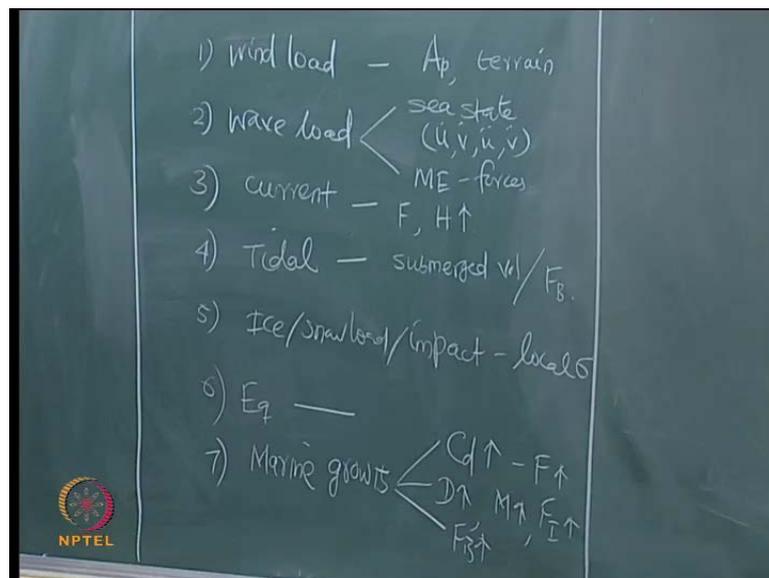


**Dynamics of Ocean Structures**  
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**Module - 1**  
**Lecture - 8**  
**Introduction to Structural Dynamics**

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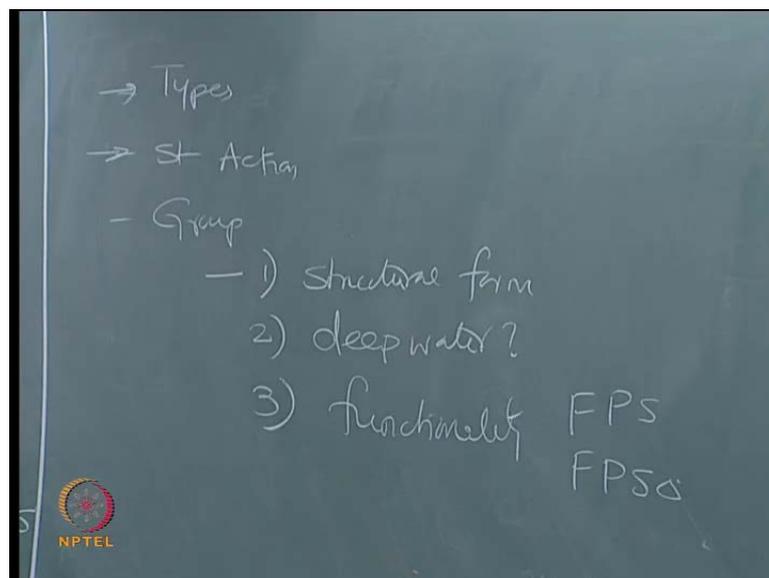


In the last lecture, we discussed about different kinds of loads as applied to offshore structures. We started with wind load. We discussed wave load, we discussed current load, we also discussed tidal forces. We also discussed ice or snow load or impact load, we also discussed earthquake load, we also discussed load indirectly caused because of marine growth. So, wind load depends on projected area and surface terrain. Wave load has many complications; you have got to define the sea state first. There are many theories available to design or define the sea state. We can use simple Airy's linear wave theory or you can use higher order wave theories to define the sea state. The moment the sea state is defined and identified, you will find out  $u \dot{\phantom{u}}$ ,  $v \dot{\phantom{v}}$ ,  $u \ddot{\phantom{u}}$ ,  $v \ddot{\phantom{v}}$ , which are water particle kinematics generated because of the sea state.

Then, you will use Morrison equation to compute the forces on any individual moment. Current will quantify the variation in the forces, the force or the wave elevation may increase. Tidal will include the submerged volume of the body, which will affect the

buoyancy force, which you want to compute. Impact load will cause local stresses on the member. So, we should estimate them. Earthquake loads are considered as part of the environmental load in offshore structures. Marine growth increases  $C_D$  because of roughness, which will increase the force because of  $C_D$  increase. It will improve the diameter, which will cause mass, which will increase the inertia force in the member. It will also increase the submerged volume, and therefore the buoyancy force. So, marine growth has got indirect effects on structural analysis.

