Module 3

Process Control

Lesson
12
P-I-D Control

Instructional Objectives

At the end of this lesson, the student should be able to:

- Write the input-output relationship of a P-I-D controller
- Explain the improvement of transient response in closed loop with P-controller
- Explain the presence of offset in presence of simple P-controller
- Define Proportional Band
- Explain the elimination of steady state error with Integral Control.
- Define the error transfer function and compute steady state error
- Explain the advantages of P-I controller over simple P and I actions
- Explain the effect of P-D controller
- Recommend a suitable controller configuration for a particular process.

Introduction

In the last lesson, a brief introduction about a process control system has been given. The basic control loop can be simplified for a single-input-single-output (SISO) system as in Fig.1. Here we are neglecting any disturbance present in the system.

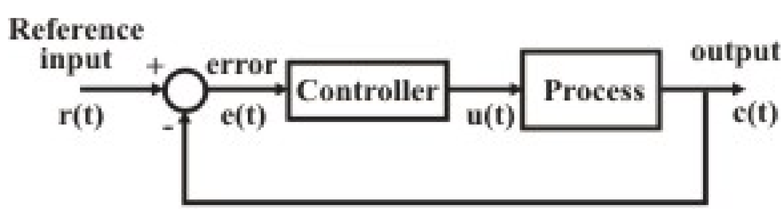

Fig. 1 A closed loop SISO system

The controller may have different structures. Different design methodologies are there for designing the controller in order to achieve desired performance level. But the most popular among them is Proportional-Integral-derivative (PID) type controller. In fact more than 95% of the industrial controllers are of PID type. As is evident from its name, the output of the PID controller u(t) can be expressed in terms of the input e(t), as:

$$u(t) = K_p \left[e(t) + \tau_d \frac{de(t)}{dt} + \frac{1}{\tau_i} \int_0^t e(\tau) d\tau \right] \qquad (1)$$

and the transfer function of the controller is given by:

$$C(s) = K_p \left(1 + \tau_d s + \frac{1}{\tau_i s} \right) \qquad (2)$$

The terms of the controller are defined as:

K_p = Proportional gain

τ_d = Derivative time, and

τ_i = Integral time.

In the following sections we shall try to understand the effects of the individual components- proportional, derivative and integral on the closed loop response of this system. For the sake of simplicity, we consider the transfer function of the plant as a simple first order system without time delay as:

$$P(s) = \frac{K}{1+\tau s} \quad (3)$$

Proportional control

With the proportional control action only, the closed loop system looks like:

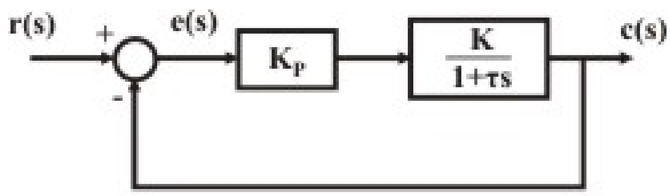

Fig. 2 Proportional Control

Now the closed loop transfer function can be expressed as:

$$\frac{c(s)}{r(s)} = \frac{\frac{KK_p}{1+\tau s}}{1+\frac{KK_p}{1+\tau s}} = \frac{KK_p}{1+KK_p+\tau s} = \frac{KK_p}{1+KK_p}\frac{1}{1+\tau' s} \quad (4)$$

where $\tau' = \frac{\tau}{1+KK_p}$.

For a step input $r(s) = \frac{A}{s}$,

$$c(s) = \frac{KK_p}{1+KK_p}\frac{A}{s\left(1+\tau' s\right)}$$

or, $$c(t) = \frac{AKK_p}{1+KK_p}\left(1-e^{-s\tau/\tau'}\right) \quad (5)$$

The system response is shown in Fig. 2.

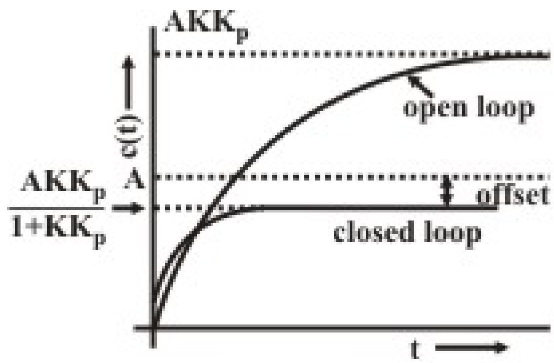

Fig. 3 Response with a proportional controller

From eqn. (5) and Fig. 2, it is apparent that:

1. The time response improves by a factor $\frac{1}{1+KK_p}$ (i.e. the time constant decreases).
2. There is a steady state offset between the desired response and the output response = $A\left(1-\frac{KK_p}{1+KK_p}\right)=\frac{A}{1+KK_p}$.

