

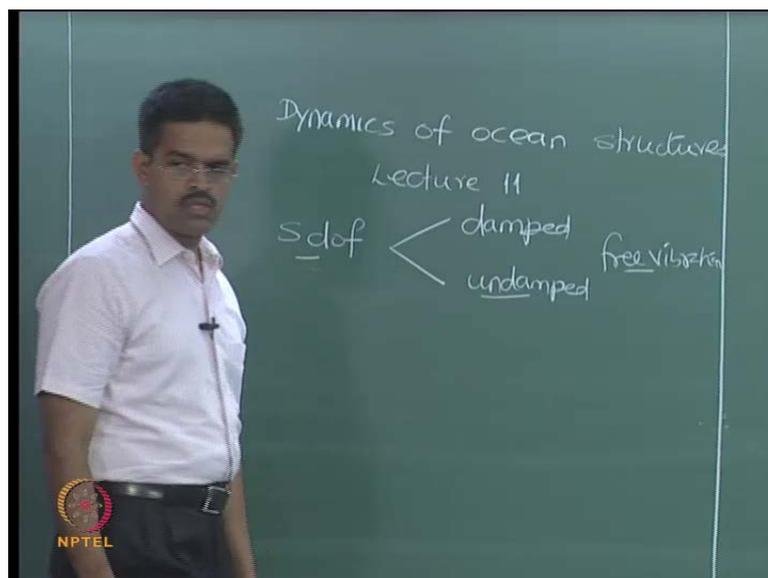
**Dynamics of Ocean Structures**  
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**Module - 1**  
**Lecture - 11**  
**Free and Forced Vibration of Single Degree - of - Freedom Systems**

So, in the last lecture, we discussed about the characters of single degree freedom system. We also understood how to write the equations of motion using five different principles simple harmonic motion, Newton's law, energy method, Raleigh's method and d'Alembert's principle. As we said d'Alembert's principle remains as the concept, because de'Alembert's principle converts the dynamic problem, the equivalent static problem at any given instant of time.

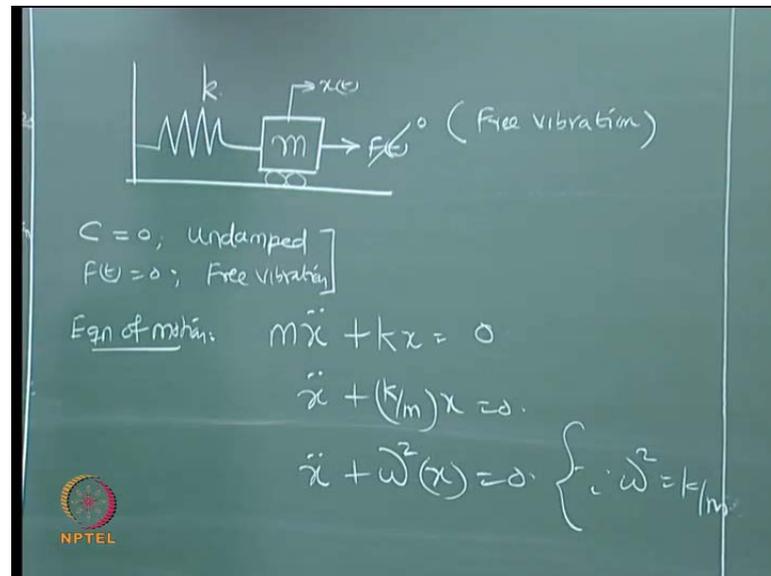
So, it is actually a concept not a reality and it can be successfully employed only for single degree freedom system models, because it is very difficult for you to understand on which modes what initial force has got to be proportionately applied. So, for single degree, there is only one mode of operation therefore, it is easy for us to understand that this can be put for single degree of freedom system models successfully.

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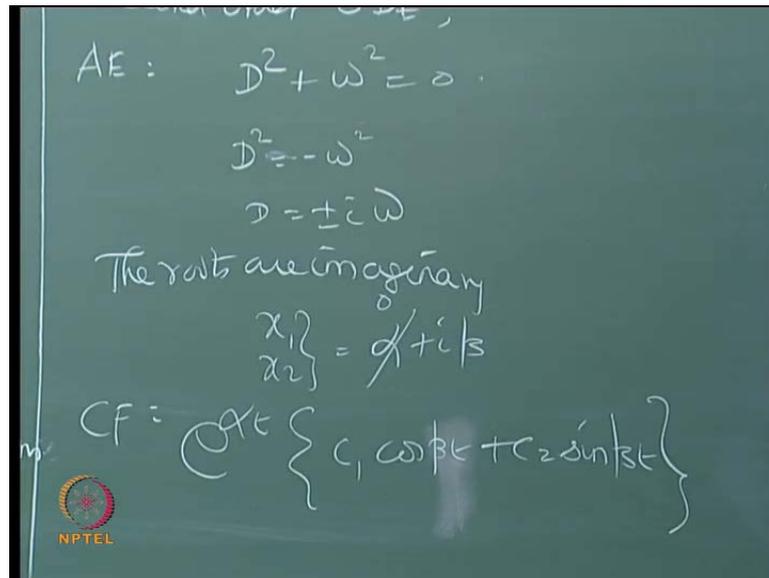
Now you pick up a single degree freedom system undamped free vibration model, and see how to estimate the essential characteristic which is fundamental frequency of vibration of this model.

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So, we all understand that single degree freedom system model idealize spring mass system looks like this. So, this is my  $F$  of  $t$ , I am measuring my  $x$  of  $t$  from the  $c$   $g$  of the mass, I am setting this to 0 I call this as free vibration. Now setting  $c$  to 0, so  $c$  is set to 0. So, it is un-damped and  $F$  of  $t$  is set to 0. So, it is free vibration. So, we already know the equation of motion for this system is given by  $m \ddot{x} + kx = 0$ . So, we can say  $\ddot{x} + k/m \cdot x = 0$ . So,  $\ddot{x} + \omega^2(x) = 0$  where  $\omega^2 = k/m$ .

