Dynamics of Ocean Structures

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Module - 1
Lecture - 12
Undamped and Damped Systems I

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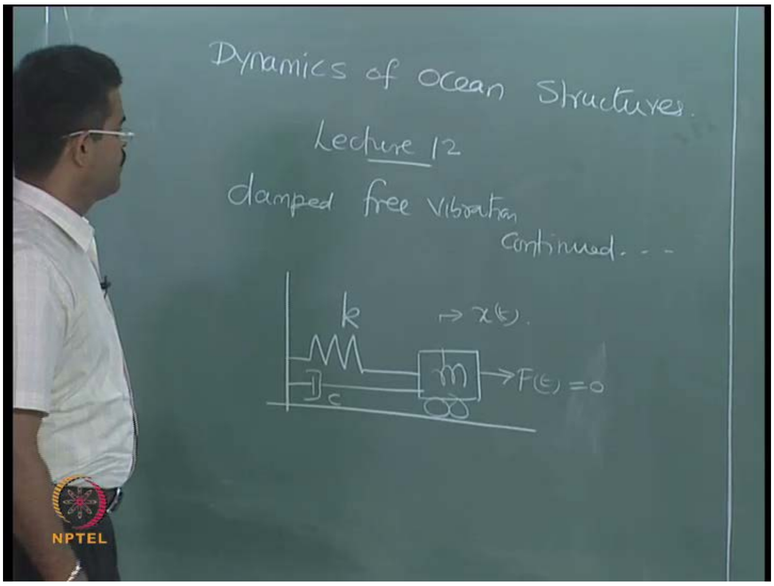

In the last lecture, we discussed about the damp free vibration. We had a model having the inertia component having the elastic restoring component, also a dash having the c component. And of course, we said side F of t will be set to 0 and a measuring the response at the point where the mass is lumped.

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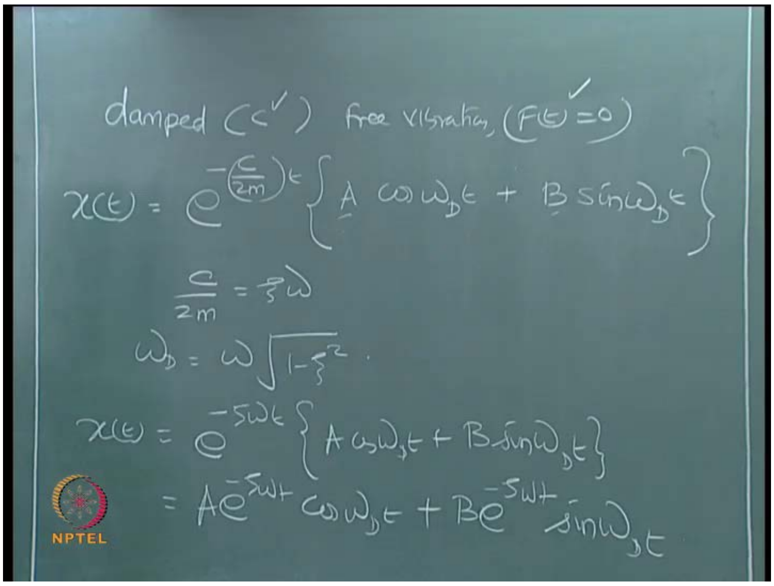

So, we are talking about damped, because c is present free vibration, because F of t is absent. So, we found out the expression for x of t in the last class which is e to the power of minus c by 2 m of t, we will use as A and B as constants A cos omega D t plus B sin omega D t. So, in the last lecture, we have used c 1 and c 2 one and the same, but I am deliberately using A and B because c 1 and c 2 may get confused with the damping constant here. Now as usual we have got two unknowns here A and B, I want to eliminate or evaluate them. I will use initial conditions and when I know already that c by 2 m can be expressed as zeta omega, where zeta is the damping ratio, and of course, omega D is given by omega root of 1 minus zeta square. So, I can rewrite this expression as e to the power of minus zeta omega t of A cos omega D t plus B sin omega D t. Now let me expand this as A e zeta omega t cos omega D t plus B…