This offset can be reduced by increasing the proportional gain; but that may also cause increase oscillations for higher order systems.

The offset, often termed as "steady state error" can also be obtained from the error transfer function and the error function *e(t)* can be expressed in terms of the Laplace transformation form:

$$e(s)=\frac{1}{1+\frac{KK_p}{1+\tau s}}\frac{A}{s}=\frac{1+\tau s}{1+KK_p+\tau s}\frac{A}{s}$$

Using the final value theorem, the steady state error is given by:

$$e_{ss}=\underset{t\to\infty}{Lt}\ e(t)=\underset{s\to 0}{Lt}\ s\,e(s)=\underset{s\to 0}{Lt}\frac{1+\tau s}{1+KK_p+\tau s}\frac{A}{s}=\frac{A}{1+KK_p}$$

Often, the proportional gain term, K_p is expressed in terms of "Proportional Band". It is inversely proportional to the gain and expressed in percentage. For example, if the gain is 2, the proportional band is 50%. Strictly speaking, proportional band is defined as the %error to move the control valve from fully closed to fully opened condition. However, the meaning of this statement would be clear to the reader afterwards.

Integral Control

If we consider the integral action of the controller only, the closed loop system for the same process is represented by the block diagram as shown in Fig. 3.

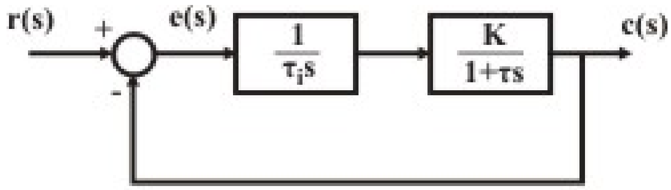

Fig. 4 Integral control action

Proceeding in the same way as in eqn. (4), in this case, we obtain,

$$\frac{c(s)}{r(s)} = \frac{\dfrac{K}{\tau_i s(1+\tau s)}}{1+\dfrac{K}{\tau_i s(1+\tau s)}} = \frac{K}{K+\tau_i s+\tau\tau_i s^2}$$

From the first observation, it can be seen that with integral controller, the order of the closed loop system increases by one. This increase in order may cause instability of the closed loop system, if the process is of higher order dynamics.

For a step input $r(s)=\frac{A}{s}$,

$$e(s) = \frac{1}{1+\dfrac{K}{\tau_i(1+\tau s)}}\frac{A}{s} = \frac{\tau_i s(1+\tau s)}{\tau_i s(1+\tau s)+K}\frac{A}{s}$$

$$e_{ss} = \underset{s\to 0}{Lt}\, s\,e(s) = 0$$

So the major advantage of this integral control action is that the steady state error due to step input reduces to zero. But simultaneously, the system response is generally slow, oscillatory and unless properly designed, sometimes even unstable. The step response of this closed loop system with integral action is shown in Fig. 4.

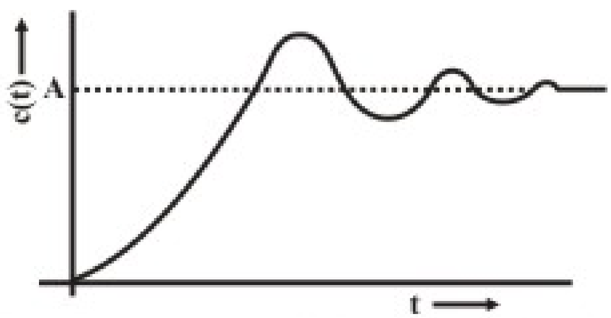

Fig. 5 Step response with integral control action

Proportional Plus Integral (P-I) Control

With P-I controller the block diagram of the closed loop system with the same process is given in Fig. 5.

