Dynamics of Ocean Structures
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Module - 1
Lecture - 14
Undamped and Damped Systems III

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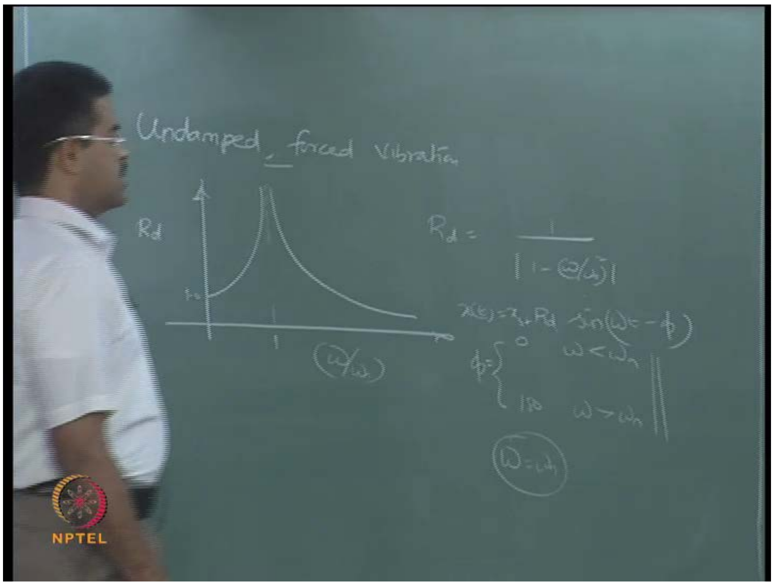

So, in the last lecture, we discussed about undamped forced vibration. We already said that if you try to plot the deformation response factor versus the ratio of forcing frequency with that of natural frequency, you get a plot similar to this. So, at a bandwidth closer to one that is when the forcing frequency is equal to that of the natural frequency of the system; the governing equation is not able to quantify the value here; whereas this value remains as 1.0. And the closed form expression what we had for this was R d was given by 1 by mod of 1 minus omega by omega n square. And x of t in general was expressed as R d sin omega t minus phi of x st. And of course, phi had two values: 0 and 180 depending upon whether omega is less than omega n or omega greater than omega n. Whereas, when omega equals omega n, this equation does not quantify the value in this band. So, we will extend this discussion further now when omega equals omega n what happens.

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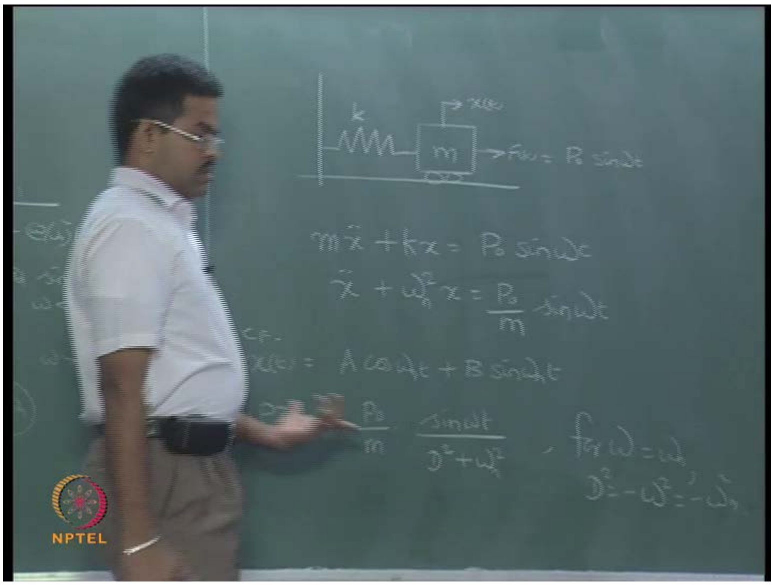

The system what I am discussing is this. I am applying F of t, which is P naught sin omega t. I am measuring x of t from the CG of the mass and I am applying a restoring constant k. That is an undamped system. So, when omega equals omega n, we had the equation of motion this way – m x double dot plus k x is P naught sin omega t; x double dot plus omega n square of k is P naught by m sin omega t. So, the x of t as the complimentary function is A cos omega n t plus B sin omega n t. And the particular integral is P naught by m sin omega t by d square plus omega n square. When I substitute omega equals omega n, the denominator will become 0 here, because I have to substitute D square as minus omega square or minus omega n square; denominator will become 0.

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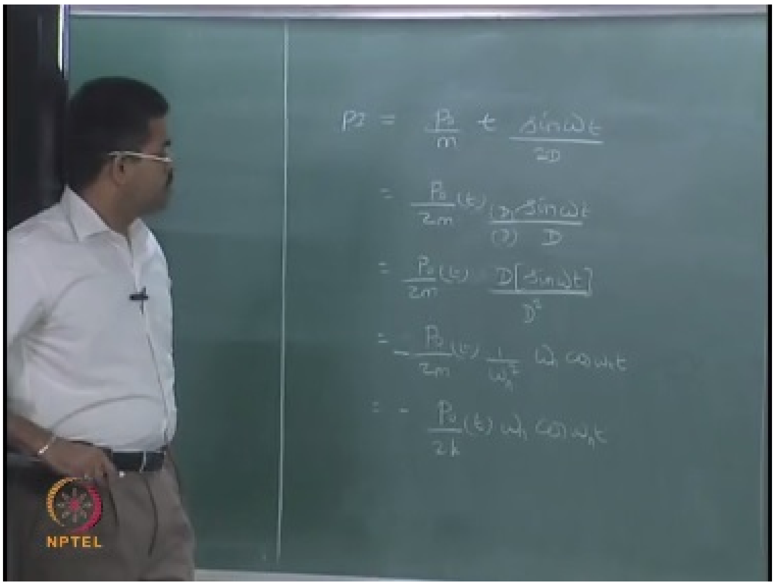

So, in that case, my particular integral can be P 0 by m t of, because t is the variable here – sin omega t. I can even use omega n here, because I am using the same and differentiate the denominator. So, I can now, say P 0 by 2 m t sin omega t by D; where, D is the differential operator, that is, d by dt of the argument. So, I now multiply this by D and D below; so, P by 2m t of differential operator of sin omega t by D square. I substitute D square as minus omega n square – minus P 0 by 2m t 1 by omega n square; I am having minus sign here – omega n cos omega n t. So, omega n is k by m minus P naught by 2k t omega n cos omega n t.