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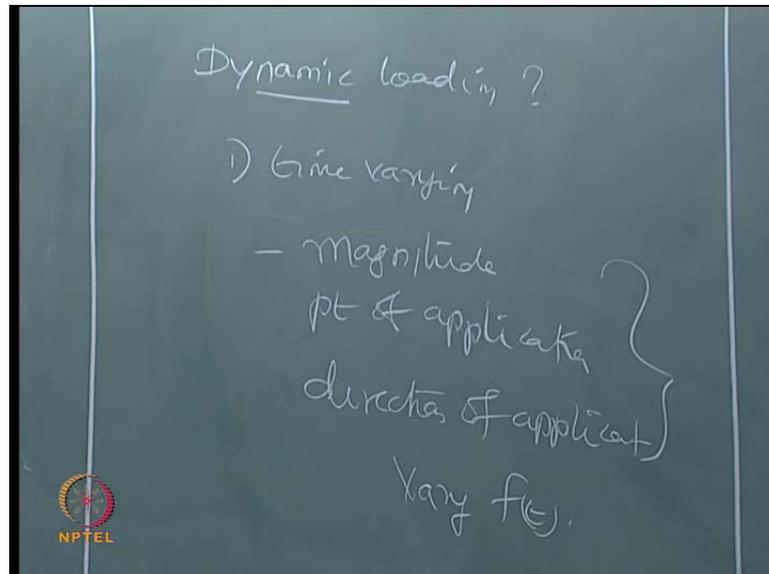


So, in the last lectures, we have discussed about different types of offshore platforms, and their structural action. I can also group them depending upon, 1, their structural form; 2, depending upon at water depth where they are fixed and 3, of course, depending upon the functionality. For example, FPS, FPSO, FSO, etcetera. Depending upon the functionality also you can change. But, all these loads what we saw, environmental loads are common to all these groups of structures. There is no isolation or there is no exemption of these forces acting on any one of the group. It means, I do not select the form or a type of structure of an offshore platform to avoid any kind of forces. This is a very interesting statement.

Generally, if you want to avoid any specific type of action on a specific component or any specific form of a number, you design the structural design like that, whereas in offshore structures, the form selection does not depend on isolation or avoidance of any

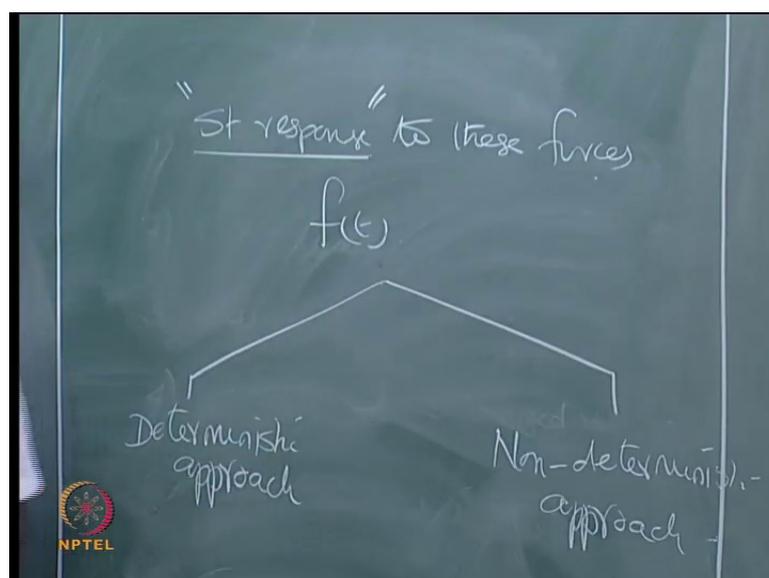
of these forces. All of these forces will be predominantly present and will act on all types of offshore and coastal structures.

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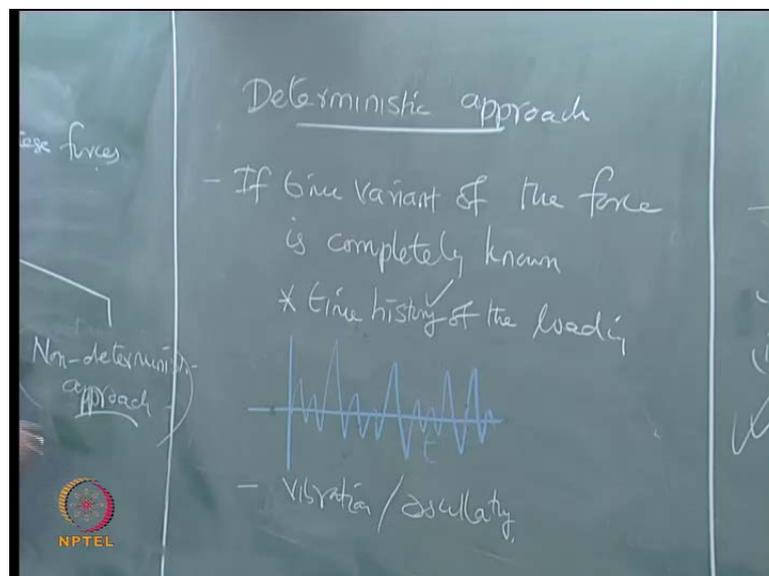
Now importantly, the moment I say action of forces on structures, we will talk about dynamic loading. What is a dynamic loading? When we will say the loading is dynamical actually? One, it is time varying. It means, its magnitude, point of application, direction of application, all may vary as a function of time.