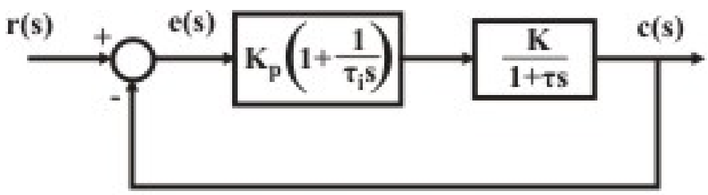

Fig. 6 Proportional plus Integral Control action

It is evident from the above discussions that the P-I action provides the dual advantages of fast response due to P-action and the zero steady state error due to I-action. The error transfer function of the above system can be expressed as:

$$\frac{e(s)}{r(s)} = \frac{1}{1+\dfrac{KK_p(1+\tau_i s)}{\tau_i s(1+\tau s)}} = \frac{\tau_i s(1+\tau s)}{s^2\tau\tau_i + (1+KK_p)\tau_i s + KK_p}$$

In the same way as in integral control, we can conclude that the steady state error would be zero for P-I action. Besides, the closed loop characteristics equation for P-I action is:

$$s^2\tau\tau_i + (1+KK_p)\tau_i s + KK_p = 0\,;$$

from which we can obtain, the damping constant as:

$$\xi = \left(\frac{1+KK_p}{2}\right)\sqrt{\frac{\tau_i}{KK_p\tau}}$$

whereas, for simple integral control the damping constant is:

$$\xi = \left(\frac{1}{2}\right)\sqrt{\frac{\tau_i}{K\tau}}$$

Comparing these two, one can easily observe that, by varying the term K_p, the damping constant can be increased. So we can conclude that by using P-I control, the steady state error can be brought down to zero, and simultaneously, the transient response can be improved. The output responses due to (i) P, (ii) I and (iii) P-I control for the same plant can be compared from the sketch shown in Fig. 6.

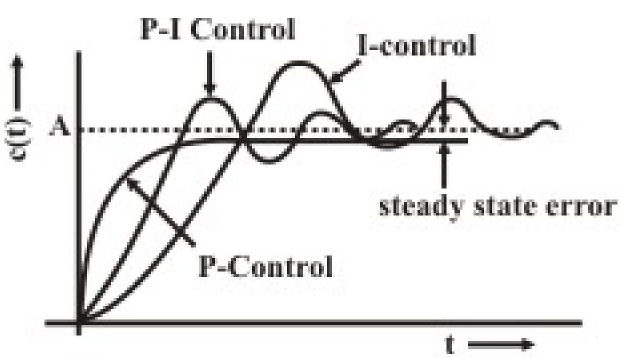

Fig. 7 Comparison among the transient responses with P, I and P-I control

Proportional Plus Derivative (P-D) Control

The transfer function of a P-D controller is given by:

$$C(s) = K_p(1+\tau_d s)$$

P-D control for the process transfer function $P(s) = \frac{K}{1+\tau s}$ apparently is not very useful, since it cannot reduce the steady state error to zero. But for higher order processes, it can be shown that the stability of the closed loop system can be improved using P-D controller. For this, let us take up the process transfer function as $P(s) = \frac{1}{Js^2}$. Looking at Fig.7, we can easily conclude that with proportional control, the closed loop transfer function is

$$\frac{c(s)}{r(s)} = \frac{\frac{K_p}{Js^2}}{1+\frac{K_p}{Js^2}} = \frac{K_p}{Js^2 + K_p}$$

and the characteristics equation is $Js^2 + K_p = 0$; giving oscillatory response. But with P-D controller, the closed loop transfer function is:

$$\frac{c(s)}{r(s)} = \frac{\frac{K_p(1+\tau_d s)}{Js^2}}{1+\frac{K_p(1+\tau_d s)}{Js^2}} = \frac{K_p(1+\tau_d s)}{Js^2 + K_p(1+\tau_d s)}$$

whose characteristics equation is $Js^2 + K_p\tau_d s + K_p = 0$; that will give a stable closed loop response.

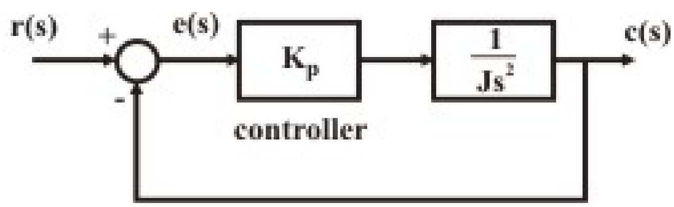

Fig. 8 Control action with a higher order process

The step responses of this process with P and P-D controllers are compared in Fig.8.