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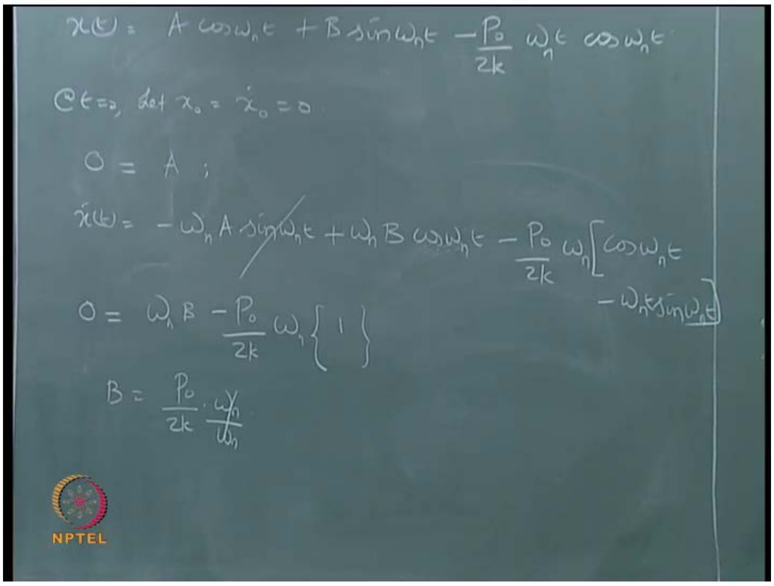

So, my total result, which will be a combination of complimentary function and particular integral will now become A cos omega n t plus b sin omega n t minus P naught by 2k omega n t cos omega n t. It is one and the same, because I am using omega or omega n same. So, let us eliminate A and B. At t is equal to 0, let x 0 and x dot 0 is set to 0. So, x of t is here. I substitute this as 0. That will give me A. And I am putting 0; this term will go away. So, x dot of t will be a differential of this minus omega n A sin omega n t plus omega n B cos omega n t minus P naught by 2k omega n of cos omega n t minus omega n t sin omega n t. So, substituting this as 0, this term will go away. So, omega n of B minus P naught by 2k omega n; this term will become 1; and this term has a t value here. So, it goes away. So, that way, B becomes P 0 by 2k omega n by omega n.

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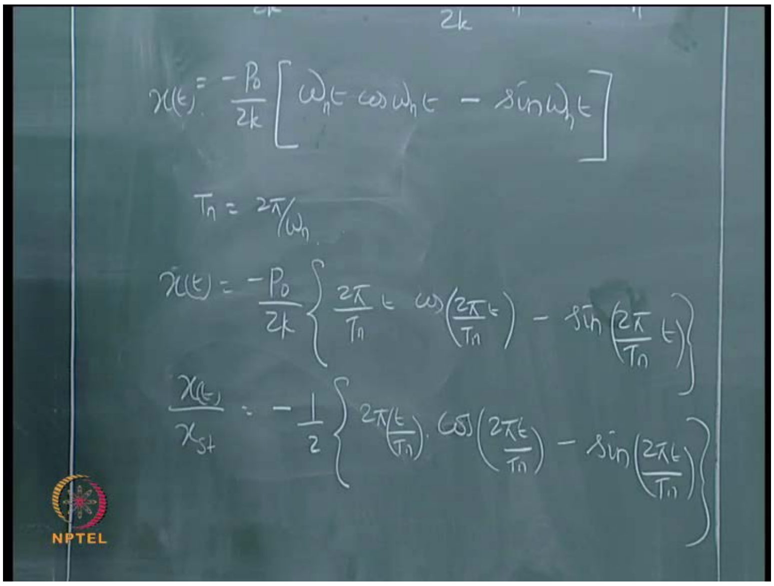

So, my final answer of x of t is P naught by 2k sin omega n t minus P naught by 2k omega n t cos omega n t; is equal to P naught by 2k of omega n t cos omega n t minus sin omega n t. That is x of t. We already know, T n is 2 pi by omega n; I have omega n here. Let me substitute back here as 2 pi by T n. So, x of t now becomes minus P naught by 2k 2 pi by T n of t of cos 2 pi by T n of t minus sin 2 pi by T n of t. I can also say x of t by x static; x static is nothing but P 0 by k. So, I will get this value as minus half of the whole equation back again 2 pi t by T n of cos of 2 pi t by T n minus sin 2 pi t by T n. I wish to plot this for different values of t by T n and see what happens to my ratio of x t to x static.

