



Jet Aircraft Propulsion

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Lecture 41

Lecture with Numerical Examples of Ramjet, Pulsejet and Scramjet

Problem-1 Ramjet

A ramjet is flying at Mach 1.818 at an altitude 16.750 km (Pa = 9.122 kPa, Ta = -56.5°C = 216.5 K., sonic speed, a = 295 m/s). The flow is assumed to enter the intake of the ramjet through a normal shock standing at the intake face. No pre-entry loss or friction loss inside the engine is assumed to exist. Combustion delivery temperature is 1280 K. and the fuel –air ratio is 1:40. The area at the intake face is $A_1 = 0.0929 \text{ m}^2$ and at the Combustion chamber , $A_3 = 0.1858 \text{ m}^2$

Calculate :

- i) Mass flow rate through the engine
- ii) Throat area in the nozzle, A_5
- iii) Combustion related pressure drop in the combustion chamber
- iv) If the nozzle expands to ambient pressure – find the thrust produced
- v) If the nozzle expands only in a convergent nozzle – find the thrust produced
- vi) Calculate the propulsive efficiencies for (iv) and (v)
- vii) Calculate TSFC in both the cases
- viii) Complete and draw the cycles for the cases –
(a) with C-D nozzle and (b) Convergent nozzle

Solution :

Flight velocity=Intake velocity of air= $M_1 \cdot a = 536$ m/s

From isentropic relations,

Total temperature at entry, $T_{0a} = 360$ K

Total Pressure at entry, $P_{0a} = 53.85$ kPa

Ambient air density, $\rho_1 = P_a/R \cdot T_a = 0.147$ kg/m³

Mass flow through the Intake = $\rho_1 \cdot V_a \cdot A_1 = 7.3$ kg/s

Normal shock : From shock tables :

At intake face, for $M_1 = 1.82$, $M_2 = 0.612$, $T_2 = 334.8$ K

$P_{02} = 0.803 \cdot P_{01} = 43.25$ kPa, $T_{02} = T_{01} = 360$ K

Since the duct losses due to friction etc are zero,

$P_{03} = P_{02} = 43.25$ kPa, $T_{03} = T_{02} = 360$ K

Using isentropic tables, at stn 2 behind the shock,
 $M_2 = 0.6121$, the area may be computed from

$$A_2/A_1 = 1.16565;$$

$$\text{Then , } A_{cc}/A_1 = (A_{cc}/A_2).(A_2/A_1) = 2 \times 1.16565 = 2.33$$

Inside the combustion chamber, the Mach number may be computed from isentropic tables or relations as : $M_3 = 0.26$

Combustion chamber calculations :

Due to accompanying heat addition, **Rayleigh flow** tables or relations need to be utilized for calculation of parametric variations.

Flow in the C-D nozzle may be assessed as :

Starting from Mach 0.83 (CC delivery) assuming isentropic flow, $A_4/A_t=1.027$ in the convergent duct

The exit area may be calculated from

$$A_t = A_4 / (A_4/A_t) = 0.1858 / 1.02696 = 0.181 \text{ m}^2$$

As no duct loss is prescribed, $P_{0t} = P_{04}$, $P_t = 0.25 \times P_{04}$

At the exit, after flow through the divergent duct,

$$M_e = 1.55 ; A_t/A_e = 1.211, T_e/T_{0e} = 0.675$$

$$A_e = 1.211 \times 0.181 = 0.22 \text{ m}^2, P_e = P_a \text{ as prescribed}$$

From $T_{0t} = T_{0e} = T_{04}$, as no heat / work is transacted

Since, $T_{0t} = T_{0e} = T_{04} = 1280 \text{ K}$, then $\overline{T_e} = 864.5 \text{ K}$

Jet Vel is now calculated $V_e = M_e \sqrt{\gamma \cdot R \cdot T_e} = 916.5 \text{ m/s}$

For fuel-air ratio of 1:40 thrust is calculated as :

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Jet Vel is now calculated $V_e = M_e \sqrt{\gamma \cdot R \cdot T_e} = 916.5 \text{ m/s}$

For a fuel-air ratio of 1:40 the thrust may now be calculated as :