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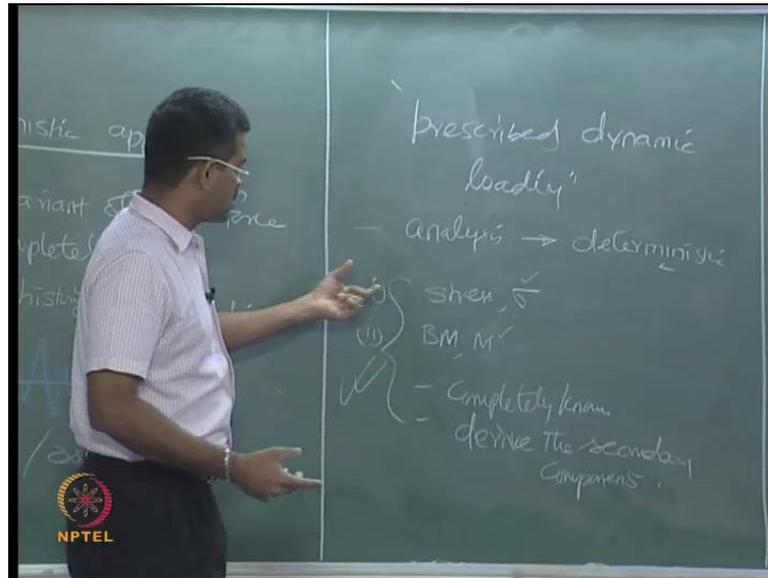
So, when we have a force, which is time variant, then the structural response to this force or these forces will also be a function of time. The moment you understand the structural response can be a function of time, I can divide this into two classifications. One is what we call deterministic approach. Another is what I call non deterministic approach. Remember, I am classifying the response to these forces as deterministic and non-deterministic. We already understand both of them are a function of  $t$ . Why because the structural response will remain a function of time because the loading applied on the structure is a function of time. So, though they are function of time, still I can classify them as deterministic and non-deterministic in terms of its approach only.

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What is then deterministic approach? If the time variant of the force is completely known. What do you understand by the time variant of the forces completely known? It means, you know the full time history of the loading. I can give an example. Let us say, I have a time history of the loading as shown here. This is time. May be, this is magnitude of the load and it varies. You know the complete quantified variation of this magnitude at any instant of time for the full history, which is applied as a function of time on the structure, whose response is also a function of time. Now, what is the difference between the loading, which looks like random in my picture. But, I still call this as deterministic because it looks like random variation. Now, though it is vibration or oscillation, oscillatory, as long as the full time history is known to me, I call this loading as not a random loading, but a different name, which is called prescribed dynamic loading.

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So, I do not call this a random load. I call this as prescribed dynamic load, because the full dynamic loading or the full time history of the loading is known to me. Once I have this prescribed dynamic loading, then the analysis what you do to find out, the action or the response of the structure for this loading will also be then deterministic. It means, you will have a close form answer of the response. That is why I call this as deterministic.

In general term, deterministic means known. Non deterministic means unknown, in general terms. Structure is determinate. The reactions of the structural support system are known to me using the standard set of equilibrium equations. Indeterminate, some of them are not known to me. With the help of standard equations, I have to solve them using some techniques. So, deterministic is related to knowledge of knowingness of the time history. Now, one can wonder why the response of a structural system on prescribed dynamic loading will become deterministic. The answer is very simple. Since the force history is completely prescribed and known to me, the response corresponding to the history will also be known to me.

Remember, importantly the force varies with time. The response will also vary with time. But, still I call them as deterministic because I know the full variation completely. That is why I call this as prescribed dynamic loading or the analysis is called deterministic approach. Now, there are many advantages of doing a deterministic

analysis. Number one, for example, let us look at the responses of the structural system encountered by these forces. What could be the common responses what we want to know from a structural system under these kinds of loads? Can you name few of them? What are these responses to be interested to know?

Let us start with simple. I want to know the stress  $\sigma$ . I want to know, for example, the bending movement, may be  $m$ , etcetera. Now, all of these values are now completely known and interestingly, one can be derived from the other. For example, if I know stress, I can find the, if I know the bending load, I can find the bending stress and so on. If I know the deflection, I can find it.

So, all of them are related. I can derive the secondary components from this analysis. So, deterministic approach has one great advantage. You may not have to arrive at all the values. If you find some of them can be determined or derived from the previous value, you can keep on doing it. This is considered as one of the advantage of deterministic approach. You may wonder, sir, why I am calling this as a deterministic approach. Why not deterministic method? Do you have any answer for this? Why I am calling this deterministic approach? Why not method? Does it look like a method or as an approach? No. We are not assuming the response determinant. We are estimating the response time history, which is completely known to me.

So, since we do not have a particular algorithm, you call this is an approach. Is it not? Is it so or it is not a method. What is the difference between method and an approach? Method is a solution technique; approach is devising that solution technique. I am looking at this problem as if it is a closed form problem. I may look at the same problem as if it is a random variable problem. It is only my view. Therefore, it is not a method. It is my view. I am looking at this as a closed form. If I look at this problem as a closed form problem, the advantages are, any derivable values can be worked out successfully.

