Module
3

Limit State of Collapse - Flexure (Theories and Examples)

Lesson 4

Computation of Parameters of Governing Equations

Instructional Objectives:

At the end of this lesson, the student should be able to:

- identify the primary load carrying mechanisms of reinforced concrete beams and slabs,
- name three different types of reinforced concrete beam with their specific applications,
- identify the parameters influencing the effective widths of *T* and *L*-beams,
- differentiate between one-way and two-way slabs,
- state and explain the significance of six assumptions of the design,
- draw the stress-strain diagrams across the depth of a cross-section of rectangular beam,
- write the three equations of equilibrium,
- write and derive the expressions of total compression and tension forces *C* and *T*, respectively.

3.4.1 Introduction

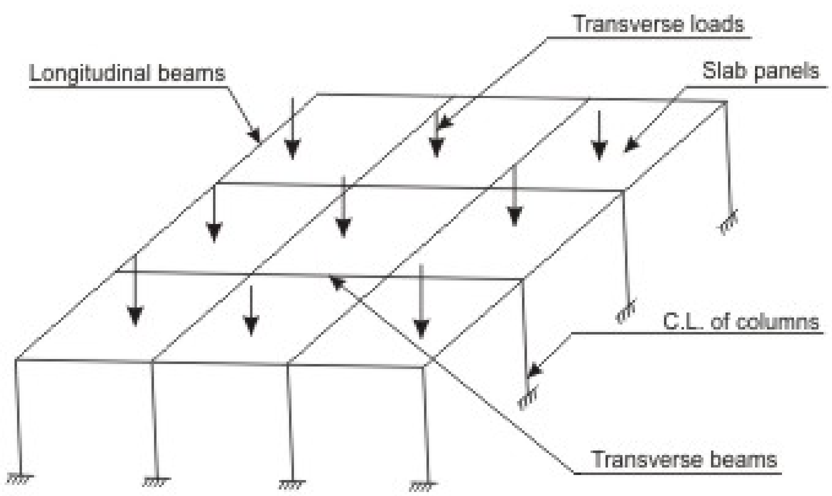

Fig. 3.4.1: Slab-beam-column system under transverse loads

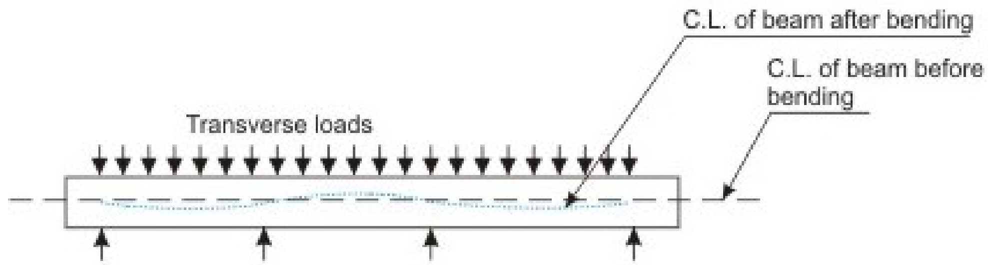

Fig. 3.4.2 : Beam under bending

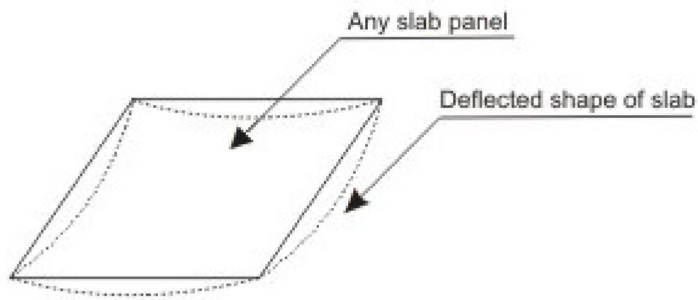

Fig. 3.4.3: Slab under bending (supported at corners)

Reinforced concrete beams and slabs carry loads primarily by bending (Figs. 3.4.1 to 3). They are, therefore, designed on the basis of limit state of collapse in flexure. The beams are also to be checked for other limit states of shear and torsion. Slabs under normal design loadings (except in bridge decks etc.) need not be provided with shear reinforcement. However, adequate torsional reinforcement must be provided wherever needed.