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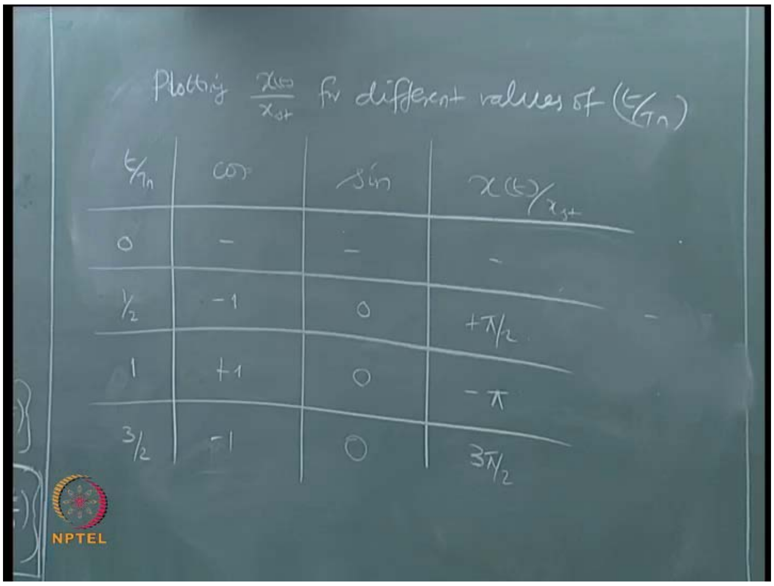

So, plotting x t by x static for different values of t by T n, let us try to plot this here in this format first – t by T n; let us see what will happen to the cos component; what will happen to the sin component; then what will happen to x of t by x static. Let us at least have samples of 3-4 values. Let us start with 0. So, this is my equation. So, see what happens when t by T n is 0. So, this term anyway goes away; there is a multiplier here. This term also goes away because sin component of 0 is 0. So, there is nothing; the value is not there. The next value I can start. Already there is a pi multiplier here inside the argument of both sin and cos. I no need to multiply by pi here again; I want to eliminate this to see a phase lag. So, I will start with half. So, if I substitute half, let us look at the cos argument here and see what happens. What will be the value of cos argument? Minus 1. Sin argument – 0. So, what will happen to x of t?

Student: Plus pi by 2.

Plus pi by 2; let us be louder. Then let us start with 1. What happens to the cos argument? Cos argument…

Student: 0, 1…

It is interestingly, you can either have 0 or 1 only; let us not try another value. It will be plus 1. And what happens to the sin argument? 0. What happened to the ratio?

Student: Minus pi.

Minus pi. So, half, 1, 1 and a half. So, what happens to the cos argument? You can look at the scenario; 0, negative, positive, negative; so, minus 1. Look at the scenario; it can keep on jumping like this. Sin argument – 0; and ratio? 3 pi by 2. Is it all right? 3 pi by 2. Let us try to plot this.

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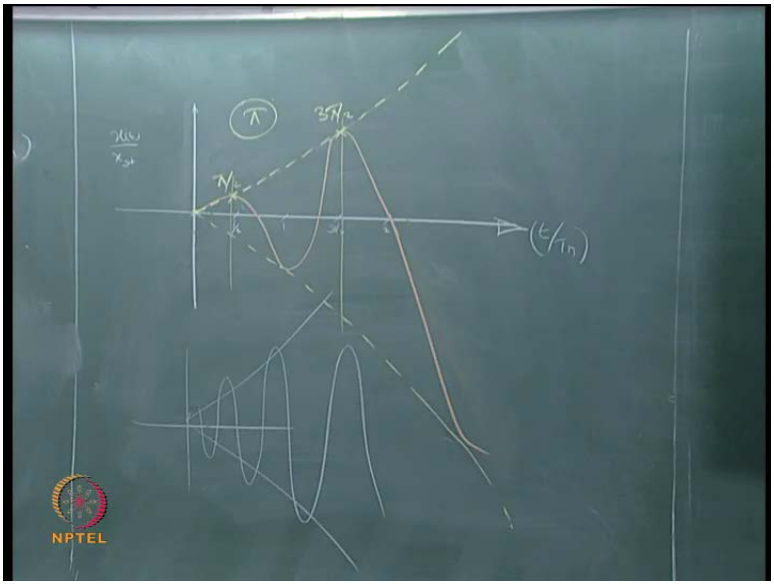

This is going to be a ratio of t by T n, which I am plotting. This is going be the ratio of x of t by x static. So, let me mark these points as 0, half, 1, 1 and a half, 2 and so on. This is 0; this is half; this is 1; this is 3 by 2; this is 2 and so on. We already know the point here is 0. At half, it is plus pi by 2. So, it is here. And at 1, it is minus pi. So, it is here. And at 3 by 2, it is plus 3 pi by 2. It is here. So, let us try to draw… So, you can see here that, the response is getting increased like a bell; the positive value here is pi by 2; the next positive value here is 3pi by 2. So, for one cycle, let us say from positive to negative to positive – one cycle, the jump is pi. For one cycle, it will happen in the negative also; can plot it. So, actually, the plot will look more realistically like this. Keep on expanding. So, we can infer certain things from this data. Let us remove that. What are those inferences we get from this figure?

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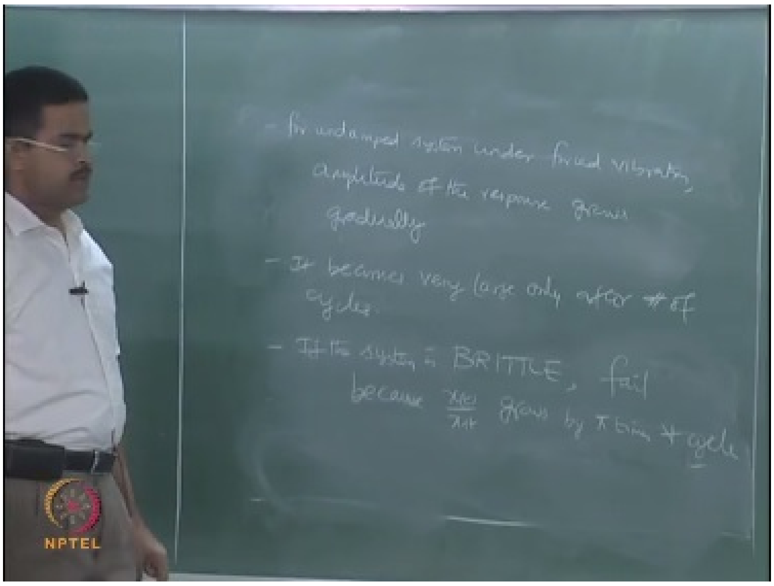

For an undamped system under forced vibration, the amplitude of the response grows gradually. It becomes very large only after number of cycles. It takes number of cycles to become very large. Now, when the system is brittle; if the system is brittle; during these number of cycles of response amplitude growing, the system will fail, because x t by x static, that is, the response ratio grows by pi times for every cycle. It is about 3.14 times. So, every cycle; one jump – three times; second jump – about six times and so on; keep on increasing. If ductile… I will remove this.