Thrust is given by : $F = \dot{m}[(1+f)V_e - V_1] + A_e(P_e - P_a)$

C-D nozzle Thrust is, $F = 4090 \text{ N}$

Propulsive Efficiency,

$$\begin{aligned}\eta_p &= F.V_a / F.V_a + \frac{1}{2}.\dot{m}.[(1+f)V_e^2 - V_a^2] \\ &= 51.2 \%\end{aligned}$$

For fuel-air ratio prescribed as $f = 1/40$

Sp. Fuel Consn., TSFC = $f/(F/\dot{m}) = 0.16 \text{ kg/N-hr}$

If the nozzle is only a convergent nozzle:
The exit face is the throat of the nozzle.

convergent nozzle

Pressure ratio necessary for choking : 1.893

Available pr. ratio across the nozzle = $P_{04}/P_a = 3.96$

The nozzle is choked, and exit pressure, $P_e = 19.1$ kPa

$T_e = T_t = T_{04}/[(\gamma+1)/2] = 1280/1.2 = 1067$ K

Exit jet velocity, $V_e = V_t = \sqrt{\gamma \cdot R \cdot T_e} = 654$ m/s

From isentropic tables, exit (throat) area, $A_4/A_t = 1.027$

Whence, exit area $A_e = 0.1809$ m²

Thrust $F = \dot{m}[(1+f)V_e - V_1] + A_e(P_e - P_a) = 3587$ N

Sp. Fuel Consn., TSFC = $f/(F/\dot{m}) = 0.1834$ N

Prop Efficiency $\eta_p = F \cdot V_a / F \cdot V_a + \frac{1}{2} \dot{m} \cdot [(1+f)V_e^2 - V_a^2] = 54.8\%$

Problem 2 – Pulsejet

An aircraft powered by a pulsejet engine is flying at 12 km altitude, at Mach 2. The engine parameters are given as : inlet area = 0.084 m². Combustion chamber pressure development, $P_{03}/P_{02} = 9.0$, heating value of fuel, $Q = 43,000$ kJ/kg, combustion efficiency = 0.96. Assuming ideal (no loss) flow through the intake, find:

- (i) The air mass flow rate,
- (ii) Maximum Temp.
- (iii) fuel-air ratio, f
- (iv) Exit velocity, V_e
- (v) Thrust of the engine
- (vi) TSFC

At 12 km $P_{0a} = 18.75$ kPa ; $T_{0a} = 216.65$ K

The flight velocity is : $V_a = M_a \sqrt{\gamma \cdot R \cdot T_a} = 590$ m/s

The air mass flow rate is : $= \rho_a \cdot V_a \cdot A_1 = 15$ kg/s

Total temp across the intake diffuser remains

constant, $T_{01} = T_{0a} = T_a [1 + (\gamma - 1) M^2 / 2] = 390$ K = T_{02}

Total pressure, $P_{01} = P_{0a} = P_a (T_{01} / T_{0a})^{\gamma / (\gamma - 1)} = 147$ kPa

In the combustion chamber pressure rises from P_{02} to P_{03} by a prescribed ratio 9.0, and the temp ratio (temp change across the CC) is same by gas laws.

The combustion delivery temp is : $T_{03} = 3510$ K

The fuel air ratio may be calculated

$$f = \dot{m}_f / \dot{m}_a = (c_{p\text{-gas}} \cdot T_{03} - c_{p\text{-air}} \cdot T_{02}) / Q \cdot \eta_{cc} = 0.0975$$

In the jet pipe:

$$\begin{aligned} \text{The exhaust velocity is } V_e &= \sqrt{2 \cdot c_{p\text{-gas}} \cdot T_{03} \cdot [1 - 1/9^{0.25}]} \\ &= 2297 \text{ m/s} \end{aligned}$$

$$\text{Specific Thrust, } F/\dot{m}_a = (1 + f)V_e - V_a = 1931.5 \text{ N-s/kg}$$

$$\text{The thrust. } F = 28950 \text{ N} = 28.95 \text{ kN}$$

$$\text{The TSFC} = f/(F/\dot{m}_a) = 50 \text{ mg/N-s} = 0.180 \text{ kg/N-hr}$$

$$\begin{aligned} \text{Propulsive Efficiency } \eta_p &= F \cdot V_a / (F \cdot V_a + \frac{1}{2} \dot{m} \cdot [(1 + f)V_e^2 - V_a^2]) \\ &= 0.418 = 41.8\% \end{aligned}$$