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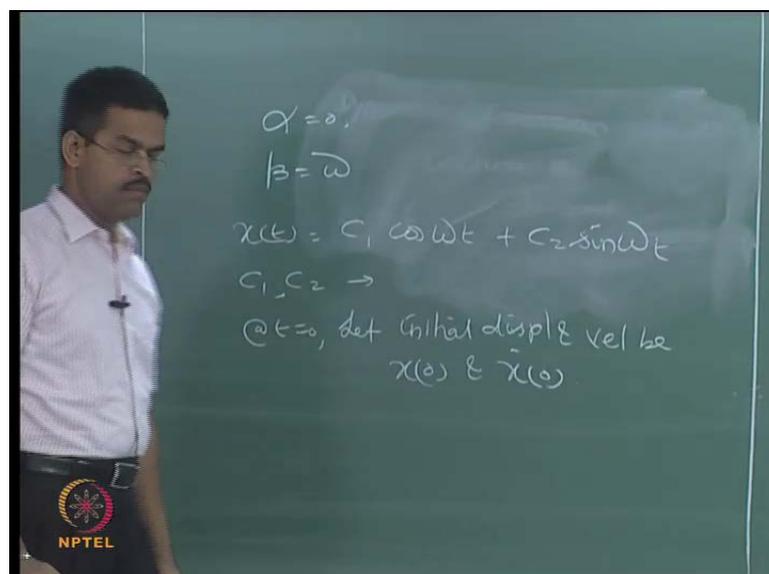


AE:  $D^2 + \omega^2 = 0$   
 $D^2 = -\omega^2$   
 $D = \pm i\omega$   
The roots are imaginary  
 $\left. \begin{matrix} \alpha_1 \\ \alpha_2 \end{matrix} \right\} = \alpha \pm i\beta$   
CF:  $e^{\alpha t} \{ c_1 \cos \beta t + c_2 \sin \beta t \}$

NPTEL

So, this is actually a second order ordinary differential equation, the variable is time. So, it is a time variant problem. So, I must write the auxiliary equation for this problem. I can say this is D square plus omega square 0. So, D square is minus omega square D is plus or minus omega. Therefore, the roots are imaginary. So, one can write basically the root should be alpha plus i beta that is  $\alpha_1 \alpha_2$  will be alpha plus i beta in my case the real part is 0, because it is not there. So, the complementary function for a second order differential equation is simply given by e power alpha t of  $c_1 \cos \beta t$  plus  $c_2 \sin \beta t$  valued in alpha 0 and beta is omega from my comparison.

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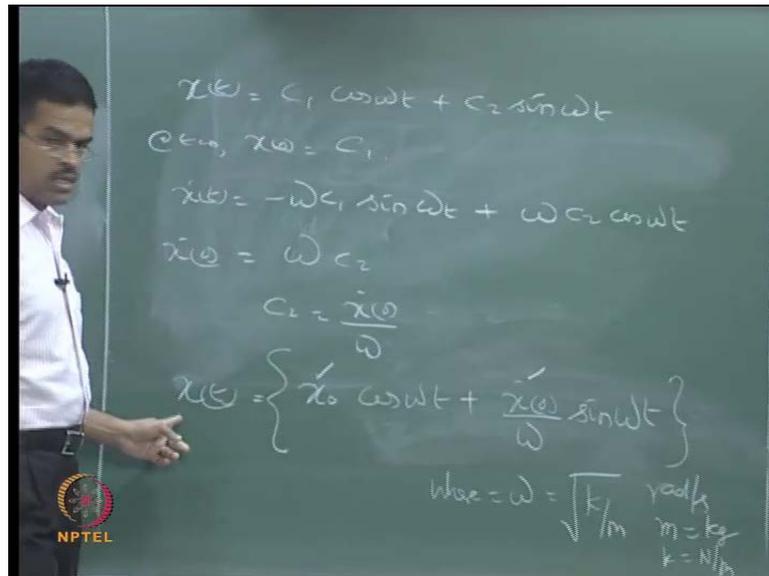


$\alpha = 0$   
 $\beta = \omega$   
 $x(t) = c_1 \cos \omega t + c_2 \sin \omega t$   
 $c_1, c_2 \rightarrow$   
@  $t=0$ , det initial displ & vel be  
 $x(0)$  &  $\dot{x}(0)$

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So, I must say  $x$  of  $t$  is  $c_1 \cos \omega t$  plus  $c_2 \sin \omega t$  now  $c_1$  and  $c_2$  are constants which should be eliminated. So, at  $t$  equal 0 let initial displacement and velocity be  $x_0$  and  $\dot{x}_0$  respectively. So, substitute here and tell me what is your  $x$  of  $t$  ultimately substituting for  $c_1$  and  $c_2$ .

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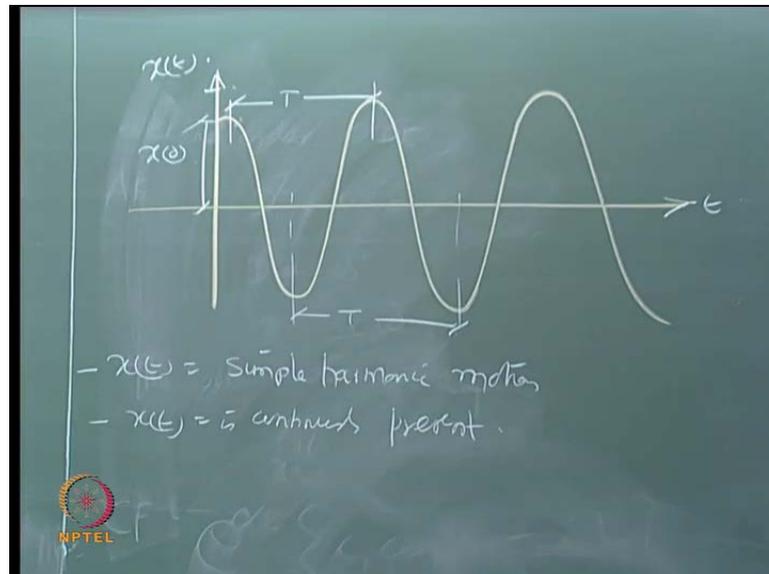


So, my  $x$  of  $t$  is  $c_1 \cos \omega t$  plus  $c_2 \sin \omega t$ . So, at  $t$  equals 0  $x_0$  is the initial displacement I have here. So, that is equal to  $c_1$  because this function goes away. So,  $\dot{x}$  of  $t$  is minus  $\omega c_1 \sin$  of  $\omega t$  plus  $\omega c_2 \cos \omega t$ . So,  $\dot{x}$  of 0 which is the velocity at  $t$  equal to 0 the  $\sin$  function goes away it becomes  $\omega c_2$ . So,  $c_2$  is  $\dot{x}$  of 0 by  $\omega$  therefore,  $x$  of  $t$  a full function will be  $x_0 \cos \omega t$  plus  $\dot{x}_0$  by  $\omega \sin \omega t$ .