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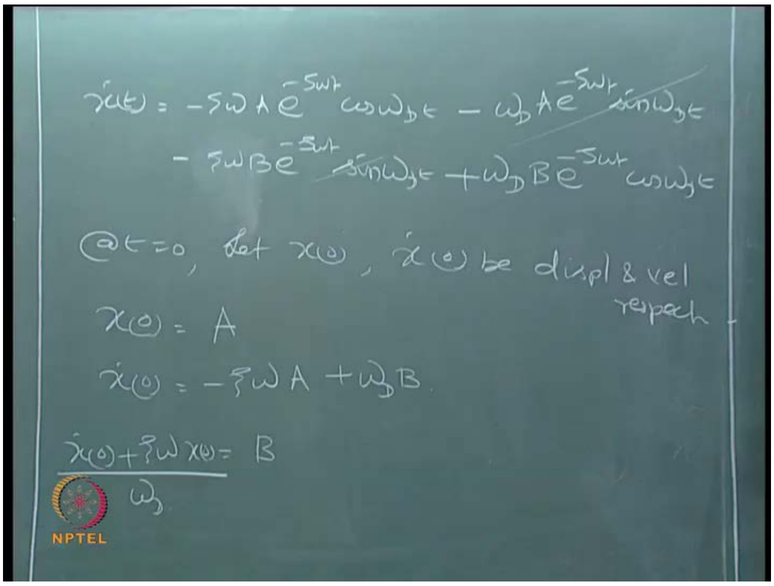

Let me find the first derivative of this, minus zeta omega A e zeta omega t cos omega D t minus omega D A omega D t sin omega D t minus zeta omega d. So, let us use initial condition at t is equal to 0; let x(0) and x dot of 0 be displacement and velocity respectively. So, let us substitute in this expression here at t is equal to 0, x 0 in this equation. I get this as A; and in the second equation, I say x dot 0 will be minus zeta omega A plus omega D B, is it not? Because the sin term any way goes away. So, minus zeta omega a plus omega D B. Second evaluate B as x dot 0 plus zeta omega x 0 by omega D substituting for a as x(0). So, now my equation is complete, I can use these values of A and B here, I can write x of t in the complete form, if I know the initial velocity and displacement at time t is equal to 0, I know the whole equation. Let us now see, what could be the interesting feature of this equation at various t values along the curve.

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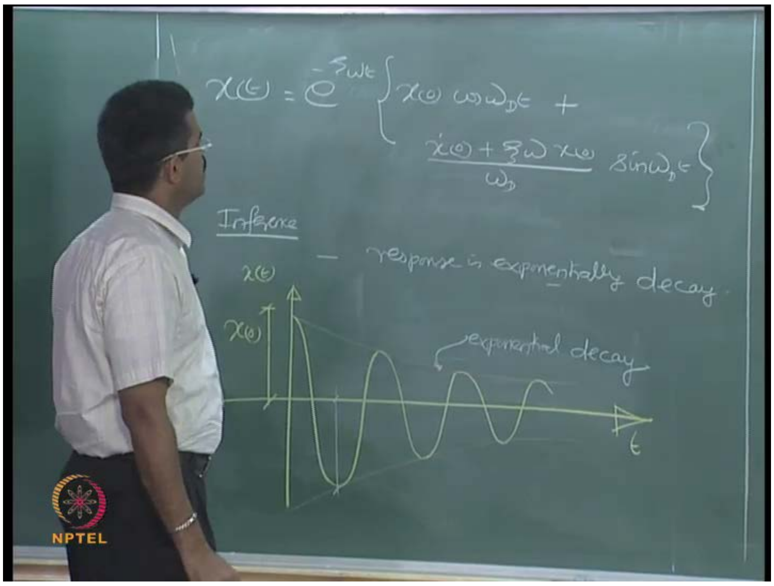

So, the complete equation what I have here is, x of t is equal to e to the power of minus zeta omega t x (0) cos omega D t plus x dot 0 plus zeta omega x(0) by omega D of sin omega D t. So, you can have a general inference from this equation. The first inference, I can draw from this equation is, that since x of t is e to the power of minus, I can say the response is exponentially, because it is e power decay, decay because it is e power minus, so decay. So, typical value of plot, try to look at the plot. It is my t, it is my x of t, and of course, this value is x(0) that is the initial displacement I have. Now, let us say I have a point where look at this equation and see or try to evaluate this equation at different t values along this curve. So, this is an exponential decay, the response keeps on dying down.