Problem 3 – Scramjet

A scramjet powered aircraft flies at Mach 5 at 16.75 km where $T_a = 216.67$ K and $P_a = 9.122$ kPa. The intake has a shock structure of two oblique shocks with both deflection angles $\delta = 10^\circ$. By burning hydrogen fuel ($Q = 120,900$ kJ/kg), the temp is raised to 2000 K. The fuel air ratio = 0.025. The nozzle expansion ratio is $A_5/A_4 = 5.0$. The inlet and the exit areas are $A_1 = A_5 = 0.2$ m². If $c_p = 1.51$ kJ/kg.K ; $\eta_{cc} = 0.8$

Calculate :

- i) Mach number at combustion chamber inlet
- ii) Exhaust jet velocity
- iii) Overall efficiency

Flight velocity is : $V_a = M_a \sqrt{\gamma \cdot R \cdot T_a} = 1475 \text{ m/s}$

Mass flow through the engine $\dot{m}_a = \rho_a \cdot V_a \cdot A_1 = 43.3 \text{ kg/s}$

Inlet total temp $T_{01} = T_{0a} = T_a [1 + (\gamma - 1) M^2 / 2] = 1300 \text{ K} = T_{02}$

From shock relations or tables,

Across the first shock , for $M_1 = 5$ & $\delta = 10^\circ$

Shock angle, $\beta = 19.4^\circ$. $M_2 = 4.0$ and $T_2/T_1 = 1.429$

Across the second shock, $M_1 = 4$ & $\delta = 10^\circ$

Shock angle, $\beta = 22.2^\circ$. $M_3 = 3.3$ and $T_3/T_2 = 1.33$

In the combustion chamber heat is added to air flow with supersonic speed

Using Rayleigh Flow relations (or tables)

$$M_4 = 1.26, \text{ and } P_{04}/P_{01} = 1.033, T_{04}/T_{01} = 0.966$$

Combustion chamber pressure ratio, $P_{04}/P_{03} = 0.228$

$$\begin{aligned} \text{Fuel-air ratio, } f = \dot{m}_f / \dot{m}_a &= (c_{p\text{-gas}} \cdot T_{03} - c_{p\text{-air}} \cdot T_{02}) / Q \cdot \eta_{cc} \\ &= 0.01093 \end{aligned}$$

Nozzle Flow

For $M_4 = 1.26$, $T_{04}/T_4 = 1.317$, critical area ratio = 1.05

Whence, $A_e/A_t = (A_e/A_4) \cdot (A_4/A_t) = 5 \times 1.05 = 5.25$

Nozzle Flow

Nozzle outlet Mach number , $M_e = 3.23$,
for which isentropic temp ratio $T_{05}/T_5 = 3.11$

$$T_5 = \frac{T_5 \cdot T_{05} \cdot T_{04} \cdot T_4 \cdot T_3 \cdot T_2}{T_{05} \cdot T_{04} \cdot T_4 \cdot T_3 \cdot T_2 \cdot T_1} \cdot T_1 = 654.5 \text{ K}$$

The exhaust velocity , $V_e = M_5 \sqrt{\gamma \cdot R \cdot T_e} = 1560 \text{ m/s}$

Specific Thrust, $F/\dot{m}_a = (1+f)V_e - V_a = 102 \text{ N-s/kg}$

TSFC = $107.15 \text{ mg/N-s} = 0.385 \text{ kg/N-hr}$

Thrust , $F = 43.3 \times 102 = 4416 \text{ N}$

Propulsive Efficiency $\eta_p = F \cdot V_a / F \cdot V_a + \frac{1}{2} \cdot \dot{m} \cdot [(1+f)V_e^2 - V_a^2]$
 $= 0.527 \text{ or } 52.7\%$

Comparison of Rotor-less Jet Engines

	M	H km	V_e m/s	F (kN)	TSFC Kg/n- hr	η_{pr}	F/\dot{m}_a N/Kg /s
Ramjet (C-D noz)	1.8	16.7	916	4.09	0.16	51.2	560
Pulsejet	2	12	2297	28.95	0.18	41.8	1931
Scramjet	5	16.7	1560	4.41	0.38	52.7	102

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Concluding Lecture