So, if you know the initial values of  $x_0$  and  $\dot{x}_0$ , which are displacements and velocities at  $t$  is equal to 0. If you know these initial values and if you know  $\omega$  where  $\omega$  is square root of  $k$  by  $m$  which is structural characteristic it does not depend on the time variant at all. Remember  $\omega$  is not the function of time; if know the stiffness of the system or the elastic restoring force constant, and if know the mass of the system which in a share represented the value of the model. You must be able to work out  $\omega$  in radian per second where mass is given in kg and  $k$  is given in Newton per meter. So, if you know these initial values you will be able to get the full let

us try to plot this and see how it looks like. It is actually a simple harmonic motion with an initial displacement  $x_0$ . So, let me try to plot this.

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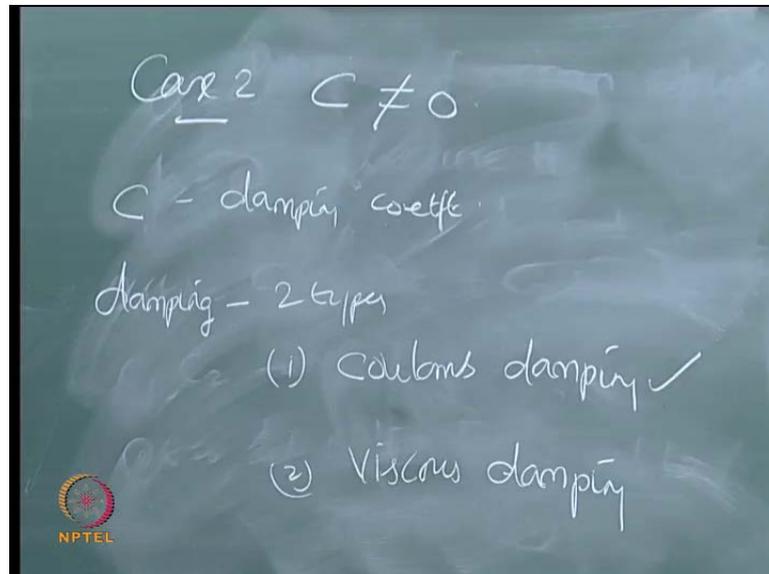


So, this is my function of time, this is my  $x$  of  $t$  and this value is what I call as  $x$  zero. So, pick up any successive  $p$  and that is my time period of vibration one full cycle. You can pick it up even in the negative peaks also, these are also  $t$ . So,  $x$  of  $t$  as you see here is a simple harmonic motion that is the first observation what I see from this figure. The second observation is  $x$  of  $t$  is continuously present and it is not dying down the value the amplitude of  $x$  of  $t$  is constant is continuously present for infinite  $t$ . Whereas in reality when you try to displace the spring mass system and release the force after any long duration of time, the response or the amplitude of the response will dye down and it will come to static state of rest.

Whereas this figure does not show anything of that order it means there is something which is missing in this figure or in this whole idealization which is not really modeling the actual behavior of my spring mass system. Because a spring mass system should come to rest after long cycles of operation, which is not happening from the analytical equation what we just now saw. So, the one which is missing is the damping which is offered by the air or the resistance to the movement of this  $x$  of  $t$ . So, we must model the damping component to it. So, it is an un-damped system I have modeled the damping

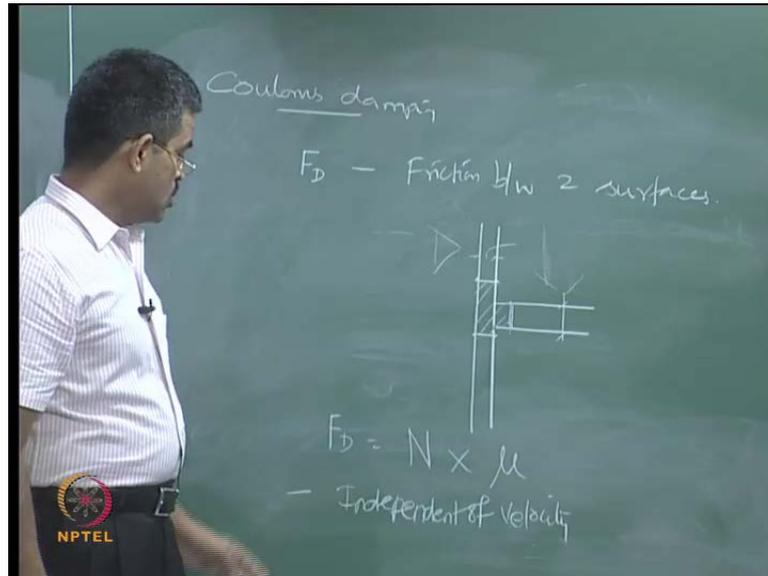
component. So, the absence of damping component gives unrealistic value of  $x$  of  $t$  which is as shown in the equation here I call this equation number one.

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Now let us set case two, let us see  $c$  is no more equal to 0. Now the question is  $c$  is what we call as damping coefficient or constant. There are two kinds of damping exists in the literature, damping are of two types. The first type what we call as viscous damping, let us put it as coulomb damping here. The second is viscous damping, there are special problems associated with these kinds of damping's. We will see them in detail after this lecture is completed. So, there are some special ca features associated with viscous damping, I mean with the coulomb damping why they are actually not popular for fluid structures interaction problems, we will see that later. So, let us quickly understand what we understand by or what do you mean by coulomb damping and viscous damping coulomb damping is a resistance of or the damping force generated, because of friction.