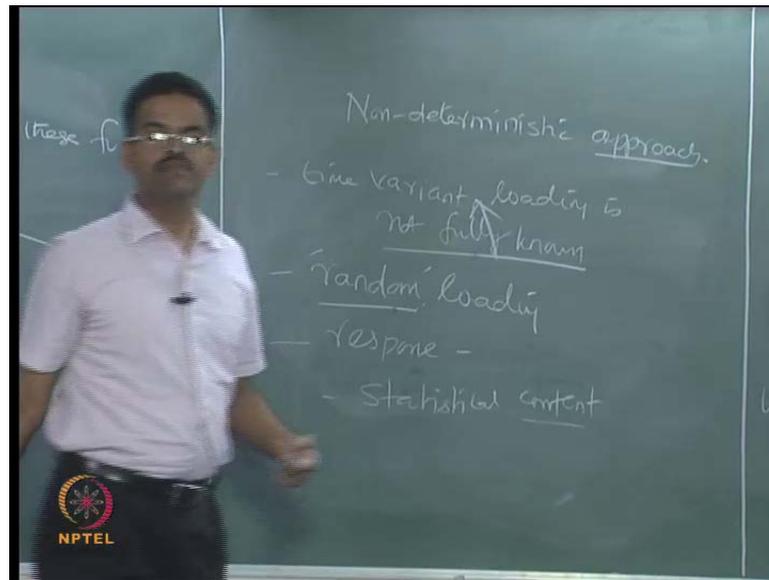
So, the analysis is slightly faster. It is faster. It is also a dynamic analysis. Why it is a dynamic analysis? You have a time varying content present in this, which is one of the important components of dynamic analysis. It is a dynamic analysis, right. But, the approach is deterministic. The structure may not be determinate. Understand the terminology very carefully. Structure may be indeterminate and the approach is deterministic because it is my view, I am looking at this problem as a close form

solution. If I looked at this problem as a closed form approach, then the advantages are, I can derive. You may wonder why this is called as a great advantage. Why I am calling them as derivable quantities as advantages in the analysis? Any answer? Fine. But, why I am calling them as, what you are answering is, why I want them to derive. Agreed, because I want to design. I need not want a bending movement or any other bending stress.

If we know the bending movement, I can find the bending stress, if we know the structural characteristics. Fine. What you are answering is, why I want to know them. What I am asking is, why I am calling them as derived quantities and what are advantages. In any method of analysis, you will not find all of them instantaneously available. Tell me any one method of analysis, where you got all the structural unknown quantities at one shot? You will not get? No method exists of that nature. You will get one set of values first and the second set can be derived from the one. Can you give me an example of any one method of analysis? Yes, Slope deflection method. Update the statistical response method techniques. But, slope deflection will give you a slope and deflection and from that, you can keep on deriving the fixed end moments or the reaction of the supports. So, first phase of values are set of values will be what you will obtain from the analysis. Remaining all will be derived from them.

So, deterministic approach will help you, to permit you to do this kind of derivability quantities easily. Therefore, the structural analysis or the dynamic analysis forming this approach is simple and easy. But, the complexities related to defining  $f$  of  $t$  and defining  $x$  of  $t$  from  $f$  of  $t$  exist. These complexities are still there. So, what does it mean is, all time variant responses you obtain, may not be obtained from non-deterministic methods. Just because you saw  $x$  of  $t$  as a response function for an  $f$  of  $t$  on a structural system, you may not clearly admit that, what method you have followed is deterministic. Remember, very interestingly in this whole discussion, we never spoke about the quantitative description of the structure, whether it is indeterminate or determinate. That is immaterial in this argument. It is only a methodology or a way, right. Then we will talk about what is non-deterministic approach. What do we do here?

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Now, in non-deterministic approach, time variant loading is not fully known. Now, there is a question here. There is confusion here. If that load history is not known, how do I do my analysis? What I mean to say is, in a given window, you pick up any one of your choice and do the analysis. You do not know what is the time history of the full stretched. It is not fully known. I am not saying it is not known at all. If you do not know this, you cannot do the analysis at all. It is not completely known. So, we call such kind of loading as random loading.

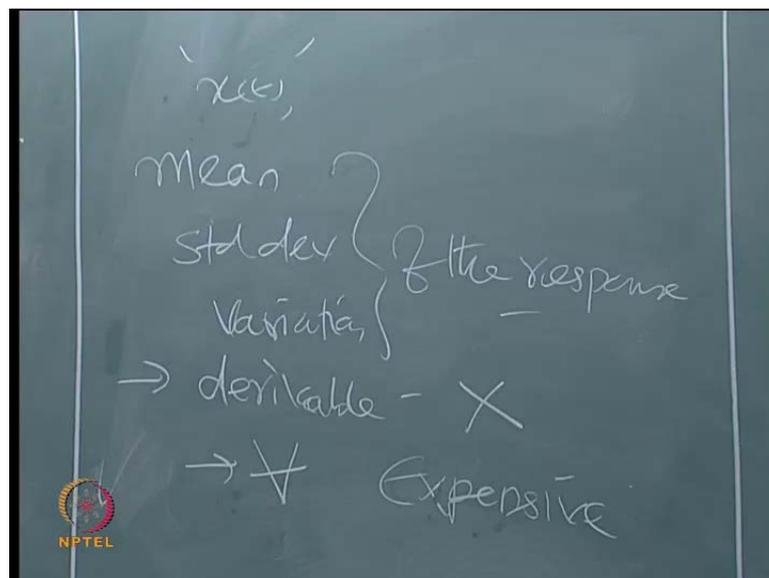
Now, there is a very interesting terminology here, random. It is very confusing. There are two things associated with randomness. People think random loading is the load which I do not know. It is not correct. Or people may say the random loading is one which keeps on varying. I do not the variation. That is not the right terminology of understanding random loading. Even a random number has a specific designated value. It does not change. From an interval of 0 to  $2\pi$ , a number of, let us say 3.157 or 2.102, which lies in this brace of 0 determines the random number. That value of the number or the magnitude of the force does not change.