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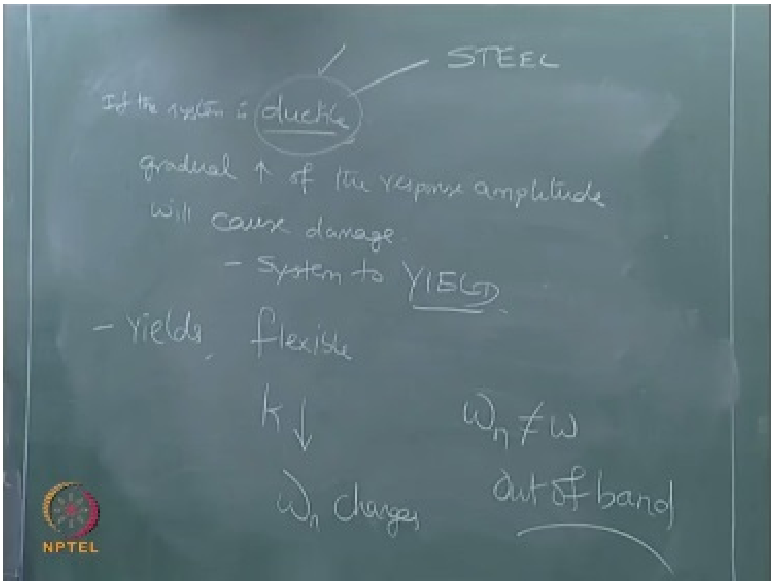

If the system is ductile, the gradual growth of the response amplitude will cause damage, which will make the system to yield. When the system yields, the system becomes flexible. When the system becomes flexible, stiffness reduces. When stiffness reduces, omega n changes and you will be out of this band; you will be out of this band actually. So, that is a great advantage. So, you must design offshore systems as ductile systems. It is because of this reason; offshore structure systems are essentially and primarily made out of steel. Then you may wonder that, where this ductility comes into play in selection of material. We have also seen that in the primitive lectures.

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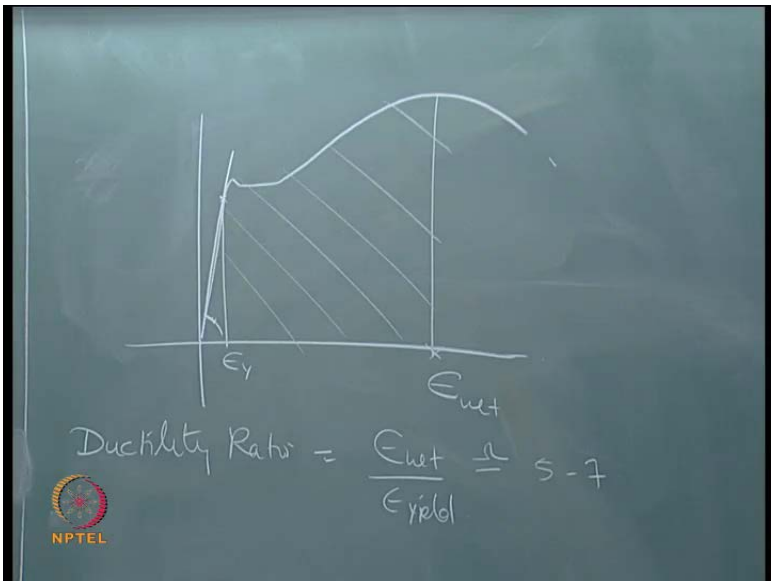

Look at the typical stress-strain curve. So, if you look at this ratio; I call this as epsilon ultimate; if you look at the yield point and call this as epsilon yield; the ductility ratio is epsilon ultimate by epsilon yield for classical steel, which is being used for offshore structural system. This is as high as 5 to 7. So, the material has got lot of reserve energy beyond the yield point before it fails. But, at the same time, the stiffness of the material; this is the initial stiffness. We understand the slope is actually the stiffness of this member or the material. The stiffness degrades. So, when the stiffness degrades, omega n changes. We are not worried whether it is going to increase or decrease; we cannot say that, because m is also playing a role here. What we are interested to say is that, when omega n changes, we are out of this band of resonance – so-called resonance. So, you will be away from the resonance band. So, system will not undergo a failure because of resonance band. That is very important information we gain even though the structure is undamped. These all argument is for undamped. So, what happens to damped? When it is damped? That is what our next argument is. Any question here?