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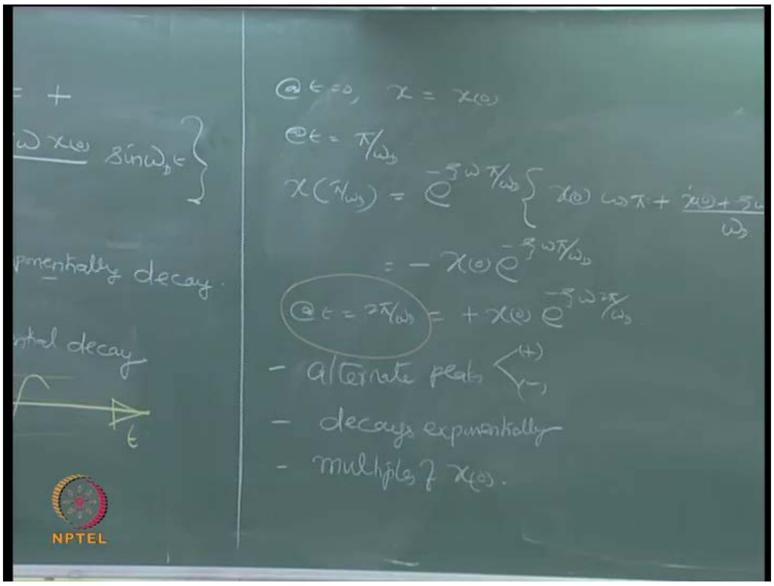

So, we already know at t is equal to 0, x is nothing but x naught from this equation or from this curve. Of course, this was an initial condition what we have also assumed. Now I want to estimate this at t is equal to pi by omega d; you may wonder that how I am picking up this number? Very important. So, you can see here the cosine and sin relationship or functions of omega D. If I try to take this value the denominator introduce pi here, I can see that face difference between this by eliminating the omega D. Keep on doing it for different values of multiples of pi within denominators omega D that is pi, 2 pi, 3 pi and so on, see what happens? You will get an interesting conclusion from this. So, I want you to evaluate the t is equal to pi omega D this equation.

So, I will start x of pi by omega D that is x at t is given by e to the power of minus zeta omega pi by omega D x 0 cos pi, because omega D and omega B will go away cos pi plus x dot 0 plus zeta omega x(0) by omega D sin of pi. And this will give me minus x(0) e to the power of minus zeta omega pi. So obviously, in this figures expected that this value is negative; I get it here. It is this value and of course, this point is pi by omega D. Now let us say at t is equal to 2 pi by omega D, I multiple of this, what happens? Do this equation again. So, do back and tell me what happens at 2 pi omega, 2 pi by omega D. So, you will get this value as a positive number which will be x(0) e to the power of minus zeta omega 2 pi by omega D is that ok? So, what we infer from the successive values of x at t 0, x at pi by omega D, x at 2 pi by omega D and so on, I am getting alternate peaks - plus minus, plus minus, plus minus and so on. I get alternate peaks.

Alternate peaks in sense positive and negative is it not? Obviously, when you use 3 pi by omega d, you will see that you will get negative peak again.

Secondly, the qualitative decay component of e power minus is maintained, so it decays. Next, the successive peaks are all factors or multiples of x naught, x naught, x naught of something x naught of something, so there are multiples of x naught that is multiples of initial displacement, clear? We may not work out further, so if you want me to mark this point, if you want to make me mark let us say this point, this point corresponds to and so on, keeps going, because positive negative, positive negative you will get successive peaks. The only clue here is you must know why we are augmenting this equation at the multiples of pi by omega t. The reason is as I said since the response is cosine and sin function, harmonic function of omega D, I am eliminating it by keeping a denominator, I am introducing a pi surface lag just see how it is happening, that is why it is shifting. I hope there is no confusion here? You are able to get this?

Now interestingly let us look at the ratio of the successive positive peaks or successive negative peaks, let us see what happens. I will remove this, let us see a call this as x naught, x suffix 0, this is x(0). I call this is as x 1, I can call this as x 2. I am sequentially numbering them as positive peaks; in call this as x minus 1, x minus 2, x minus 3 and so on. So, negative indicates, there are negative peaks; positive indicates, there are positive peaks, and the number indicates that successive peaks. So, I can rewrite it slightly in a different manner, I will remove this it is easy for us to understand. Can I remove this? Take away this is easy.

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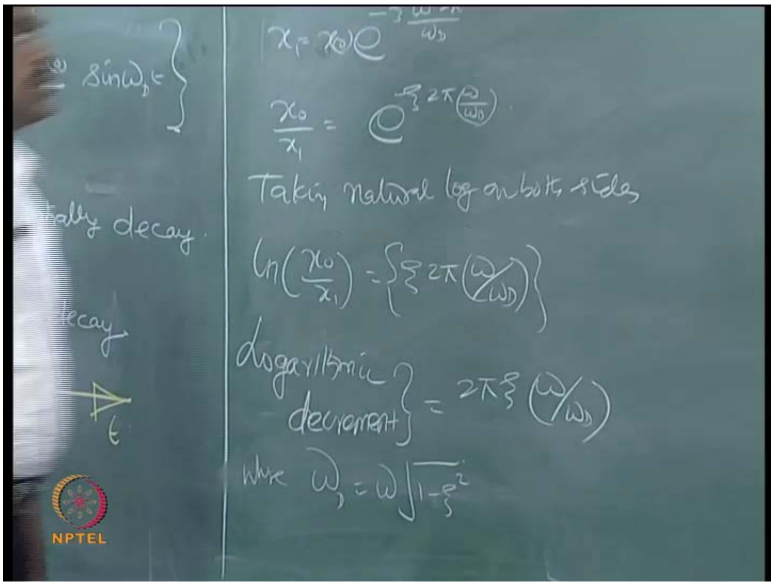