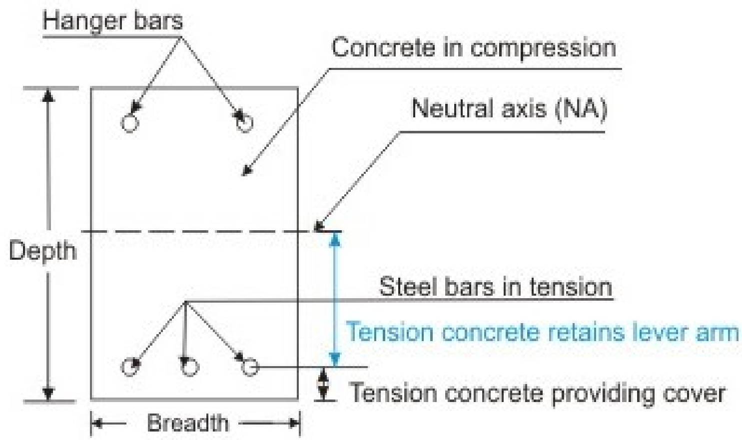

Fig 3.4.4: Singly reinforced rectangular beam under positive bending moment (near midspan)

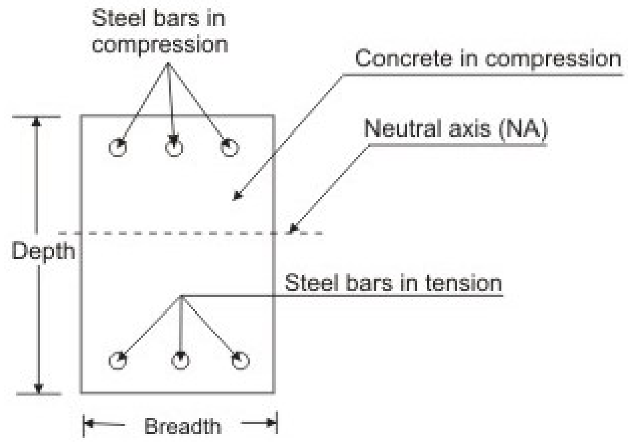

Fig 3.4.5: Doubly reinforced rectangular beam under positive bending moment (near midspan)

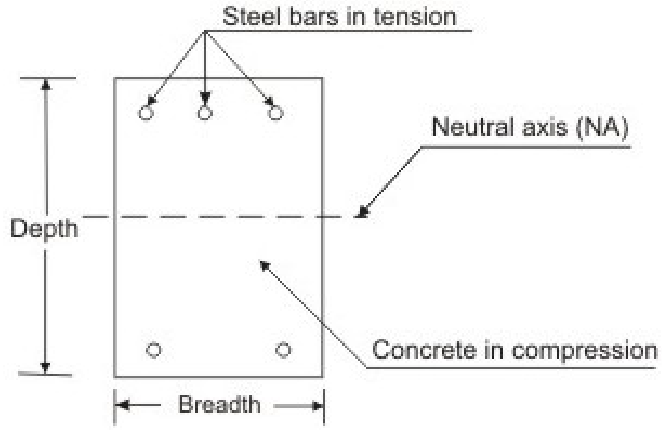

Fig. 3.4.6: Singly reinforced rectangular beam under negative bending moment (over the support)

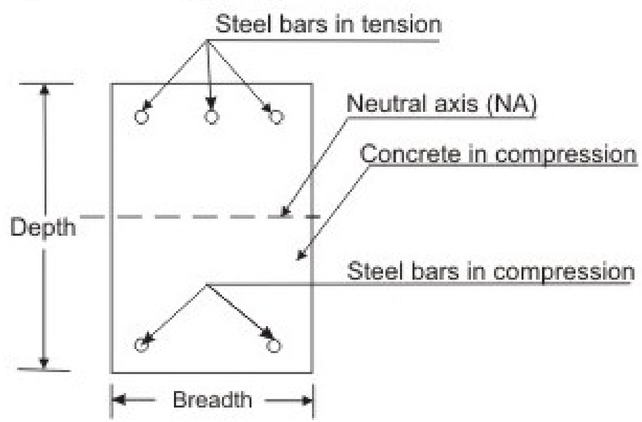

Fig. 3.4.7: Doubly reinforced rectangular beam under negative bending moment (over the support)