Student: Stiffness of the material or stiffness of the entire structure we have to…

So, what he is trying to ask is, whether the stiffness is associated to material degradation or the stiffness of the whole structure? I will give you these equations later once I complete these arguments. But still, since he has raised this question…

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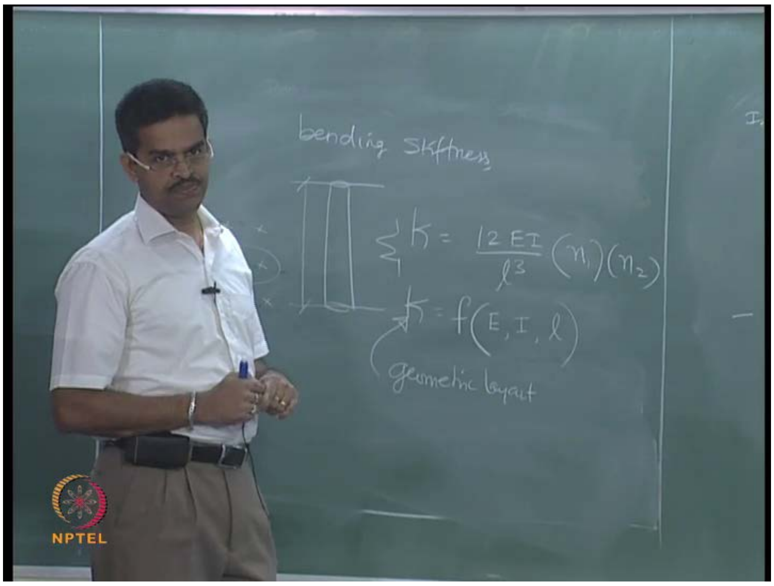

If you are talking about bending stiffness for a column member of any boundary condition, k is given by 12 EI by l cube; where, l is the distance between the supports. So, now, stiffness is a function of Young's modulus of the material, cross-sectional dimensions of the member, length of the member. Remember – it is not a function of boundary condition; it is not a function of boundary condition. Further, when you have got series of members aligned subjected to lateral force, then this k will now become a sum of multiplier of n 1 and n 2; where, n 1 will be the number of members acting along the force; and n 2 will be number of such frames, which are supporting this force. It means the whole geometric layout also becomes a function of k in addition to the material characteristic. Now, the degradation can happen in any form; it can be a local failure, where only the material yielded; it can be a global failure, where one of the frames fails. So, both of them will add and will influence the degradation of stiffness.

Student: (( ))

Now, what he is trying ask again a question is what would be the omega n value at the nu? So, we have got to talk about the tangent stiffness matrix of the global system. But, here I am taking a very simple example, where k is a linear spring and m is a single mass point. I am not talking about a frame of this type. When we talk about this in the second module, I will explain how this will help us. In this case, k is a simple linear stiffness, which is AE by l. So, how to estimate omega n in that case of failure? We will come to that. But, what we understand here is when omega n changes, the structural system or the single degree freedom system model will be away from the so-called dangerous band. So, the structure will not fail. But, omega will also chase omega n, because omega is also increasingly changing as far as we are concerned in ocean structural systems. So, we are not going to discuss that application problem now; we will talk about the next module, where we take examples of dynamic analysis of different offshore structural systems and we will see how we can interpret these results at that time.

Now, we understand very simply that, the omega is a single value, which is P 0 sin omega t. whereas, omega n is a single number, which is simply k by m; and k is not a function of geometric layout, nothing like that; we have a single value. So, when this is equal, I have a resonance band. Now, this would not be equal, because stiffness will degrade. Why? There is a yielding happening if it is a ductile system. So, that is what the catch here is. Any question here?

Now, our argument is, even when the structural system is not damped, there is an automatic phenomena, which takes the system out of the resonance band. But, one dangerous part here is, the ratio of response, that is, x t to x static keeps on increasing without any upper bound. It takes large number of cycles to reach a very large value. But, hypothetically, it can reach an infinite value also from this figure; keeps on going. It is like a bell; keeps on going; there is no upper bound on this. What happens when I introduce damping to the system? So, that will make interesting for us to understand what would be the difference. So, now, we will look at the damped system.

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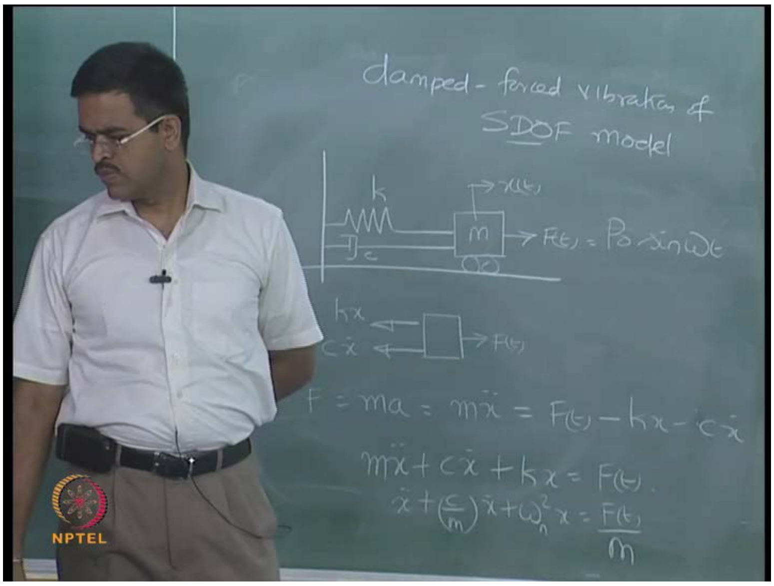

Forced vibration of of course single degree freedom system models; we are focusing on single degree. So, the conceptual diagram will look like this; which I am assuming as P naught sin omega t. I am measuring x of t from the CG or the mass center of this. I have restoring component k; I have damping component C; I am using a viscous damper model here. If I draw a free body diagram; when I apply a force F of t to the system, which is always present in the system for any instant of time t; it is a dynamic force. The restoring components will be stiffness multiplied by the displacement. And I am using a viscous damper model. So, C multiplied by the velocity; we already said this. And I know using Newton's law, this force should be mass into acceleration, which will be m x double dot; which will be equal to F of t minus of this, because they are opposite in nature. So, I should say k x minus e x dot. So, I will get m x double dot plus C x dot plus k x as F of t as the equation of motion.