So, x 0 is x naught x 1 is e to the power of minus zeta omega two pi by omega d of x naught let me put it here . So, if I try to work out the ratio of the successive peaks, I get this as e to the power of zeta twp pi omega by omega d is that fine x naught will go away. I take natural log on both sides this is what we call as logarithmic decrement is nothing, but 2 pi zeta omega by omega d where omega d is given by omega root of one minus zeta square.

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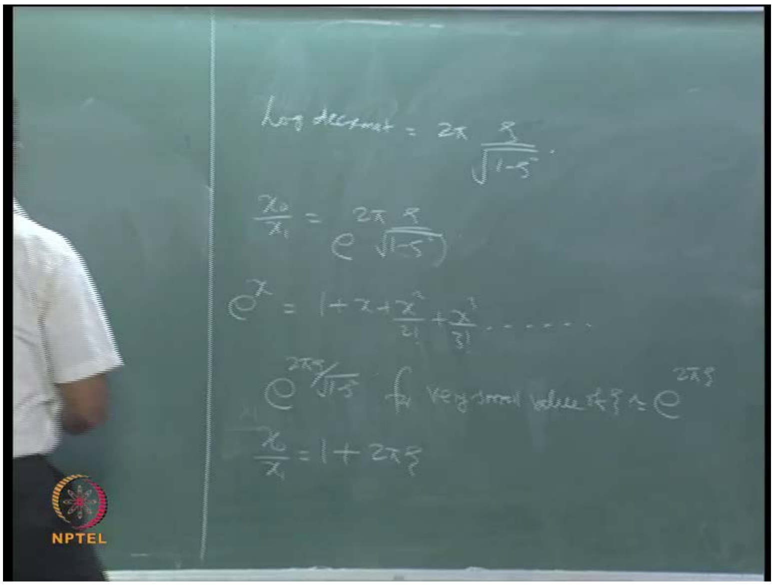

Therefore, logarithmic decrement can be substituting here, 2 pi zeta by zeta square. Let us expand this argument slightly further and see what is the benefit of this whole discussion? I also know that x 0 by x 1 is nothing but the decrement the successive decrement is 2 pi zeta 1 minus zeta square now e power now we know e power x, if you want to expand in exponential series it is given by 1 plus x plus x square by 2 factorial plus x cube by three factorial and so on that is how you expand e power x. Now e to the power of 2 pi zeta by 1 minus root of zeta of square for very small value of zeta can be approximated as e to the power of 2 pi zeta itself is it not 1 minus data square will not cause any difference in denominator. So, now, I can expand this and say that x naught by x 1 is simply 1 plus e 2 pi zeta.

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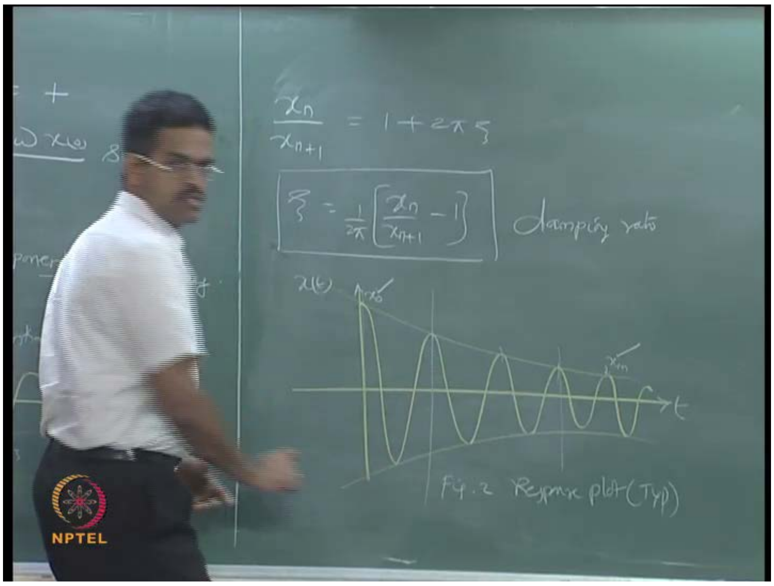

