m}{r}\right)^{\sin^2 \alpha_2}$

$$C_{w2} = C_{w2m}\left(\frac{r_m}{r}\right)^{\sin^2 \alpha_2} ;\quad C_2 = C_{2m}\left(\frac{r_m}{r}\right)^{\sin^2 \alpha_2}$$

At station 3, exit of the rotor,

$\alpha_3 = 0$; $C_{w3} = 0$

And the expression for axial velocity is

$$C_{a3}^2 = C_{a3m}^2 + 2U_m C_{w2m}\left[1 - \left(\frac{r}{r_m}\right)^{\cos^2\alpha_2}\right]$$

Tabulated results of Case B

	C_{a2}	C_{a3}	C_{w2}	C_{w3}	α_2	α_3	β_2	β_3
Hub	**218.5**	**216.7**	**378.5**	**0**	**60**	**0**	**17.9**	**54**
Mean	**200**	**200**	**346.5**	**0**	**60**	**0**	**0**	**60**
Tip	**185**	**183.3**	**320**	**0**	**60**	**0**	**-19.4**	**69**

For Case (C)

Since $C_{w2t}/ C_{w2h} = r_h / r_t$ – this is Free Vortex law

Same may be applied at rotor outlet also :

$C_{a2} = \text{const} = C_{a3}$ at mean radius

The results are summarized in the table :

Tabulated results of Case C

	C_{a2}	C_{a3}	C_{w2}	C_{w3}	α_2	α_3	β_2	β_3
Hub	200	200	389.7	0	62.8	0	22.25	57
Mean	200	200	346.5	0	60	0	0	60
Tip	200	200	311.8	0	57.3	0	-20.8	62.55

All Three cases compared : Design velocity diagrams

Stn	Case	C_{a2}	C_{a3}	C_{w2}	C_{w3}	α_2	α_3	β_2	β_3
Hub	A	261.3	200	346.5	0	53	0	8.4	57
Hub	B	218.5	216.7	378.5	0	60	0	17.9	54
Hub	C	200	200	389.7	0	62.8	0	22.25	57
Mean	A	200	200	346.5	0	60	0	0	60
Mean	B	200	200	346.5	0	60	0	0	60
Mean	C	200	200	346.5	0	60	0	0	60
Tip	A	121.3	200	346.5	0	70.7	0	-17	62.5
Tip	B	185	183.3	320	0	60	0	-19.4	69
Tip	C	200	200	311.8	0	57.3	0	-20.8	62.55

Next Lecture -----

Turbine blade cooling