This lesson explains the basic governing equations and the computation of parameters required for the design of beams and one-way slabs employing limit state of collapse in flexure. There are three types of reinforced concrete beams:

(i) Singly or doubly reinforced rectangular beams (Figs. 3.4.4 to 7)
(ii) Singly or doubly reinforced *T*-beams (Figs. 3.4.8 to 11)
(iii) Singly or doubly reinforced *L*-beams (Figs. 3.4.12 to 15)

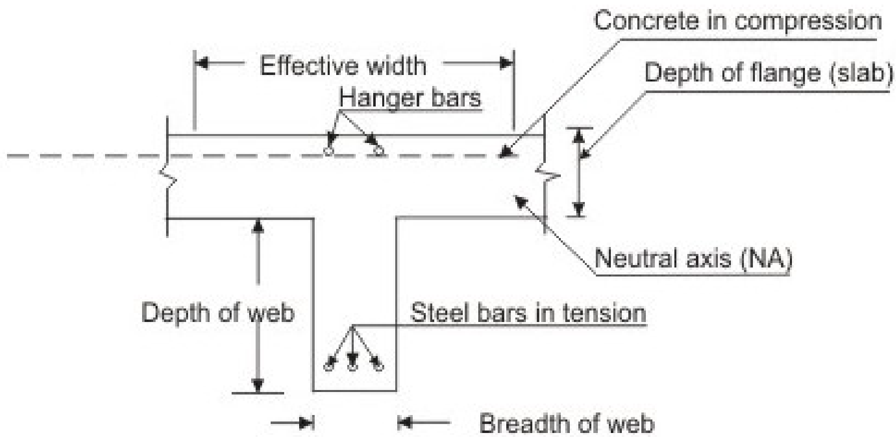

Fig. 3.4.8: Singly reinforced T- beam under positive bending moment (near midspan)

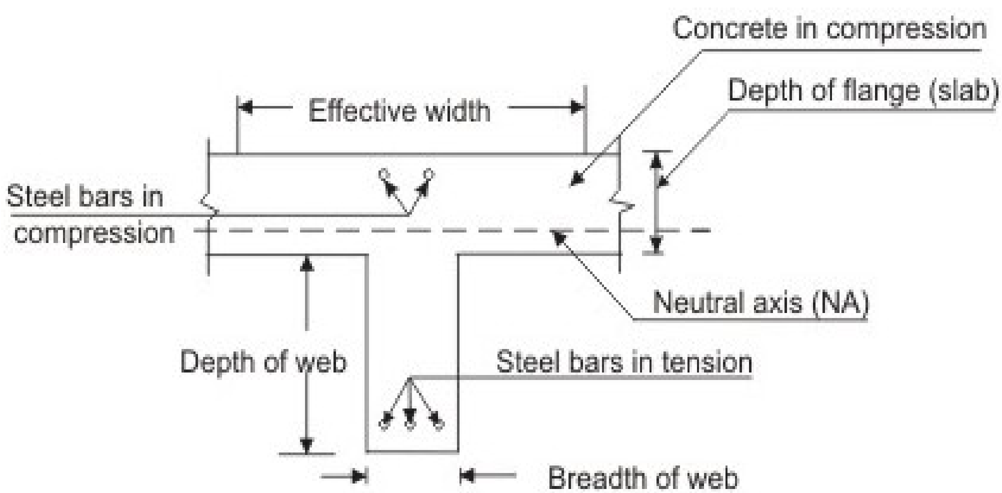

Fig. 3.4.9: Doubly reinforced T- beam under positive bending moment (near midspan)