I can also write this in a general form instead of saying x naught and x 1, I can say x n by x n plus 1, I am taking successive peaks. Any successive peak I can say it is 1 plus 2 pi z, therefore zeta can be x n x n plus 1 minus 1 1 by pi. So, this will give me interestingly the damping ratio, you may wonder what is an advantage of this damping ratio? Before understanding this, let us try to take another example and use this expression to find zeta, let us say I have a curve like this. And pick the values as x 0 here and x m here, some peak, here I am comparing successive peaks, here I am comparing n number of peaks where n will be equal to 1, 2, 3, 4. I am picking the fourth peak from the reference peak; I want to generalize this equation for zeta you may wonder why it is it necessary I will show you that?

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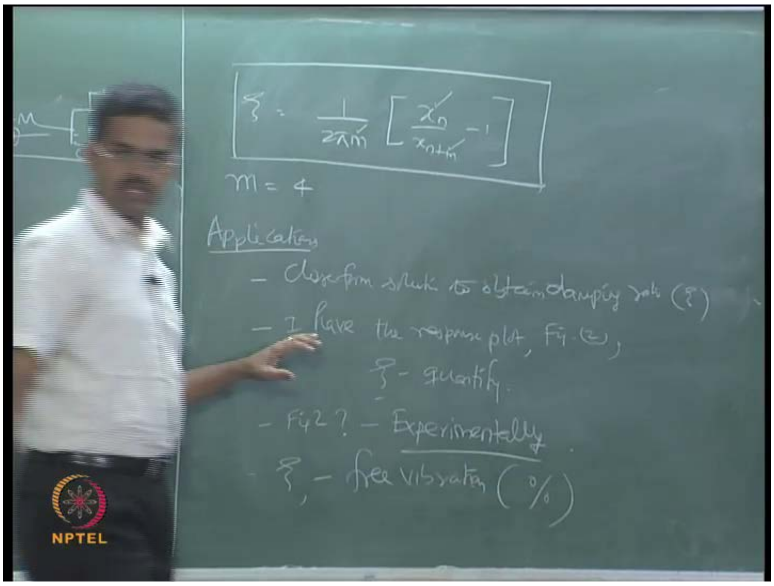

So, if I try to get a plot like this and generalize the expression for obtaining zeta then zeta can be obtained as 1 by 2 pi m of x n by x n plus m minus 1. So, in the above example here m is 4. Now let us see what is the application? This expression gives me a close form solution to obtain the damping ratio, which is zeta is it not? So, close form. If by any chance, I have the response plot as shown in figure two. Let us say figure two is this which a response plot typical, it is a typical response plot, they have a response. This is t, this is x of t, if ever response plot has shown in figure two with me I know this value I can pick up any peak down.

The line say x m, I count the successive peak number, substitute in this equation. I know every thing here, I know this, I know this, I know this, I will get zeta, I can also check this zeta by picking. Let us say this value with this picking this value with this picking this value with this will be uniformed. Now what is the advantage of this? Once I have the response plot as shown in figure two, I can find zeta what I call quantify zeta.

Now the question is where will I get this figure two? Where will I get this figure two? By any chance, by any magic, if I get this figure two, directly I can find zeta. Interestingly, my dear, this is the only way to get zeta you will not be able to get zeta otherwise by any other technique; on the other hand, you must get x of t for a given vibration system experimentally.

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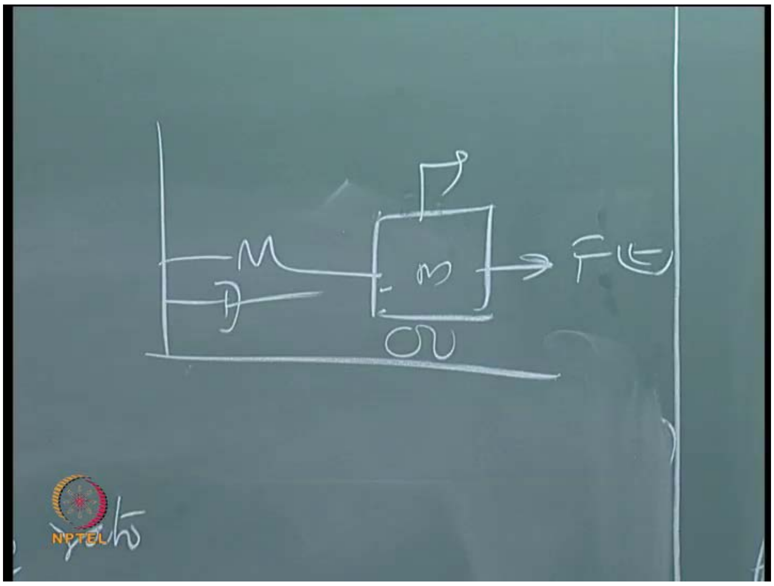

So, what you do you simply have a model which is an idealize single degree multi degree. Whatever may be apply a free vibration to this put one displacement transducer in acidometer here directly record the time history vibration of this mass in a time scale like this you will know what is the damping imposed on the mass under the given working condition. So, I can obtain this experimentally and most importantly zeta is worked out only based on free vibration.