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So, I write the auxiliary equation. Before that, let us divide this here itself; I think x double dot plus c by m of x dot plus omega n square of x is F of t by m. So, it is D square plus C by m of D plus omega n square is set to 0 always for an auxiliary equation. So, the roots: alpha 1 and alpha 2 for this to write down the complimentary function can be minus C by m plus or minus root of C by m whole square minus 4 omega n square by 2; equal to minus C by 2m plus or minus root of C by 2m whole square minus omega n square. So, we already know C by C c is the damping ratio zeta; and C c is C by 2m omega n. We already know this. Now, in this argument inside the parenthesis system what we call discriminate, there are two cases: C by 2m can be less than omega n; can be equal, can be greater. So, under damped, critically damped, and over damped.

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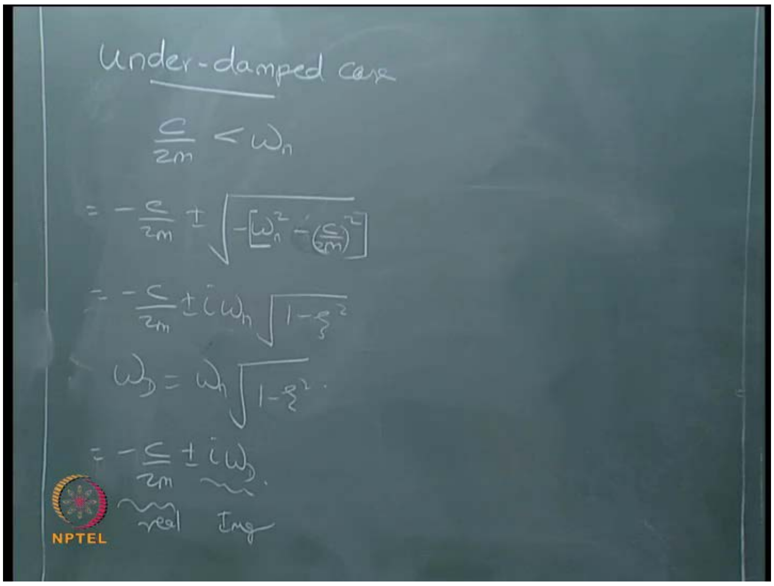

Let us talk about the first case, where C by… So, let us talk about an under-damped case, where C by 2m is less than omega n. So, I can rewrite this equation as minus C by 2m plus or minus root of minus omega n square minus C by 2m the whole square; can be minus C by 2m plus or minus i omega n root of 1 minus zeta square, because C by 2m can be written as zeta omega n. So, zeta square omega n square; omega n square common out; 1 minus zeta square. So, we already know the damped frequency is 1 minus zeta square. So, I can say minus C by 2m plus or minus i omega D. So, this becomes the real part; this becomes the imaginary part of the root.

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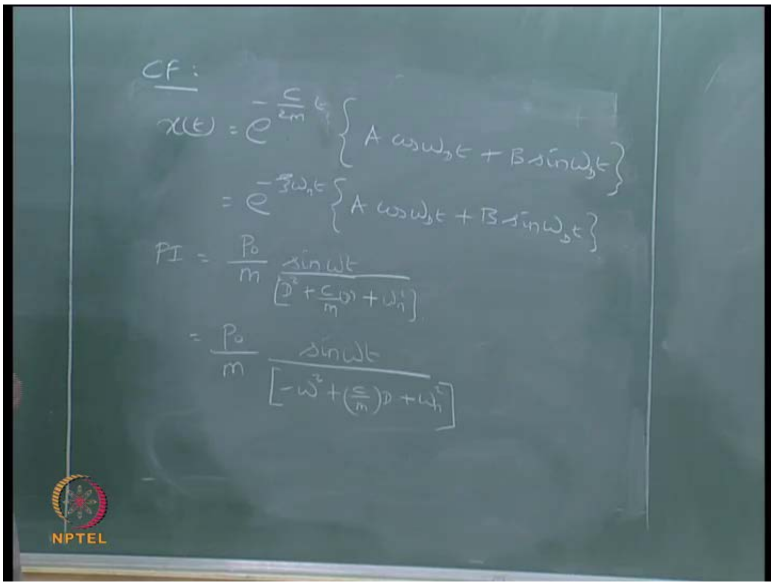

So, I can write the complimentary function as x of t – e to the power of minus C by 2m of t of A cos omega D t plus B sin omega D t. So, C by 2m from this equation is zeta omega n. So, I can say e minus zeta omega n t A cos omega D t plus B sin omega D t. So, I have to also write the particular integral for this problem, which is P 0 by m sin omega t by D square plus C by m of D plus omega n square. D is a differential operator here. So, as usual, I substitute D square as minus omega square; P 0 by m sin omega t by minus omega square plus C by m of D plus omega n square.

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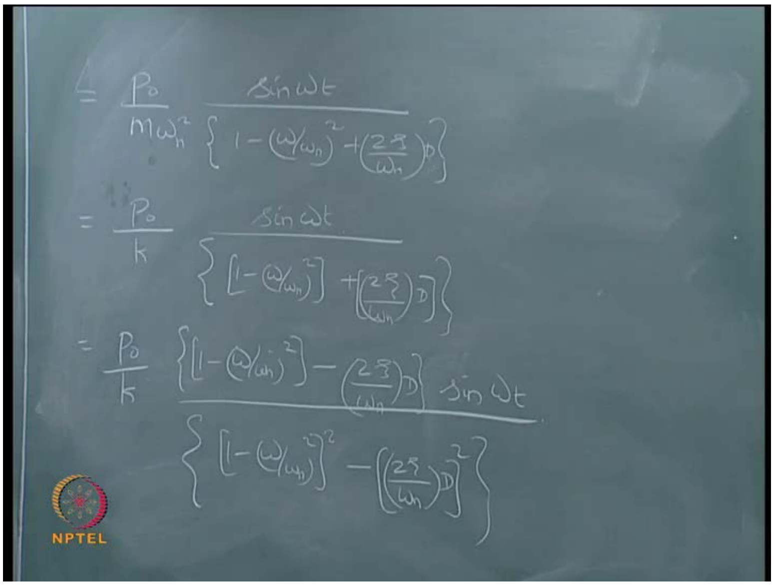