Then, why it is called random? Because, if it has got a definite value, I should not call this is random. I am calling this, because I do not know the guess actually. I do not know the value. I am guessing that value, in a given set of 0 to  $2\pi$ . That is why randomness comes from the choice of you selecting the load; it means, the road is random because

you are selecting the window where you are going to the analysis and that window may vary between a b c d analyses.

This problem does not occur in a prescribed dynamic loading because that is a full complete loading history available to you. Anybody does the analysis has to follow the time history of a total, whereas, in this case it is not so. So, random loading is what we say it is not fully known. It is not a prescribed loading. In such cases, your response will also have the randomness in its representation. The moment I say randomness coming into representation, then you look for statistical content of this. I do not look for the absolute value of this. I am looking for the statistical content of this. I am not looking for the response of, let us say  $x$  of  $t$ .

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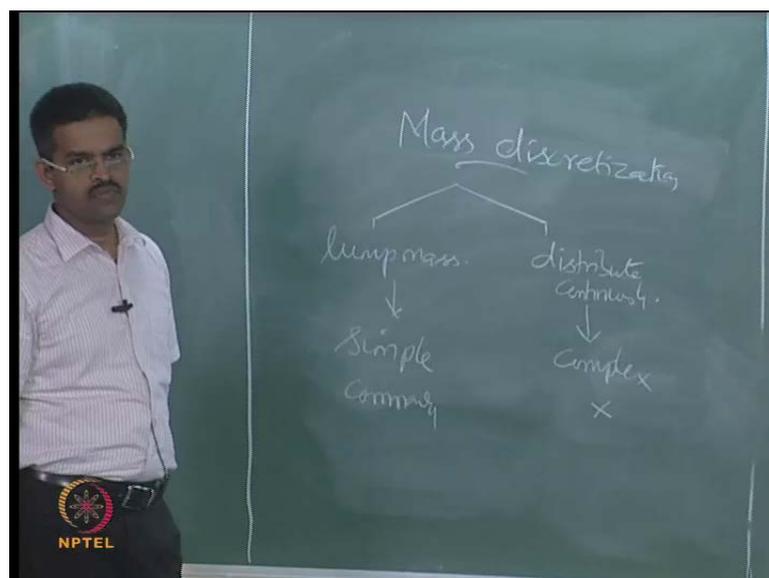


I will not look for the response of  $x$  of  $t$ . I will look for the mean, the standard deviation and the variance as the first level values of the response. Look at the statistical content of this. Depending upon the statistical content of this, I can also try to extrapolate the distributive value of this or the property of distribution of this. So, I do a statistical content of the response. Now, one of the response is, let us say displacement or deflection,  $y$  of  $t$  or  $x$  of  $t$ . From the displacement or deflection, I want to derive an additional value like rotation or from the displacement, I want to find deflection. I know I want to find the binding stress.

Let us say I want to check the binding stress. I want to derive a quantity from the displacement. If you want to do that, all the derivable phase of values cannot be done in this method. So, this method will impose a penalty on using every quantity requested by you, should be done in the same fashion as you did for the first quantity. So, this method is computationally expensive. This method becomes computationally expensive. You cannot derive a quantity from an existing quantity in this case. Is that clear?

So, that will be the fundamental difference between the non-deterministic approach of dynamic analysis to deterministic approach of dynamic analysis. Remember one important content here. Both of them talk about time, variance of the response because the loading is time variant. It is the method by which you are approaching the problem. Is that clear? Whatever complexities exist in the system; whatever complexities exist in defining  $f$  of  $t$  of the loading; whatever randomness exist and uncertainty exist in randomness of the loading prescription, all will remain there in both the methods or both the approaches. There is no doubt. So, this method is stated as computationally expensive, whereas the earlier one is not as computationally expensive as this method. There are two different techniques or approaches by which we do dynamic analysis. Now, once I start understanding the dynamic analysis, then I must also look into the other characteristic of dynamic analysis, which is inertia force.

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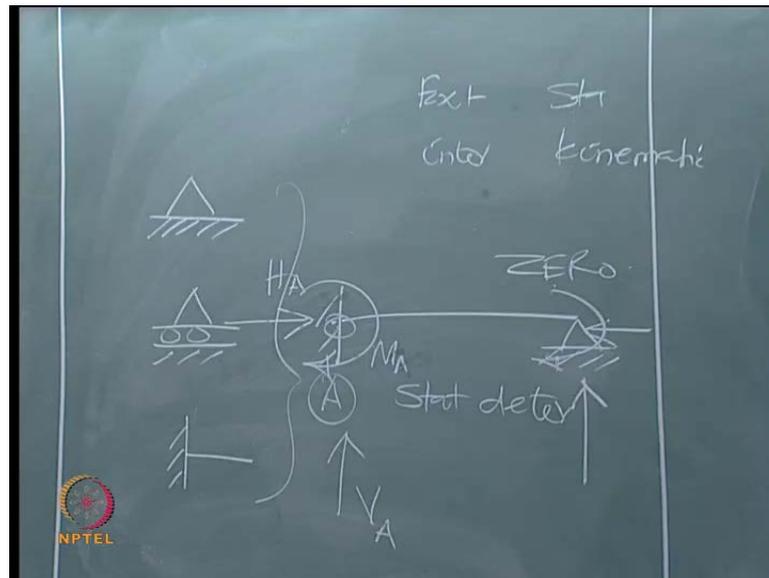
So, look at that. I must speak about the mass concentration. In a given system, I would put it as mass discretization. Because, I am going to be keen to understand, how do I study the inertia of force effects on dynamic analysis? So, when I talk about mass discretization, there are two methods here again. I can lump the mass at any point of a choice. I can also distribute the mass continuously over the entire length of the member. Both ways I can do. This is again a deviation here. I can do both ways. I can lump the mass at a specific point; I can distribute the mass continuously for the entire moment. So, I you look at the comparison quickly, this will be simple; this is complex. This is commonly followed. This is not commonly followed. Both are dynamic analysis. We are talking about the mass.