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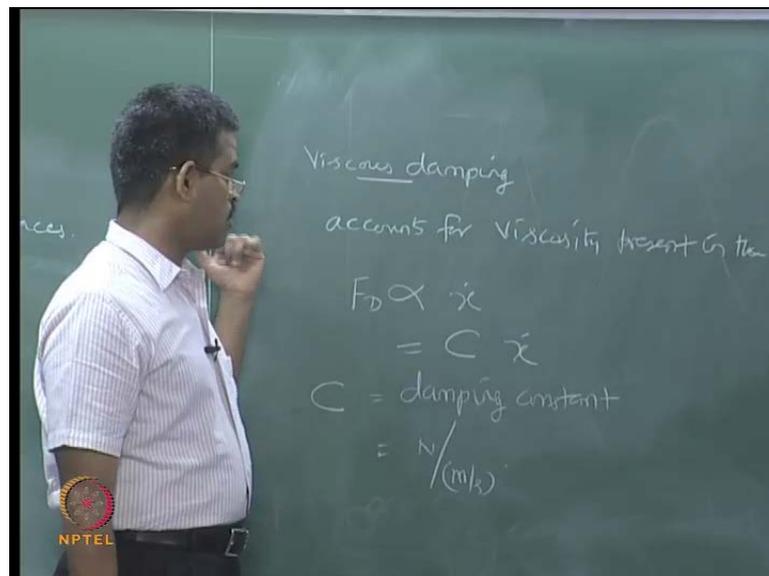
Coulomb damping offers damping force generated because of friction between two surfaces. Now you may wonder where the surfaces are coming in, can I have friction between the liquid surface and the solid surface? Interestingly let us take up a structure or a leg which is connected by a specific joint using a special, this is a joint. The diameter of this member and the diameter of this member are different and the boundary condition of this member and the boundary condition of these members are different.

The load subjected to this member and the load subjected to this member is different. It means the behavior of this member under the combination of load is different from this. Therefore, there is always a possibility that at this joint there can be relative movement between these two members. So, that relative movement can offer frictional resistance. So, frictional resistance need not always arise from the fluid and the structural interaction can even come within the members also.

So, this can also resist the damping or it can offer resistance to the response of the system, because it can dampen, it can reduce the response. So, the damping force in coulomb damping is generated by friction between the two surfaces. So, it is actually equal to the normal force multiplied by the coefficient of friction between the two surfaces that is how it is estimated. It is interesting to know that this force is independent of velocity, though you may wonder that because of the relative movement between the two members at this junction frictional force is generated.

So, that movement should have a capturing value of its displacement in velocity, but that movement will be very very minimum compared to the overall global movement of the structure. So, it is independent of velocity. It depends on the normal force acting on the member and the coefficient of friction between the two surfaces which are sliding one against the other. So, it is independent of velocity and this is how the damping force is given is a normal force is a normal force multiplied by coefficient of friction between the two surfaces of the material which are sliding against each other.

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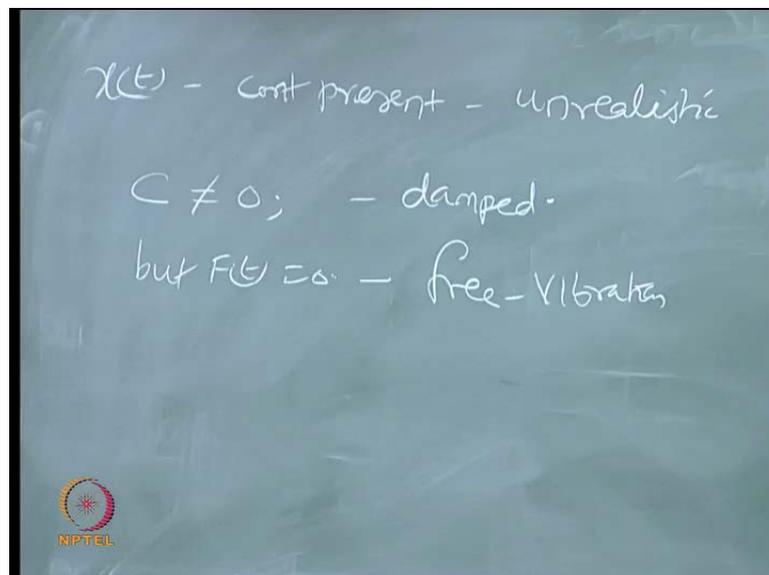
If we look at the viscous damping viscous damping is function of velocity the damping force the viscous damping accounts for, it accounts for the viscosity present in the system. So, the damping force in this case will be proportional to the velocity or can be equal to some constant on the velocity. And this constant is called the damping constant, whose unit can be Newton per meter per second, and that is the unit of this damping constant C.

And this is very famous method of damping adopted in offshore structures or ocean structural system; because we deal with fluid structure interaction and viscosity will be present in the system, therefore our damping proportional to velocity being fixed base structure. Still we will have velocity of the part body or the water particle. If it is a compliant structure, will have the relative velocity between the water particle and the structural member. So, velocity will be predominantly present in both the cases therefore,

our model of using viscous damping for structures or models which has fluid structure interaction is more relevant compared to that of coulomb damping.

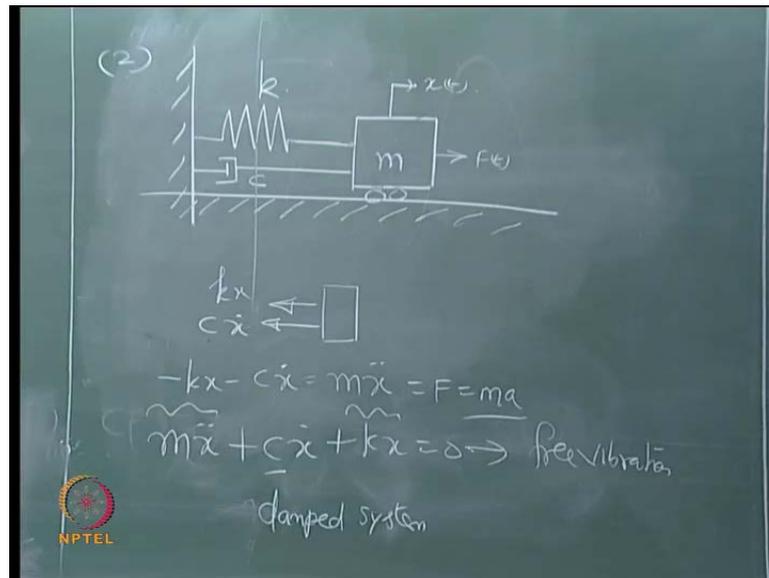
So, we will be generally using viscous damping models in our ocean structural system, that is the reason why we are using this because FSI deals with the component of velocity present predominantly in the system. You may wonder that in a fixed base structure, how the velocity of the structure is important, I am saying the water particle velocity will be anyway important. So, velocity will have a dominant role to play which can offer a damping value.