Try to simplify this further. So, I can say P 0 by m omega n square – sin omega t by 1 minus omega by omega n the whole square; C by m is 2 zeta omega n. C by m already we know is 2 zeta omega n. I have taken omega n square out to – plus 2 zeta by omega n of D. So, there is a differential operator here. I want to eliminate this. So, I can rewrite this equation slightly in a different form. I say P 0 by omega n square is k by m; I can straight away say P 0 by k. It is k by m. So, I write here as sin omega t by 1 minus omega by omega n the whole square plus 2 zeta by omega n of D. I am writing like this. So, you multiply the conjugate of this up; P naught by k into 1 minus omega by omega n the whole square minus 2 zeta by omega n into D into sin omega t by 1 minus omega by omega n the whole square of whole square minus 2 zeta by omega n into D the whole square.

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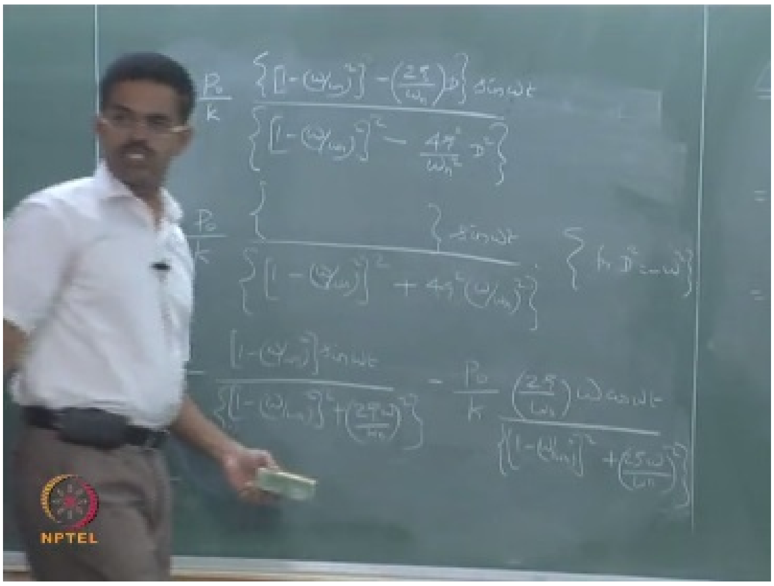

It is equal to P naught by k into 1 minus omega by omega n the whole square minus 2 zeta by omega n into D into sin omega t by 1 minus omega by omega n the whole square of whole square minus 4 zeta square by omega n into D square. So, now, I substitute this square as omega square minus omega square. So, if I do that… It becomes plus; I get 4 zeta square omega by omega n the whole square by substituting… So, let me take this component multiplied by sin omega t and expand it, because I have a D here. D is a differential operator on sin omega t.

Let us expand this. Let us do that; P 0 by k 1 minus omega by omega n the whole square of sin omega t by 1 minus omega by omega n the whole square of whole square plus 2 zeta omega by omega n the whole square of sin omega t minus P 0 by k 2 zeta by omega n; of differential operator on sin omega t; so, I should say omega cos omega t by 1 minus omega by omega n the whole square of whole square plus 2 zeta omega by omega n the whole square. So, I have got two factors here or two components here. P 0 by k 1 minus this value multiplied by this sin omega t component; other is cos omega t component. Both of them are the function of forcing frequency. So, I call this as C and I call this as D; D is not a differential operator; some constant.

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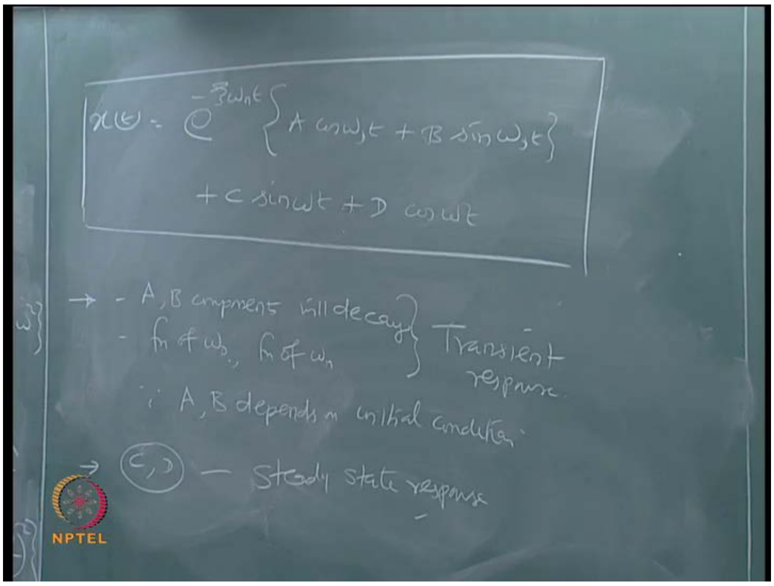

So, I can rewrite this expression as x of t e minus zeta omega n t of A cos omega D t plus B sin omega D t. That is what I have the complimentary function; plus C sin omega t – I say D – plus D; and D is nothing but minus of this value – cos omega t. That is x of t. Now, see here the inferences from this equation are the following. There are four components in this result; two of them are multiplied by an exponential decay function, because it is e power minus. So, the A and B components will decay, because there is an decaying function multiplied here. And they are also function of omega D, which is in turn function of omega n. So, I call this component as transient response, because A and B depends on initial condition. Look at the second component: C and D components. So, let us look at here. The C component – though we call this as a constant, this is only for us to easily write the equation. This is not a function of any initial condition. So, C and D component is what we address as steady state response. So, this is the case where when the model is under damped. And at that situation, if omega becomes equal to omega n, let us see what happens.