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The moment I speak about mass, then where to lump this mass, because I am interested to follow easy path. I want to do my analysis simple. Though I want to claim this is a dynamic analysis, I will follow this path, lumped mass system. When I follow this path of lumping the mass at a specific location, where will I lump them? That is important. Now, you have to lump them at places, where you are identifying what we call degrees of freedom. Then what are degrees of freedom? What do you understand by degree of freedom? Now, this is a very common confusing term in structural analysis itself. Let us talk about this. The degrees of freedom what we talk in dynamic analysis, are different from the degrees of freedom what we talk in structural analysis. I can give an example. As we understand that most of us will understand or will know structural analysis basics.

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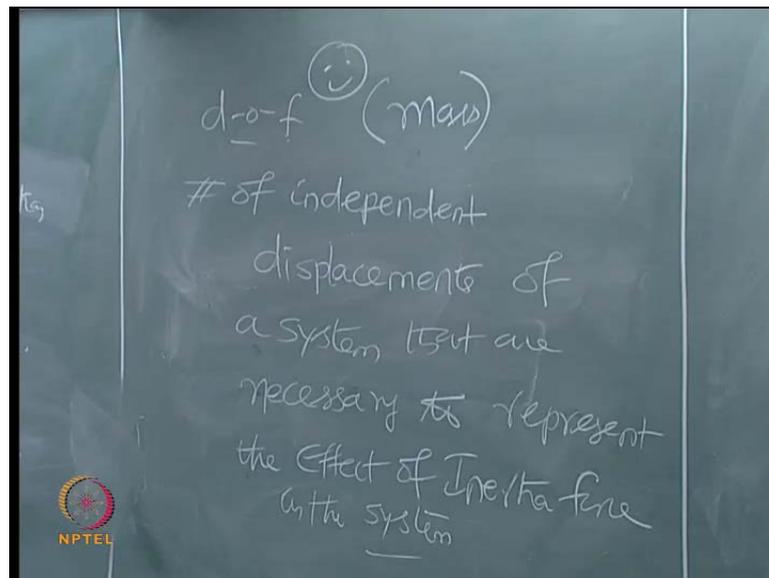
Let say, I have a hinged support, a support on roller, a support which is fixed. I can categorically say, from these kinds of supports, whether my structural system is statically determinate or indeterminate. To understand this, I also define degrees of freedom in the analysis. For example, support A. What are the degrees of freedom support A has? It has got 3 degrees of freedom. Vertical translation  $V_A$ , horizontal translation  $H_A$  and a moment on a rotational translation  $m$ . These are also degrees of freedom for joint a in a given member A.

Similarly, this will have again vertical, horizontal and rotation, but the rotation will be set to 0 in this case, because it cannot. So now, I will say the degrees of freedom, based on this system, will be 1 2 3 4 5. Five is existing. I will have 3 equations of equilibrium. I can call this as statically indeterminate to a second order, if I include the axial deformation in the member. If I exclude the axial deformation in the member, then the degree of indeterminacy will be different. So, here the degrees of freedom depend on where the support wants to move. So, it is related to displacement or action on the support. It can be either external or internal. It can be an action or can be a reaction. One is called static other is called kinematic. We can add them together to find total degree of indeterminacy.

So, while defining the indeterminacy in a given structural system, which is based on forces or internal reaction, we also define degrees of freedom in a given member. Now,

the degrees of freedom what we talk about dynamic analysis is different from what we are talking about here. Because, in dynamic analysis, degrees of freedom have a link with lumping of mass, whereas here, I never spoke about mass in identifying these V A H A and N A. Never.