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So, in the last example that is which is un-damped free vibration model, we already saw that the response of the system is continuously present which is unrealistic, because there will be some damping offered by different sources. The response will dye down after even after a long duration of time, but the analytical equation does not reflect this characteristic in my solution. To incorporate that now we should say that let  $C$  not be equal to 0, but let  $F$  of  $t$  be still remain 0. So, I am still doing now a free vibration analysis or a free vibration response for a damped system. So, I am looking now for a damped free vibration analysis. Any questions here, any questions here, any doubts?

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So, now let us write down the standard model once again. The spring mass system whose mass is known; I have elastic restoring constant which is  $k$  or the stiffness of the system. And now I have also a dash part which is the damping constant given in a system, and of course, I can apply  $F$  of  $t$  here and I can apply  $x$  of  $t$  here. You must understand, there is a very great similarity in all the problems or all the derivations, I am doing. I am measuring the coordinate of displacement of the mass only at the point where the mass is concentrated. I could measure the displacement coordinates anywhere at my origin, anywhere I can measure, I am not doing that. I am always measuring the displacement coordinates only at the C g of the mass, where the mass is lumped.

I am also applying the force only at the point where the mass is being, I can also pull the spring I can push the spring from here, I can move the support I am not doing them. It means, in general I am using the displacement coordinates and the force coordinates at the point where the mass is lumped that is a very important point. We will violate this and see what happens later in multi degree freedom system models. So, this is a single degree freedom system model, because the model is constrained to move only in the horizontal direction that you see here. The model can either jump, the model can move normal to the board already we discussed in the previous lectures, and how we are preventing by putting an additional constraint in the normal to the board mass cannot move in the other fashion. So, it is only in one degree.

So, if I use Newton's law, let us say these are my restoring forces which is  $k$  and  $x$  and this is again  $c \dot{x}$  I am using a viscous model here. So, I should say minus  $kx$  minus  $c \dot{x}$  should be  $m \ddot{x}$  because that is my mass into acceleration force as per the Newton's law. So, I will get  $m \ddot{x} + c \dot{x} + kx = 0$ ; the 0 indicates, it is a free vibration study. The presence of  $C$  indicates that it is a damped system, and this is my inertia component, this my restoring force component which will always present in all the systems.

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$$m\ddot{x} + c\dot{x} + kx = 0$$

$$\Rightarrow \ddot{x} + \left(\frac{c}{m}\right)\dot{x} + \left(\frac{k}{m}\right)x = 0.$$

$$\Rightarrow \ddot{x} + \left(\frac{c}{m}\right)\dot{x} + \omega^2 x = 0$$

$$D^2 + \left(\frac{c}{m}\right)D + \omega^2 = 0. \quad \left\{ \omega^2 = k/m \right\}$$

I am rewriting the same equation back again here. So,  $m \ddot{x} + c \dot{x} + kx = 0$ . Divide by  $m$ , so  $\ddot{x} + c/m \dot{x} + k/m x = 0$ . So,  $\ddot{x} + c/m \dot{x} + \omega^2 x = 0$  where  $\omega^2$  is considered as  $k/m$ . So,  $D^2 + c/m D + \omega^2 = 0$  that is my auxiliary equation for this second order ordinary differential equation.

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$$\alpha_{1,2} = \frac{-\left(\frac{c}{m}\right) \pm \sqrt{\left(\frac{c}{m}\right)^2 - 4\omega^2}}{2}$$
$$\alpha_{1,2} = -\frac{c}{2m} \pm \frac{1}{2} \sqrt{\left(\frac{c}{m}\right)^2 - 4\omega^2}$$
$$\alpha_{1,2} = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \omega^2}$$

So, I can simply write minus  $c$  over  $m$  plus or minus root of  $b$  square minus  $4$  omega square whole by  $2$  will be my roots  $\alpha_1$  and  $\alpha_2$ . So, I will just modify this once again. So,  $\alpha_1$  and  $\alpha_2$  which are the roots of my equation can be minus  $c$  by  $2m$  plus or minus half root of  $c$  by  $m$  the whole square minus  $4$  omega square. This can be further minus  $c$  by  $2m$  plus or minus root of we take it inside  $c$  by  $2m$  the whole square, because this  $4$  comes here minus omega square, because this  $4$  will get divide. So, these are my roots now,  $\alpha_1$   $\alpha_2$ . I will remove this.

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For the roots to be real/distinct -

a) Critical damping

set the Discriminant to ZERO.

$$\left(\frac{c}{2m}\right)^2 - \omega^2 = 0$$
$$\left(\frac{c}{2m}\right)^2 = \omega^2$$
$$\left(\frac{c}{2m}\right) = \omega$$

So for the roots to be real and distinct, I must have  $x$  of  $t$  s for the roots to be real and distinct. So, I understand from here  $\alpha_1$  will be  $-\frac{c}{2m}$  plus of this value, and  $\alpha_2$  will be  $-\frac{c}{2m}$  minus of this value. So, the roots can be distinct can be real, because there is no imaginary component in this. So, the imaginary component come into play depending upon what could be the value of these two relatively; if this value is higher than the root can become imaginary; if this value is higher the root can become positive. So, we do not know whether the roots will be real or distinct, it means to understand this roots more in detail I must work on this part mathematically what we call as discriminant of this equation. Let me pick up this. So a, I am looking for what we call in the literature critical damping critical damping mathematically says set the discriminant to 0. The moment I do that when I set this discriminant, the argument inside this parenthesis is called discriminant. If I set this to 0 the roots will become real and equal. They are not distinct, because both the roots will be  $-\frac{c}{2m}$ , because this is going to be 0. But I want to pick up that first and see what happens. So,  $\frac{c}{2m}$  the whole square minus  $\omega$  square is set to 0. So,  $\frac{c}{2m}$  the whole square is  $\omega$  square  $\frac{c}{2m}$  is  $\omega$ .