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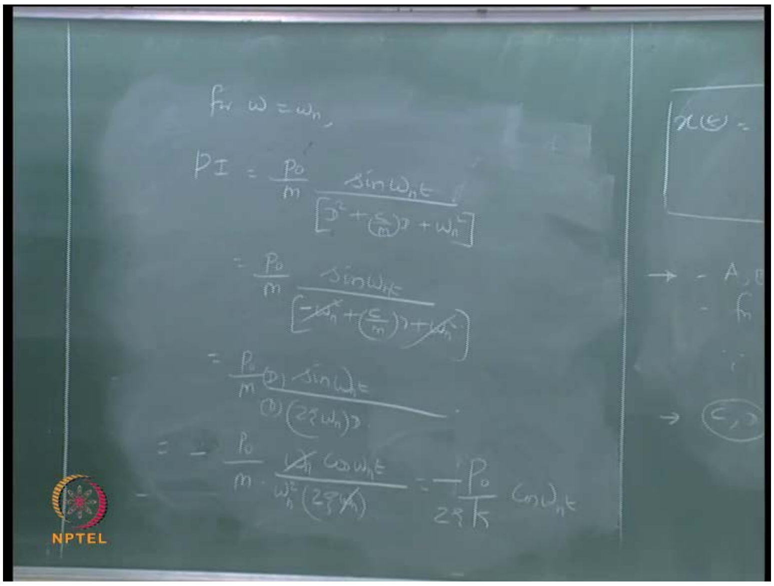

Now, I have a fundamental question to you; if I write a general expression like this, where I know the values of omega D and C's and D's and of course, A and B depending upon the initial conditions of the model; can I simply use the same expression for such conditions? Yes or no? Either you should say yes or no; keeping silent means we have to close the class.

Student: Yes.

Yes? Answer is no. The procedure can violate; the denominator may set to 0; we do not know. So, in this case, PI will now become again P 0 by m sin omega n t; I am using omega n in this case – by D square plus C by m of D plus omega n square. That is what the particular integral is. And I must say that, D square is equal to minus omega n square. In this case, it is not happening to become 0, because of the damping component present. But, I must derive it again. So, I must say P 0 by m sin omega n t by minus omega n square plus C by m of D plus omega n square. It is equal to P 0 by m sin omega n t; C by m is again 2 zeta omega n of D. I multiply this by D and D in the numerator and denominator; I get P 0 by m; I get D square here; so, differential of D, that is, omega n cos omega n t by D square is minus omega n square; so, I put minus omega n square 2 zeta omega n. This goes away. This will become k by m; m goes away. I will say this as minus P naught by k; I should say 2 zeta k cos omega n t.

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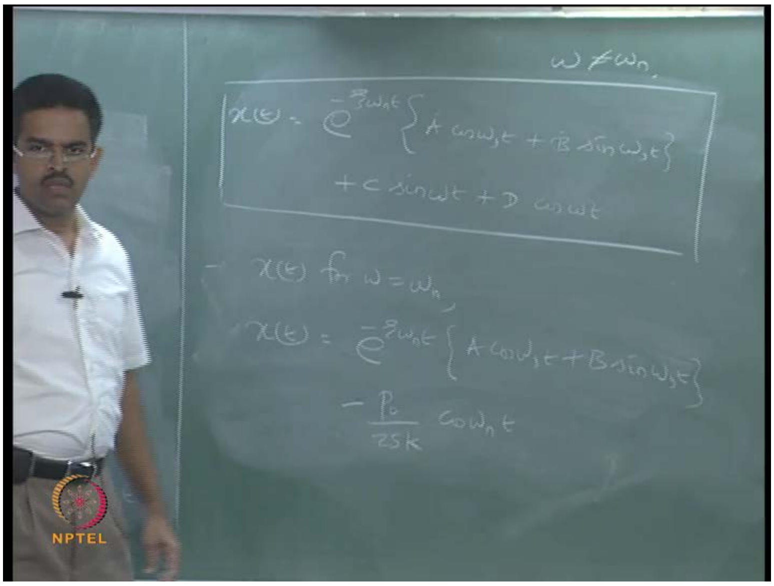

So, x of t will now get replaced as – for omega equals to omega n, that is replaced as e to the power of minus zeta omega n t of A cos omega dt plus B sin omega dt minus P 0 by 2 zeta k cos omega n t. Now, I have a question. In both these expressions, here omega is not equal to omega n; here omega is equal to omega n. The commonness is these two terms remain same; only this term is getting varied. Mathematically, because the particular integral of these two functions isdifferent – mathematically. Physically, why they are same? You can come up; any answer let us see.

Student: Initial condition is same.

No. Do you understand that, both of them are same or are we wrong somewhere? Same right? We understand that. Why they are same? This condition and these conditions are not same. Mathematically, it is accepted, because the particular integral component of the solution is different, because the procedure is different physically. Just because they are same and we have identified them as transient response; it is because of this reason, transient response does not influence any of the conditions in the reality. That is why they are not considered to be important; they are not considered to be important at all, because they do not reflect the reality in terms of the behavior of the model.

Then, again the same question, which I asked in the last lecture – why we are focusing on transient response? Same question is again coming to you back. Transient response will not influence the response of the model for a realistic condition as we discussed here. That is a physical understanding. And it is because of this reason; we do not actually give weightage to transient response in the analysis. I split this component into transient and steady state; look only at the window of the steady state to understand the more response intense functions of the model. Mathematically also, it is very clear that, they are same; it has no influence on the realistic conditions as we see here. The second question I leave it to you for homework is what will happen if omega is greater than omega n? That is over-damped systems. I have not solved this. Over damping; I want you to do it mathematically. And you will come out with only inferences and accept it. You must do this. So, we stop here. If you have any questions, we will be answering it; otherwise, we will continue in the next class.

Thanks.