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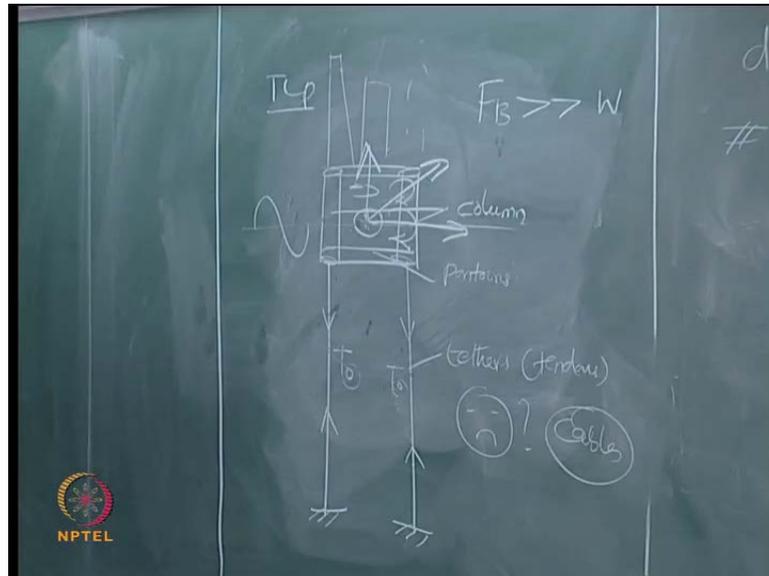


So, if we understand this, then let us define quickly, what we understand by degree of freedom. Degree of freedom classically defined as, the number of independent displacements of a system that is necessary to represent the effect of inertia force in the system. I just now said that, the degree of freedom is linked with mass that comes from here, because mass is associated with the inertia. Can we understand this in a very crude manner saying, wherever the mass is lumped, that is considered as degree of freedom? For example, I lump the mass in three places. The system will have three degrees of freedom. We lump the mass at ten places. The structure will have ten degrees of freedom. It means, since degree of freedom is related to representation of significant inertia of force on a given system, can I crudely understand that it is the number of points where the mass is lumped.

So, on the other hand, the mass is continuous, the system will have infinite degrees of freedom. If the mass is lumped at discrete points in a given structural system, those number of points you count and understand that as degrees of freedom. But, there is a catch here. You cannot associate degrees of freedom with the points where the mass is

lumped. It is to represent the motion or the inertia force of the mass in terms of its displacement. It is not only the point where the mass is lumped. It is all those freedoms, where the mass would like to move. I can give a very classical example here.

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Let us take for example, a tension leg platform. These are column numbers. These are pontoons. This is my water level in the top deck and I am having all drilling etcetera, all these things on the top. I am going to hold this structure down using tethers or tendons or, people who have forgotten cables. So, they will be always in tension. How to mark tension? It has to pull the joint. We already said that in the one of the lectures.  $t$  naught, this 0 represents initial tension given to the structural system. Why initial tension is to be given? because the buoyancy force exceeds weight in the design. So, when I try to afloat this, the buoyancy will try to push the structure up automatically.

In a very phenomenal way, I want to hold it down. I ballast them; the structure will sink or will get a drop. Connect them; de ballasts them; that ballast force will be transferred to tension in the tethers. That is how I install the TLP. Now, interestingly the pontoons and the columns and the hull, topside, all have a specific mass because all of them have a cross sectional dimension. They have a density in the material. They have a volume. Therefore, mass is there. I presume that the entire mass is lumped at a specific point. It is a magic point, which I have arrived by some calculations. There are methods to do this. But, since I do not know the method right now, as a student, I pick up any point.

For example, in an interview, somebody said it is an IAS interview. So, the person was sitting and the board was examining him and he answered all the questions. He is an IAS topper from Kerala. He answered all the questions. Then one of the board member said, "Mister x, I have got one last question to you. Please answer this question." He said, "o.k, I will answer this." He was sitting on a table of an elliptical shape, which is open ended. He is sitting here. They are sitting around. The person, one of the gentlemen asked can you tell me where is the centre of mass of this table. He just pointed his finger here and started walking out. The person said, "No. This is wrong answer. The mass is not here." He said that was the last question. So, he cannot ask the second question to me. So, I can always select this magic point as the IAS fellow did, saying this is my magic point, where the mass is lumped.

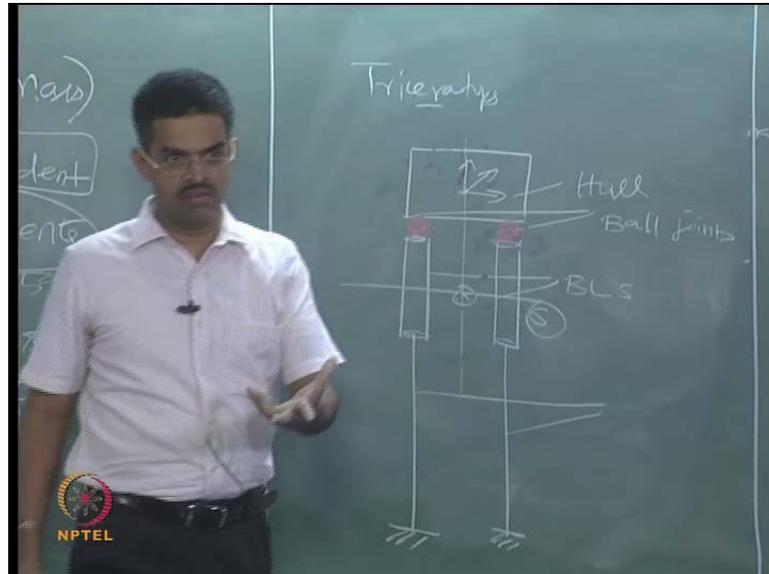
Now, the question what is troubling me is, will TLP have a 1 degree of freedom? because, there is only one mass. If you crudely understand that degree of freedom is the number of points where the mass is lumped, then we should have only 1 degree of freedom. But, TLP has 6 degrees of freedom. It means, the mass has a liberty to move along x, to move along y, to move along z, to rotate about x, to rotate about y and to rotate about z. There are specific names given in the literature, which we have discussed in the previous lectures.