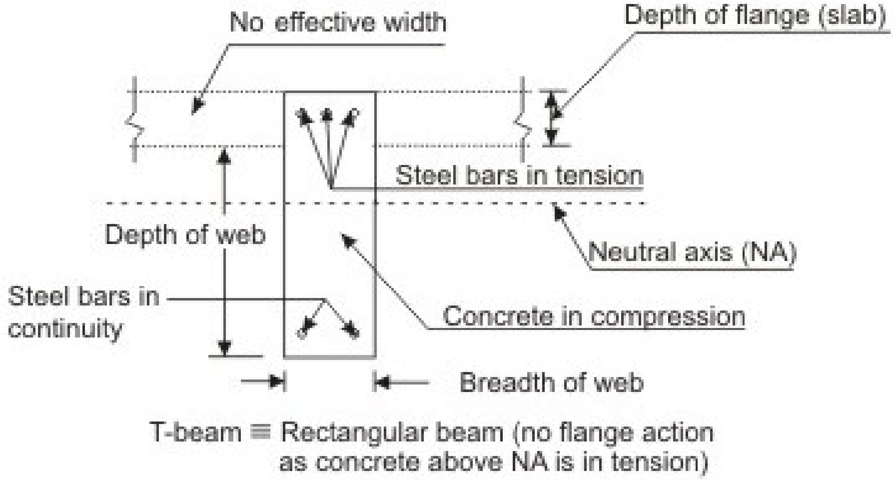

Fig. 3.4.10: Singly reinforced T-beam under negative bending moment (over the support)

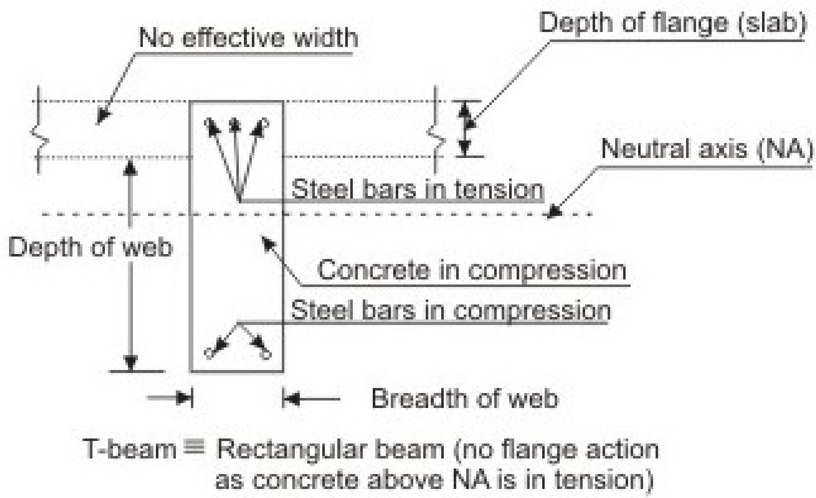

Fig. 3.4.11: Doubly reinforced T-beam under negative bending moment (over the support)

During construction of reinforced concrete structures, concrete slabs and beams are cast monolithic making the beams a part of the floor deck system. While bending under positive moments near midspan, bending compression stresses at the top are taken by the rectangular section of the beams above the neutral axis and the slabs, if present in *T* or *L*-beams (Figs. 3.4.4, 5, 8, 9, 12 and 13). However, under the negative moment over the support or elsewhere, the bending compression stresses are at the bottom and the rectangular sections of rectangular, *T* and *L*-beams below the neutral axis only resist that compression (Figs. 3.4.6, 7, 10, 11, 14 and 15). Thus, in a slab-beam system the beam will be considered as rectangular for the negative moment and *T* for the positive moment. While for the intermediate spans of slabs the beam under positive moment is considered as *T*, the end span edge beam is considered as *L*-beam if the slab is not projected on both the sides of the beam. It is worth mentioning that the effective width of flange of these *T* or *L*-beams is to be determined which depends on:

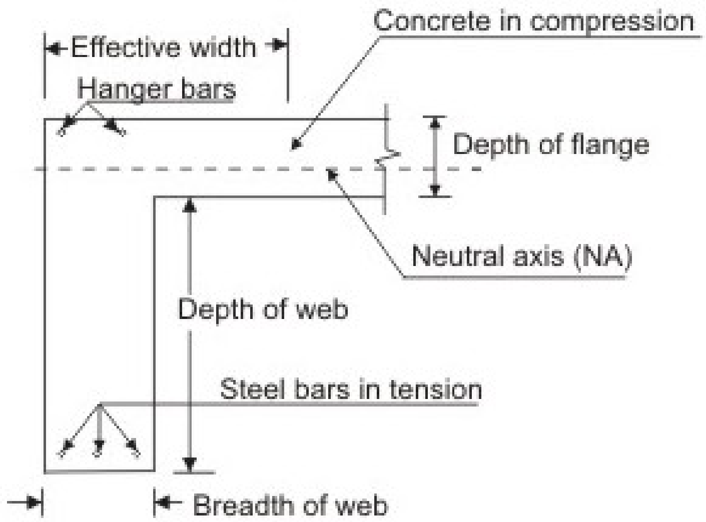

Fig. 3.4.12: Singly reinforced L-beam under positive bending moment (near mid-span)

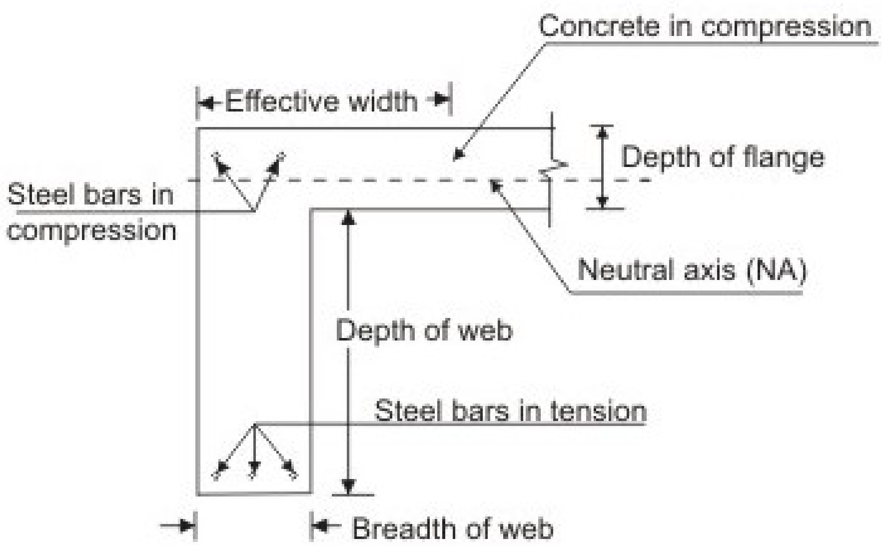

Fig. 3.4.13: Doubly reinforced L-beam under positive bending moment (near mid-span)

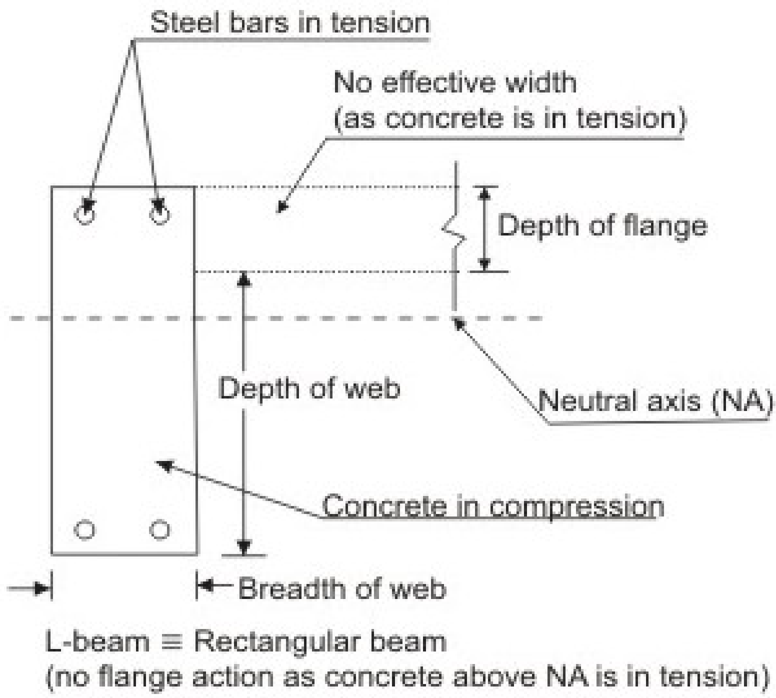

Fig. 3.4.14: Singly reinforced L-beam under negative bending moment (over the support)