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$$c_{cr} = 2m\omega$$

$$c_{critical} = 2m\omega \quad (3)$$

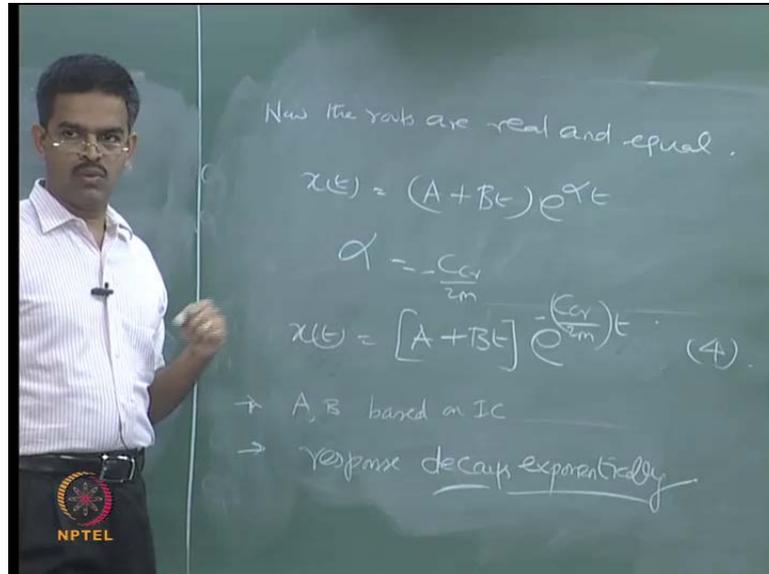
$$c_{cr} = 2m\sqrt{\frac{k}{m}} \quad \left\{ \because \omega = \sqrt{\frac{k}{m}} \right.$$

$$c_{cr} = 2\sqrt{k m}$$

So, I say  $c_{cr}$  is  $2m\omega$ . I am using this  $c_{cr}$  saying  $c_{critical}$  is  $2m\omega$ . Of course, this equation was two. One can also expand this slightly, because  $\omega$  is known as root of  $k$  by  $m$ . So,  $c_{cr}$  can be  $2m$  root by  $k$  by  $m$  I can say this as  $2\sqrt{k m}$  that is also there. Some literature will give you  $c_{cr}$  as in this format run at the same.

Already we know omega is root of k by m. So, one can write in either form. Since the discriminant is 0, now the roots are real and equal.

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Now the roots are real and equal, therefore x of t can be simply said as A plus B t of e alpha t where alpha in my case is C c r by 2 m. So, C c r by 2 m, therefore x of t can be A plus B t of e C c r by 2 m of t minus ok. The root is minus c by 2 m is it not? The roots where alpha 1 and alpha 2 where minus c by 2 m plus or minus this is 0. So, there is minus here right that is the minus c.

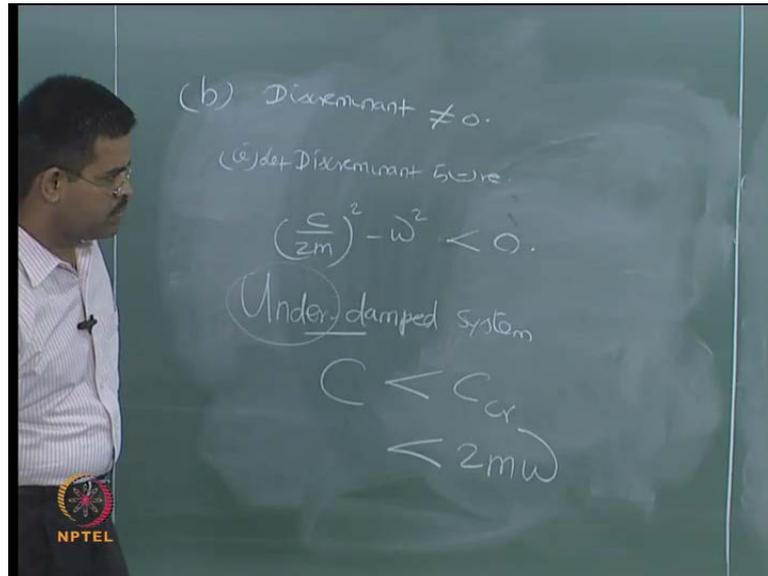
So, this indicates very simple, this is my fourth equation I am having. This indicates some inferences to me. Of course, A and B are constants which can be eliminated depending upon the initial conditions, based on initial conditions. As we used some initial condition is the last example; same way at t equal to 0 x 0 ,and x dot 0 can be displacement velocities, I can use and substitute them I will get A and B. Any two conditions, because there are two constants I can use that. The most importantly, the moment I introduce damping in the system, you will see that the response is decaying exponentially is very important. The response decays, because minus exponentially because it is e power minus, so it is decaying exponentially which is the realistic value. Where the response will dyed on after some instant of time or after some span of time which is happening from the equation number four as shown here.

Whatever may be the initial condition, whatever may be the values of A and B, it is very clearly understood that after a long span of time, and after some time  $t$  you will be keep on decreasing and at one given point of time  $x$  will become 0. Ok, which was expected in the original behavior, which was not reflected in the earlier equation of free vibration response where un-damped system was shown. So, damping is a inherent component of my single degree freedom system model, I cannot eliminate it, because it shows to me an unrealistic behavior otherwise. So, I have to include damping; and in this model, I picked the viscous damping, I will speak about coulomb damping slightly later once I complete this experiment or this exercise.

So, as I said why we are using coulomb damping in ocean structural systems, because we have got lot of interest towards viscous application in this FSI model, therefore we using it. Does not mean that we cannot use coulomb damping, there are some difficulties associated with coulomb damping. We will see them. Any doubt here or anything to be added to the inference from this equation? Are you all getting it how we are getting  $x$  of  $t$  so far? There is no confusion right.

So, if you look at this equation, you will see there is minus  $c$  by  $2m$  here. The original, I have rubbed it off, minus is there means that minus is here  $\alpha$  will be negative. Instead of  $c$  by  $2m$ , using  $C$   $c$   $r$  by  $2m$ , because I am calling this as critical damping. I mean name given in the literature. Remaining I am not changing anything; I am using the same equation back. Any doubt, shall we proceed further? Now critical damping systems have some demerits, we have seen only one case in the discriminant setting it to 0 discriminant may not be set to 0 also. So, let us do the remaining two case also discriminant can be negative it can be positive also. So, let us see the remaining two cases then will dialogue on what would be the comparison between critically damped that is  $d < 0$   $d$  negative and  $d$  positive will compare possibly in the next lecture, so now will take away.