So, free vibration tests are very important dynamics especially in motion structures because you will have an idealist model which has wave effect or water body effect surrounding the mass. For example, install a system in the test bid or a wave tank commission the system for example, may be TLP or a spa or a gravity platform or an articulated tower commission, it do not apply any excitation force from the wave or anything.

Let their structure remain static at t is equal 0 apply a free body motion to the mass point and get the response plot. From the response plot can find the damping imposed on the model by water surrounding the body material deterioration. All put together that is how we can easily apply the free vibration response and based on which I will get zeta and zetas of course, express in percentage of what is the ratio percentage of with respect to critical document under damp system? So, percentage respect to critical is what you evaluate we can easily get this value.

So, in this model, we have assumed that see is the viscous damping component, because it is that damping force is proportional to the velocity now it is not proportional to velocity. Let us say for example, a coulomb damping model, let us see the great difference a coulomb damping model or coulomb damp response in comparison to viscous damp response as we just now saw. So, the first observation what you understood from viscous damp response is the response decays exponentially, it means the decay is faster because exponential decay is faster than a linear decay, is it not. It is very high when as a coulomb damping, it will be the other way will see what is it. Any questions here, any doubt, any questions, any difficulty here? Because few of you may know this, but still this important for us to understand the continuous flow for that you will remember how we are proceeding.

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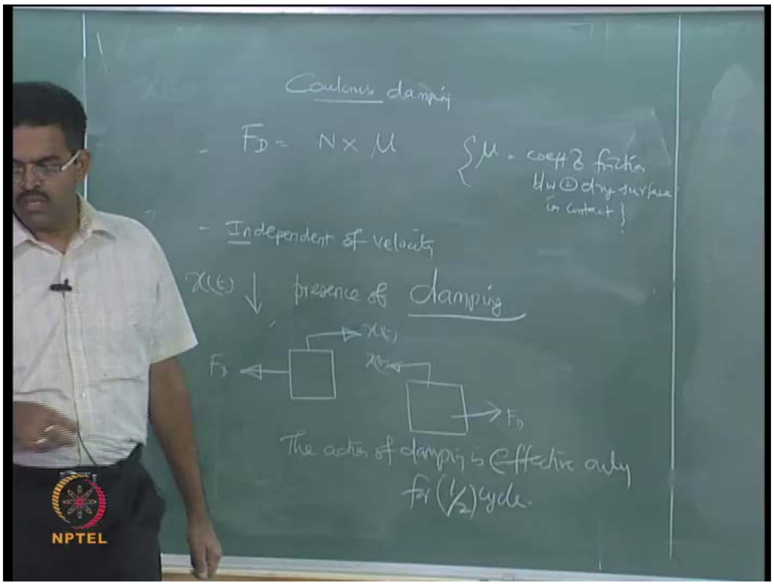

So, I am talking about coulomb damping more in detail. We held it another coulomb damping force is normal force acting on the system multiplied by the coefficient of friction between the two dry surfaces. Two dry surfaces where mu is in content n is the normal force. Therefore, we already said this is independent of velocity is independent of velocity, but it is a function of time. Now most importantly look at this curve here, we agree that the response decay because of the presence of what is that, the response decay because of the presence of damping. It means when I have a body which wants move to its right, the damping force should move the body to its left. When I have any time instance where the body is moving to its left, the damping force will move the body to its right. And they have very clearly seen here the movement of the body is alternate. Some where it is moving positive negative, positive negative, it means the damping cycle is valid only for half is it not, after half it is changes its sign is it clear. So, we cannot simply apply the damping force for the full cycle because within one full cycle there is change of sign happening in the response.

So, we must look only at the half cycle first to capture the qualitative variation of the response occurring because of damping. If you capture the full cycle you will not know, if you is only looked at the x 0 and x 1 you possibly thought that the variation was simply like this you never know it is coming down to negative. You look at x naught x minus 1 and x 1, you will realize that the damping is getting change of sign I mean the force getting change of sign the response is changing its cycle right. So, we look at the half cycle.