So, it is the number of independent displacements. There is no coupling between these two. independent displacements of the mass point, because I am talking about the inertia effect of the structure. So, it cannot be crudely understood as the number of mass points or the lumped points where it is. Therefore, if we have got a system, which is got continuous mass distribution, the degree of freedom cannot be infinite. Because, for all those mass points of continuous system, I will always discuss and note the independent coordinates or the displacement coordinates, where the mass would like to move. Is that clear?

So, the number of points, where the mass is lumped may not be same as that of the degrees of freedom of the system. We can also give another example, where, to understand the independent term here. Now, I have triceratops, a new generation structure. As I said, offshore structural systems are not functional driven. They are form driven. I select a form, which should remain innovative and which should alleviate the forces coming on the structural system, by way of arrangement of the members, by a way

of holding member in sea or by way of assembling them, so that, they attract the minimum forces encountered by the environment. So, I have a triceratops.

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So, triceratops has a top hull and it is having a ball joint. Then a BLS, which is resting below and then tethers. So, these are what we call as buoyant leg structures. This is a top side hull, may be a crude way of drawing a hull. These are ball joints and of course, these are tethers; tendons or tethers. They are not cables. They are not wires. Let us quickly look at, how do we define the degrees of freedom for this problem, because, now I have got a BLS here. The BLS will have some mass. I will lump this mass at a magic number here, the magic coordinate here. Now, I want to take the advantage of providing a ball joint in this connection. It is an advantage of ball joint is not seen in the design. I may not have it here at all. I can have a structure like this.

So, provision of the ball joint, say structural innovation in the form, which will make or help the structure to reduce the forces coming from the environment. So, what is the advantage by providing a ball joint here? The advantage is that, the ball joint will not pass the rotation from the sub structure to super structure. It will only filter the rotation, but it will pass the translations. So, if we have got 6 degrees of freedom here, all the 6 will not go here. Only 3 will be there. So, those 3, which I have here as displacements, will be independent of this. So, independent displacement components will define the degrees of freedom. It is not the mass where they are lumped. Is this clear?

So, the degree of freedom explanation is different in dynamic perspective compared to static analysis of structures. There we talk about forces and actions or reactions of the given member. We never bother about the mass lumping there. But here, we talk about displacements, which will have effect of the inertia force of the system. When I say inertia, I will talk about mass. When I say mass, I have two systems. Lumped and continuous or discretization of mass procedure.

So, I cannot call a degree of freedom as a definition of the points, where the mass is lumped. So, it is actually the displacements. What is the commonness between the degree of freedom in dynamic analysis and degree of freedom compared to the static analysis? If there is no commonness, then the name should not be common. There should be some commonness. What is that commonness? There are two things which are common. One, the degree of freedom what you explain in static analysis or structural analysis is also talking about independent displacements and it talks about displacements. May be translational or may be rotational. What is not spoken in that is the inertia component addresses the displacements. That is the difference here.

So, this becomes a very interesting sense. Even if we have a platform, where the mass is lumped at two points or where the mass is lumped at one point, though it has got 6 and has got 9 degrees of freedom, but as long as the material is paper, which has no mass, I may not do dynamic analysis for this. So, that is a very important component here. I must have significant effect of inertia on the system. Otherwise, I will not do a dynamic analysis. So, degree of freedom explains all these complexities in a very simple statement. Saying that, look at the independent displacements, where mass is lumped and look also for the significant contribution of inertia at these points.

So, there is no point in doing all these exercises, if your material does not predominantly show inertia force. Now, the question comes, why we talk about inertia force in dynamics? Why importance of inertia is dominantly had sorry. So, it is coupled with the acceleration term. So, mass and acceleration will give me the inertia force. That is fine. so, why not any other term?

For example, stiffness is coupled with displacement. That is also a derivative of acceleration. What poor thing the displacement has done, so that people talk only about the inertia. Why inertia is important in dynamic analysis? That is a very interesting

question is it not? You must know this. What he tries to say is the coupling effect between the inertia and stiffness is very least. Therefore, we either talk about inertia or stiffness. In fact, he may be right, but I am sorry, I am not able to understand. He may be right, but I cannot understand what you are trying to say. My question is very simple. Why we focus inertia in dynamics analysis? Why not displacement and velocity? Of course, they are all derivatives. They are connected. Why we do not bother about them?

Very good. We are talking about one of the important dynamic characters of a system, which is frequency of vibration. Frequency of vibration is related to mass and mass is indirectly connected to inertia. So, degree of freedom is related to inertia force represented significantly on any point of your choice. It is not the point where the mass is lumped. So, we are talking about, ultimately the frequency of vibration or vibration characteristics of a given system under the given loading. So, we worry about mass. Now, there is a very critical question here. When we talk about inertia, can I replace this term as acceleration components? Why I am talking about displacement? I can also say independent acceleration components of a given structure because we are talking about inertia. Inertia is mass; mass is acceleration. So, can I replace this? Why I cannot replace this? So, we close it, right. You think about it.