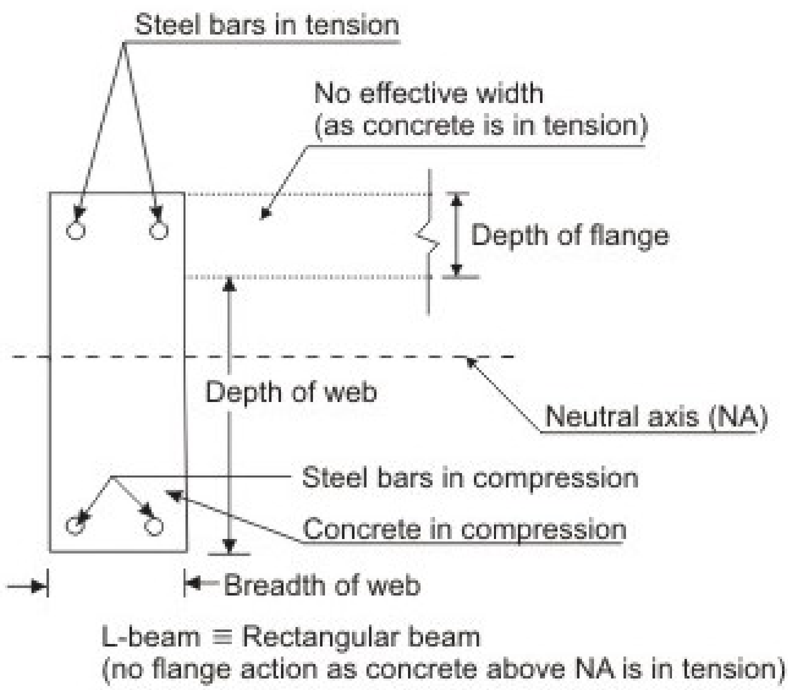

Fig. 3.4.15: Doubly reinforced L-beam under negative bending moment (over the support)

(a) if it is an isolated or continuous beam
(b) the distance between points of zero moments in the beam
(c) the width of the web
(d) the thickness of the flange

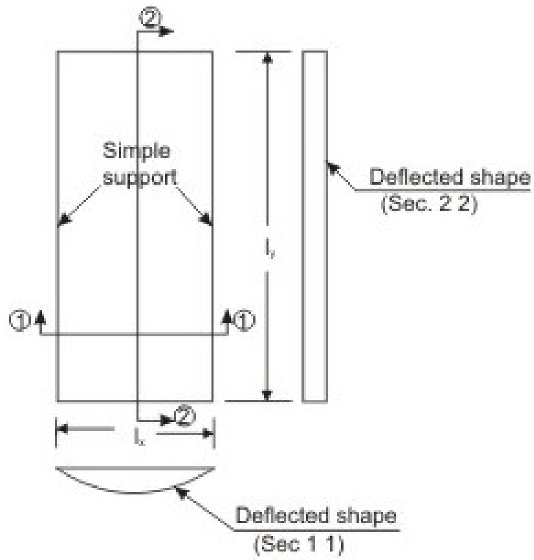

Fig. 3.4.16: One way slab (l_y/l_x >2)

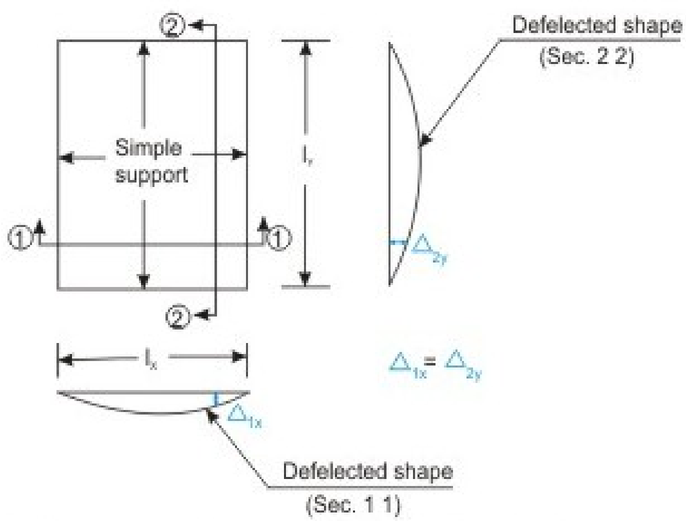

Fig. 3.4.17: Two way slab (l_y/l_x <=2)