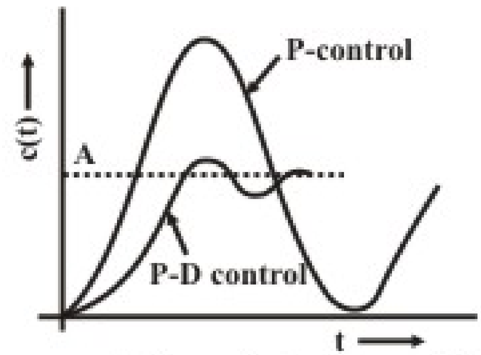

Fig. 9 Improvement of transient response with P-D control

Proportional-Integral-Derivative (PID) control

It is clear from above discussions that a suitable combination of proportional, integral and derivative actions can provide all the desired performances of a closed loop system. The transfer function of a P-I-D controller is given by:

$$C(s) = K_p\left(1 + \tau_d s + \frac{1}{\tau_i s}\right)$$

The order of the controller is low, but this controller has universal applicability; it can be used in any type of SISO system, e.g. linear, nonlinear, time delay etc. Many of the MIMO systems are first decoupled into several SISO loops and PID controllers are designed for each loop. PID controllers have also been found to be robust, and that is the reason, it finds wide acceptability for industrial processes. However, for proper use, a controller has to be tuned for a particular process; i.e. selection of P,I,D parameters are very important and process dependent. Unless the parameters are properly chosen, a controller may cause instability to the closed loop system. The method of tuning of P,I,D parameters would be taken up in the next lesson.

It is not always necessary that all the features of proportional, derivative and integral actions should be incorporated in the controller. In fact, in most of the cases, a simple P-I structure will suffice. A general guideline for selection of Controller mode, as suggested by Liptak [1], is given below.

Guideline for selection of controller mode

1. Proportional Controller: It is simple regulating type; tuning is easy. But it normally introduces steady state error. It is recommended for process transfer functions having a pole at origin, or for transfer functions having a single dominating pole; for example with

$$P(s) = \frac{K}{(1+s\tau_1)(1+s\tau_2)(1+s\tau_3)}; with\ \tau_1 \gg \tau_2, \tau_1 \gg \tau_3.$$

2. Integral Control: It does not exhibit steady state error, but is relatively slow responding. It is particularly effective for:

(i) very fast process, with high noise level
(ii) process dominated by dead time
(iii) high order system with all time constants of the same magnitude.

3. Proportional plus Integral (P-I) Control: It does not cause offset associated with proportional control. It also yields much faster response than integral action alone. It is widely used for process industries for controlling variables like level, flow, pressure, etc., those do not have large time constants.

4. Proportional plus Derivative (P-D) Control: It is effective for systems having large number of time constants. It results in a more rapid response and less offset than is possible by pure proportional control. But one must be careful while using derivative action in control of very fast processes, or if the measurement is noisy (e.g. flow measurement).

5. Proportional plus Integral plus Derivative (P-I-D) Control: It finds universal application. But proper tuning of the controller is difficult. It is particularly useful for controlling slow variables, like pH, temperature, etc. in process industries.

Conclusion

In this lesson, the basic functions of a P-I-D controller have been explained. Most of the industrial controllers are P-I-D in nature. The major reasons behind the popularity of P-I-D controller are its simplicity in structure and the appilicability to variety of processes. Moreover the controller can be tuned for a process, even without detailed mathematical model of the process. However, proper tuning of the controller parameters requires extensive experimentation. The methods for controller tuning would be discussed in the next lesson.
Crudely speaking, the desired closed loop performances, such as fast response, zero steady state error and less overshoot are achieved through incorporation of P,I and D actions respectively. But the choice of P-D, P-I or P-I-D structure depends on the type of the process we intend to control. A brief guideline for selection of controller is provided in this lesson.
There are few more issues those need to be addressed while using P-I controller. The most important among them is the anti-windup control. Further details about anti-windup would be discussed in Lesson 14.

References

1. B. Liptak: Process Control: Instrument Engineers Handbook

Review Questions

1. A P-I controller has a proportional band of 50% and integration time of 2sec. Find the transfer function of the controller.
2. The transfer function of a first order plant is $G(s) = \frac{2}{1+2s}$. It is used in a unity feedback system as shown in Fig. 2 with a proportional controller of proportional band 100%. Find the steady state error for a unit step input, and the time constant of the closed loop system.
3. Repeat problem 2 if proportional band is 50%.
4. What would be the steady state error for the plant in problem-2 if the transfer function of the controller is $G_c(s) = 2(1+\frac{1}{2s})$?
5. Incorporation of P-I action may lead to instability in the closed loop performance- justify.
6. How does incorporation of derivative action in the controller improve the closed loop performance?
7. Why derivative control is not recommended for a flow control process?
8. What type of controller would you recommend for control of pH level in a liquid?