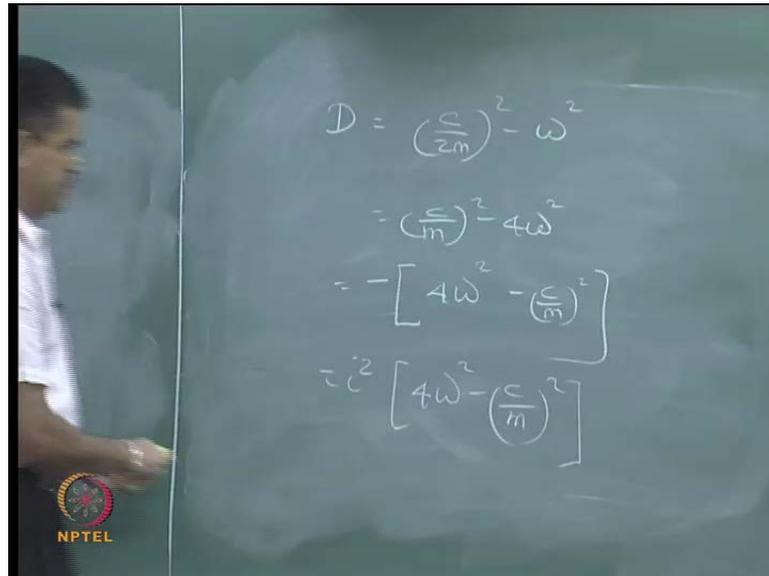
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Now, when  $d$  is not 0 that is case b, the discriminant not set to 0. Case a was discriminant set to 0, they have argued and we got the value. Now I do not want the discriminant to set to 0. So, under this, there are two cases the discriminant can be negative, it can be positive. We will take discriminant negative that is discriminant is negative. What does it mean is let us write down the equation  $c$  by  $2m$  the whole square minus  $\omega$  square that is my discriminant; this is my discriminant -  $c$  by  $2m$  the whole square minus  $\omega$  square is less than 0, negative. So, I will call this as under damped system.

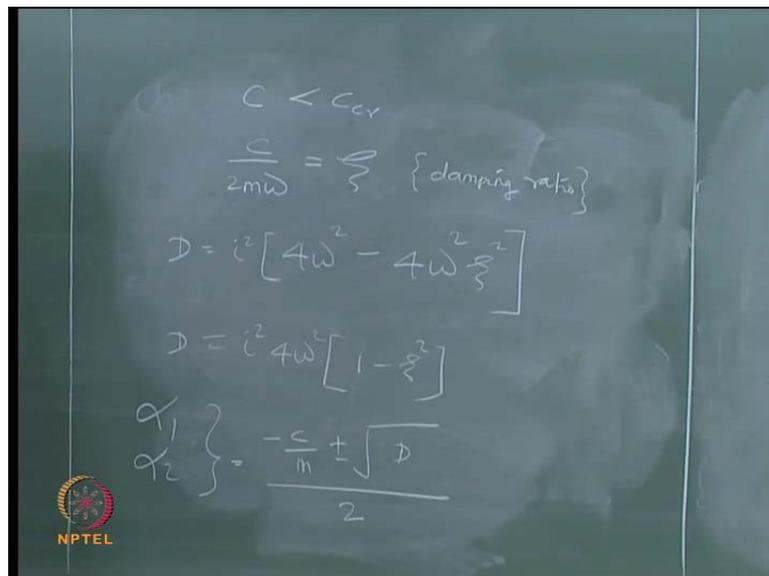
So, I call the damping constant as less than that of  $c$  critical, because it is under damped. So, the damping constant will be lesser than the critical damping; obviously, one can expect if you put discriminant as positive I call that as over damped system. And I understand that  $c$  will be greater than  $C c r$ , and the discriminant will become positive. In this case discriminant is negative, I am classifying it myself I am taking it. Let be negative, there also it was let  $d$  be 0, because we are setting it. We are imposing a condition. It is not happening by itself, we are setting a condition and seeing what is happening to my solution procedure. So, in that case I call this as under damped system and my damping constant  $c$  will be lesser than  $C c r$ . So, I should say that  $c$  will be lesser than  $2m\omega$ , because already we know  $C c r$  is  $2m\omega$ ;  $C c r$  is  $2m\omega$ . We already know this.

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$$\begin{aligned} D &= \left(\frac{c}{2m}\right)^2 - \omega^2 \\ &= \left(\frac{c}{m}\right)^2 - 4\omega^2 \\ &= -\left[4\omega^2 - \left(\frac{c}{m}\right)^2\right] \\ &= -i^2 \left[4\omega^2 - \left(\frac{c}{m}\right)^2\right] \end{aligned}$$

So, let us pick up the discriminant D slightly modify this is going to be c by 2 m the whole square minus omega square. I can modify this as c by m the whole square minus 4 omega square or I can say minus of 4 omega square. So, I can rewrite this as I square of 4 whole squares. So, I will take away this.

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$$\begin{aligned} c &< C_{cr} \\ \frac{c}{2m\omega} &= \zeta \quad \{\text{damping ratio}\} \\ D &= i^2 \left[4\omega^2 - 4\omega^2 \zeta^2\right] \\ D &= i^2 4\omega^2 \left[1 - \zeta^2\right] \\ \left. \begin{matrix} \alpha_1 \\ \alpha_2 \end{matrix} \right\} &= \frac{-\frac{c}{m} \pm \sqrt{D}}{2} \end{aligned}$$

Now we already know that c is less than C c r or C by 2 m omega that is nothing but C c r I consider this as a ratio call zeta is called damping ratio. So, getting it back D - discriminant D will be I square of 4 omega square minus c by m. So, I should say 2

omega zeta the whole square that is 4 omega square zeta square is that ok, c by m is already here. So, c by m is 2 omega zeta, I have a square value I have a square value here substitute that I will get 4 omega square zeta square . So, now, simplifying I can say i square 4 i square 4 omega square one minus zeta square that is D. So, alpha 1 and alpha 2 which are my roots will be minus c by m plus or minus root of D that is discriminant divided by 2. Now substitute back and tell me what are alpha 1 and alpha 2.