Reinforced concrete slabs are classified as one-way or two-way depending on if they are spanning in one or two directions (Figs. 3.4.16 and 17). As a guideline, slabs whose ratio of longer span (l_y) to the shorter span (l_x) is more than two are considered as one-way slabs. One-way slabs also can be designed following the procedure of the design of beams of rectangular cross-section. Again, slabs may be isolated or continuous also.

3.4.2 Assumptions

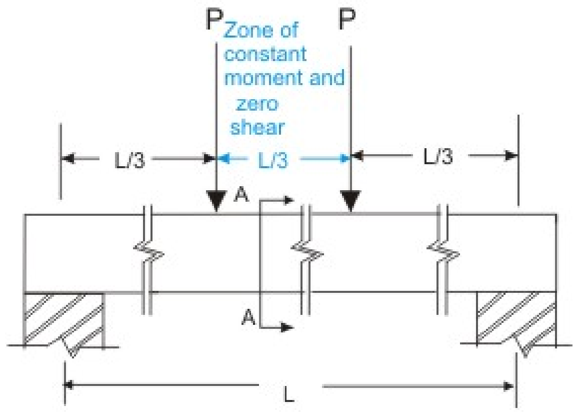

Fig. 3.4.18: Rectangular beam under flexure

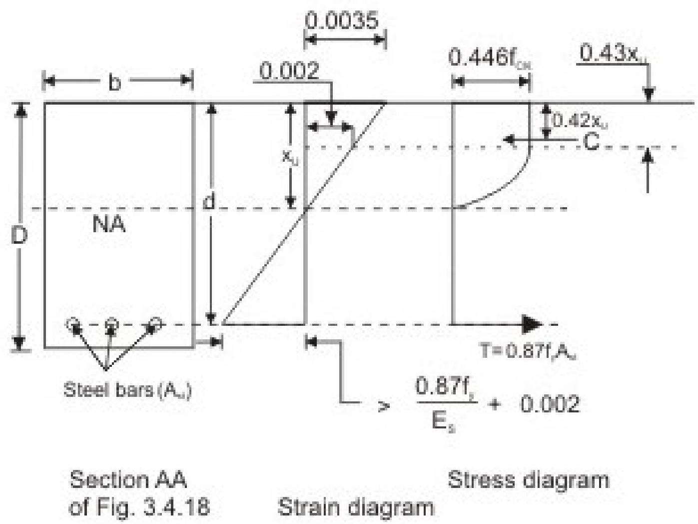

Fig. 3.4.19: Rectangular beam under flexure when $x_u < x_{u,max}$

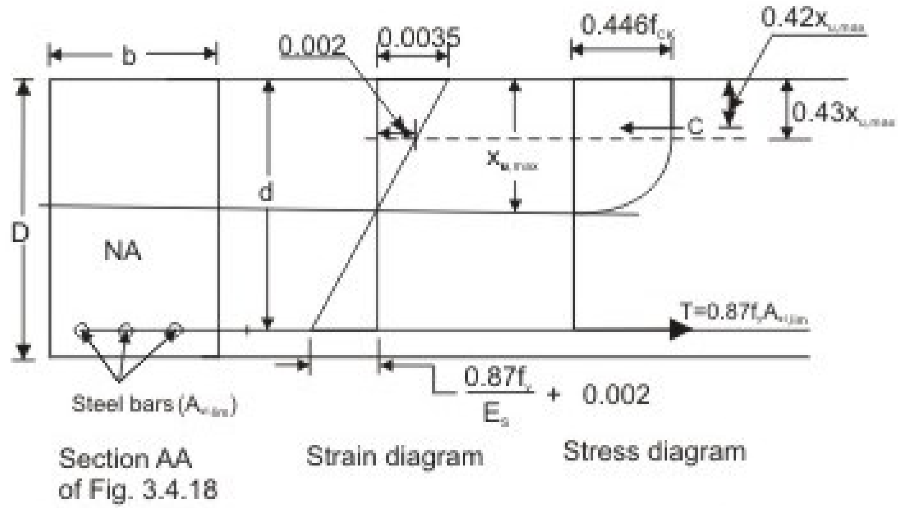

Fig. 3.4.20: Rectangular beam under flexure when $x_u = x_{u,max}$

The following are the assumptions of the design of flexural members (Figs. 3.4.18 to 20) employing limit state of collapse:

(i) Plane sections normal to the axis remain plane after bending.

This assumption ensures that the cross-section of the member does not warp due to the loads applied. It further means that the strain at any point on the cross-section is directly proportional to its distance from the neutral axis.

(ii) The maximum strain in concrete at the outer most compression fibre is taken as 0.0035 in bending (Figs. 3.4.19 and 20).

This is a clearly defined limiting strain of concrete in bending compression beyond which the concrete will be taken as reaching the state of collapse. It is very clear that the specified limiting strain of 0.0035 does not depend on the strength of concrete.

(iii) The acceptable stress-strain curve of concrete is assumed to be parabolic as shown in Fig. 1.2.1 of Lesson 2.

The maximum compressive stress-strain curve in the structure is obtained by reducing the values of the top parabolic curve (Figs. 21 of IS 456:2000) in two stages. First, dividing by 1.5 due to size effect and secondly, again dividing by 1.5 considering the partial safety factor of the material. The middle and bottom curves (Fig. 21 of IS 456:2000) represent these stages. Thus, the maximum compressive stress in bending is limited to the constant value of 0.446 f_{ck} for the strain ranging from 0.002 to 0.0035 (Figs. 3.4.19 and 20, Figs. 21 and 22 of IS 456:2000).

(iv) The tensile strength of concrete is ignored.