So, I am interested if this is my response this is my damping force if this is my response, this is my damping force that is what I understood from this figure. It means the action of dumping is effective only for half cycle is it not then afterward changes its sign. So, let us capture the half cycle response for a coulomb damp model and see what happens. Any questions here? Is this is clear? What we are trying to say, this I have inferred only from the figure what I just plotted.

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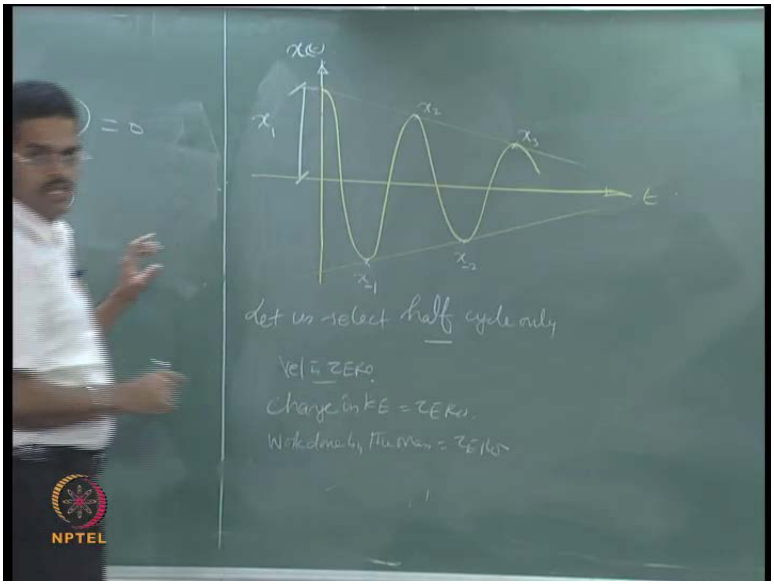

So, let me draw a typical coulomb response x of t. So, let us call this as initial displacement call this as x 1, x of minus 1, x of 2, x of minus 2, x of 3 and so on. This is x of t. So, let us select half cycle. So, now I can say if the velocity 0, you have to draw this curve very carefully, there is no slope here, there is no slope here. So, the velocity is 0, therefore change in kinetic energy will be 0 and hence work done by the mass will also be 0. I will remove this.

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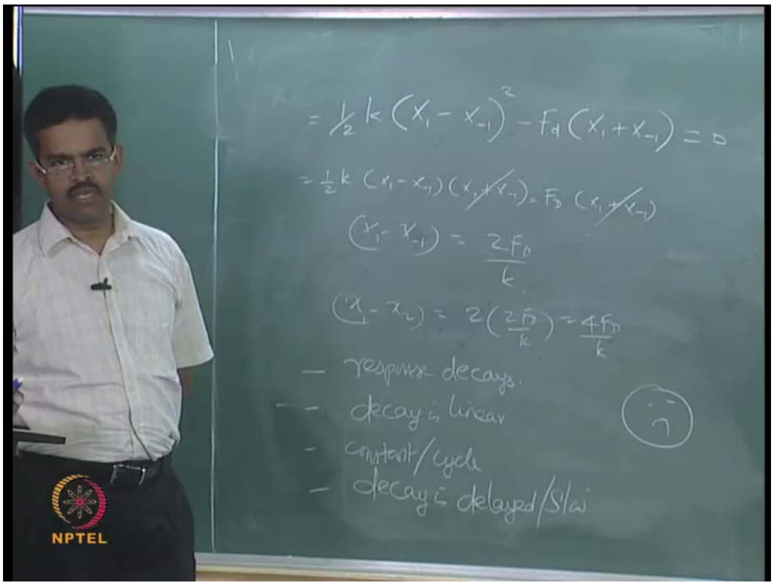

Hence half k x 1 minus x 1 the whole square minus f damping of x 1 minus of x minus 1 should be set to 0, but this is I have using minus sign here. So, if we expand this, and this is going to be my F, if we expand this half k x 1 minus x minus 1 x 1 plus x minus 1 is equal to F D of x 1 minus x minus 1 x 1 minus x minus 1 is 2 F D by k. So, what does it mean the response reduction with in half cycle from x 1 to x minus 1. The response reduction within half cycle from x 1 x minus 1 is equal to twice of F D by k which is the damping force. If you apply the same algorithm back from x minus 1 to x 2, you will again get the same relationship you can try. On the other hand, x 1 to x 2 that is the response decay from positive x 1 to positive x 2, the successive peak is twice of 2 F D by k which 4 F D by k.