(Refer Slide Time: 37:22)

The image shows a chalkboard with the following handwritten content:

$$\alpha_{1,2} = -\frac{c}{2m} \pm i \omega \sqrt{1 - \zeta^2}$$

$\omega_D$

$\omega_D = \text{damped frequency}$

$$\alpha_{1,2} = \underbrace{-\frac{c}{2m}}_{\text{real}} \pm i \underbrace{\omega_D}_{\text{imag}} = \alpha \pm i\beta$$

$$C.F. = e^{\alpha t} \left\{ c_1 \cos \beta t + c_2 \sin \beta t \right\}$$

An NPTEL logo is visible in the bottom left corner of the chalkboard image.

So, I get alpha 1, alpha 2 as minus c by 2 m plus or minus. So, if I take root of this i 2 omega, two 2 goes away, i omega one minus zeta square is it all right. I call this as omega D where omega D is called damped frequency. So, now I can say alpha 1, alpha 2 which are the roots can be minus c by 2 m plus or minus i omega D. So, this is my real part; this is my imaginary part. Now I can write a complementary function for this second order differential equation as e to the power of alpha t c 1 cos beta t plus c 2 sin beta t where the roots are expressed as alpha plus or minus i beta; alpha is the real part and beta are the imaginary part. Substitute back write the equation x of t, so what is x of t, x of t - e to the power of minus c by 2 m of t c 1 cos omega d t.

(Refer Slide Time: 39:17)

$$x(t) = e^{-\left(\frac{c}{2m}\right)t} \left\{ c_1 \cos \omega_D t + c_2 \sin \omega_D t \right\} \quad (5)$$

-  $c_1, c_2$  - IC  
-  $c \neq 0$ ,  $x(t)$  - exponentially decaying  
 $e^{-\left(\frac{c}{2m}\right)t}$

So,  $x$  of  $t$  -  $e$  to the power of  $\alpha$  is minus  $c$  by  $2m$  of  $t$   $c_1 \cos \omega_D t$  plus  $c_2 \sin \omega_D t$  that is my equation number, is it 6? 5. Now again there are constants  $c_1$  and  $c_2$  which are in this equation which can be eliminated by providing some initial conditions. So,  $c_1, c_2$ , remember the  $c_1$  and  $c_2$  are not damping constant if you are confused with the terms  $c$  you can use it  $a$  and  $b$  also.  $C$  what we are using here is a damping constant or coefficient,  $c_1$  and  $c_2$  are some constants for the differential equation if you are confused with the terms  $c_1, c_2$  with respect to  $c$  say them as  $a$  and  $b$  that is up to you.

So,  $c_1$  and  $c_2$  are the constants can be eliminated based on some initial conditions. You can apply some initial conditions and we can eliminate them, we can estimate them. What is  $c$  here is the moment you introduced damping when  $c$  is not 0, the response of  $x$  of  $t$  is again exponentially decaying, because of this term  $e$  to the power of minus  $c$  by  $2m$  of  $t$ . So, it is exponentially decaying which is again a realistic representation of the behavior of the spring mass system under free vibration. Now I want to explore this response in detail and see actually what happens how this response looks like on successive peaks, well that is important for me to estimate the value of damping. Let us see how you do that. Any questions here, any doubt?

Generally if you look at dynamics books, all the references given in the web site, all of them talk about the structural dynamics, and in all structural dynamics books this preliminary derivations are given. Very few books will make it clear to understand, I am

sorry to say this, but that is a fact - very few books. And most of the books will skip many of the sequential steps, which is very dangerous for person to understand. The moment you do not understand the sequence of steps being followed, either you will skip reading that or you will feel sleepy, either thing will happen. And it will result to a very interesting phenomena that you will start memorizing things. The moment you are memorizing because we have extra ordinary memory and remembrance capacities which has been generated by a very good diet system and sports system. So, we are not able to reproduce at any given point of time in particular during the examination very badly during comprehensive and very very badly during interviews, I mean they will ask something, you will be telling something else.

And since there is a mathematics flavor available here people are extra ordinarily brilliant in mathematics. So, therefore, they have a very strong affinity towards love and affection towards mathematics. So, they develop a much inbuilt fear for memorizing or remembering these equations; with that fear if you open any book and try to understand, sir can I understand the derivations again? you will never. If you want to understand them, you must understand them again by your own handwriting from the beginning, start from writing equation of motion and start doing it on your own.

The best thing is this only any time during your life till your death somebody asks you what would be the characteristic of my un- damped free vibration or force vibration system derive it from the beginning and then tell what it is. Because generally you will understand many things we have discussed in the black board that will come to your memory; this is what I have practiced. So, I am making it compulsory that I am deriving it on the board, I could also project in a power point and show you only the equations. So, you will have the same extra ordinary super flavor which we have been having earlier what will type to continue to have in future from the respective people which you will meet or whom you meet.

So that it may look very simple, but once you do it without my assistance, you will find the vulnerability on this equations. It does not stop here, it had started here; it will go further deeper and deeper and expand horizontally and vertically you will understand the gravity of the problem once you move deeper and deeper, it has just started. So, there is a problem you will not be able to appreciate it, we have got only five minutes. I do not want to hurry up this discussion; it is a very important discussion for me. It is a very

important discussion, I do not want to hurry up about this equation, because I want to look into the successive peaks in this decay and then I want to derive what I call the damping ratio from this. And I want to speak about the experimental validation of these data's in reality. So, I do not want to hurry up in couple of minutes. So, I will I mean, I prefer to stop the lecture here, I will continue in next class; no hurry.

If you have any questions, I will be happy to answer; if you do not have questions, I have some very strong recommendation for you whatever derivation we have so far done. We have done only for two cases rather second case a and b. So, if you consider that three cases, re derive them again then follow any one book of your choice; any one book then pick up the book at random ones, because all books will speak about this. All books on dynamics will talk about this which are available in references; any one book also look into any one research paper which addresses dynamics of offshore structures and see are you familiar with the terms being used in the paper, parallelly do this. Because in the papers, they will talk about viscous damping characteristics, some papers talk about the initial conditions, some paper talk about the damping ratio depending upon what the author is addressing.

So, start reading them parallelly, because every paper on dynamics will talk about free vibration. So, read them parallelly, and try to understand and follow one book at least. And then if you have difficulty in the next class, sir you have done something wrong, there is no minus there the author says it is plus and the author explains why it is plus, we will again will re discuss them here re visit them here. Because in my opinion, I think, I have not done any mistake, but if you fell there is a mistake done, we will re visit again this derivation in the next lecture. Otherwise, we proceed further for more interesting mathematics coming up in the next lecture. So, any doubt, any questions?

Thanks.