So, I can really write that the response reduction between the two successive peak is 4 F D by k that is true for negative also, x 2 also. So, there are some inferences I can draw from this understanding, they are the following. One - response decays no doubt even when we use coulomb damping, but the decay is linear, it is a linear number is it not. And it is constant per cycle every cycle of positive is only 4 d by k, another cycle again 4 d by k. So, if you want really reduces responds from x 1 to x n, you have got under go so many number of cycles to bring down the response from this peak to the desired value of your choice.

So what do you understand by making, so many number of cycles you are alternately creating a response which will induce a fatigue problem into a model, because the model has to undergo n number of cycles before it reaches the desired response of your level. You have you want the response not x 1, you want the response x n, let us say here somewhere. Since you know every cycle, response decay is constant for coulomb damping, you must make the model undergo that many numbers of cycles to bring the response from x 1 to x n of your choice. Therefore, the decay is delayed or slow, and hence it is not desired, it is not desired one, where the exponential decay. The decay is exponential is linear here that is very fast, it is linear where as exponential it is very fast.

I have a question for you. When this response will stop decaying, it has to stop somewhere right? When will it stop decaying from further decay? When the elastic restoring force in the given system is not capable of overcoming and nullifying the damping force will stop is it not? So the shutdown of this is automatic. It will not damp beyond that, therefore any damage cost to the elastic component in your structure will affect the damping cycle directly, there is another demerit in this model.

Any damage you have caused to the elastic restoring component in your system, of the stiffness, weakness or degraded stuff fitness in your structure, because of some loading phenomena, which will happen, I will show you later in the next module of lectures. When your stiffness gets degraded, this will directly affect the damping philosophy or principle in your structure that is why this kind of damping is not recommended and commonly used for ocean structures. We are not saying it is good or bad. There are two important points; one - the decay is there, but its delayed and slow. You need to undergo certain number of cycles to ensure a specific level of decay by any chance if the elastic restoring component k or the stiffness degradation happens in the structure by damage to the structure will directly affect the damping relationship of the structure. Just now, we saw the damping the decay will stop when there is an understanding between k and F D is it not that will affect. So, because of this reasons it is not acceptable and commonly used in ocean structures.

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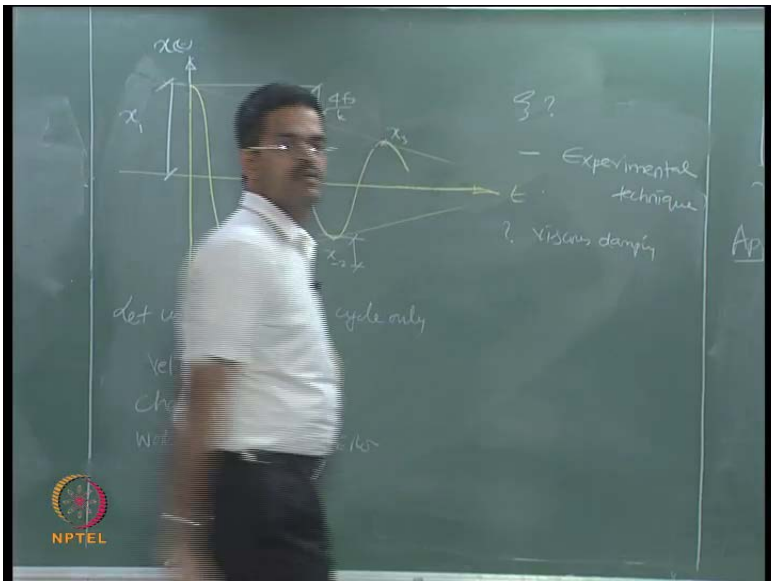

So, we have learnt two things from this class saying that how to estimate damping for a given structural model. The most common popular easy method is experimental and why we use viscous damping. So, we have discussed about damp and un-damped free vibration models the next class we talk about damp and undamped forced vibration model where F of t will not be equal to 0. Any questions here?

So, all are single degree, they will be move on to two degree and multi degree quickly and then find out the characteristics that is where we will end the twelfth lecture is running, we will finish this module in eighteenth lecture. We will have 6 more lecture in this module, and then I will move on to application of dynamics to ocean structures. I will pick up modules of ocean structure gravity platform, star, TLP, tension like I mean articulated towers (( )), I will show you how dynamic analyses done for these kinds of modules. They will carry around another 15 lectures then we will talk about stochastic dynamics in the third module that is what the program is the whole program is available on the website of NPTEL of this course. You can download the syllabus and see how we are covering the first module for eighteen lectures, we have finished twelfth lecture today. Any questions?

Thank you.