Concrete has some tensile strength (very small but not zero). Yet, this tensile strength is ignored and the steel reinforcement is assumed to resist the tensile stress. However, the tensile strength of concrete is taken into account to check the deflection and crack widths in the limit state of serviceability.

(v) The design stresses of the reinforcement are derived from the representative stress-strain curves as shown in Figs. 1.2.3 and 4 of Lesson 2 and Figs. 23A and B of IS 456:2000, for the type of steel used using the partial safety factor γ_m as 1.15.

In the reinforced concrete structures, two types of steel are used: one with definite yield point (mild steel, Figs. 1.2.3 of Lesson 2 and Figs. 23B of IS 456:2000) and the other where the yield points are not definite (cold work deformed bars). The representative stress-strain diagram (Fig. 1.2.4 of Lesson 2 and Fig. 23A of IS 456:2000) defines the points between 0.8 f_y and 1.0 f_y in case of cold work deformed bars where the curve is inelastic.

(vi) The maximum strain in the tension reinforcement in the section at failure shall not be less than $f_y/(1.15\ E_s) + 0.002$, where f_y is the characteristic strength of steel and E_s = modulus of elasticity of steel (Figs. 3.4.19 and 20).

This assumption ensures ductile failure in which the tensile reinforcement undergoes a certain degree of inelastic deformation before concrete fails in compression.

3.4.3 Singly Reinforce Rectangular Beams

Figure 3.4.18 shows the singly reinforced rectangular beam in flexure. The following notations are used (Figs. 3.4.19 and 20):

A_{st} = area of tension steel
b = width of the beam
C = total compressive force of concrete
d = effective depth of the beam
L = centre to centre distance between supports
P = two constant loads acting at a distance of L/3 from the two supports of the beam
T = total tensile force of steel
x_u = depth of neutral axis from the top compression fibre

3.4.4 Equations of Equilibrium

The cross-sections of the beam under the applied loads as shown in Fig. 3.4.18 has three types of combinations of shear forces and bending moments: (i) only shear force is there at the support and bending moment is zero, (ii) both bending moment (increasing gradually) and shear force (constant = P) are there between the support and the loading point and (iii) a constant moment (= $PL/3$) is there in the middle third zone i.e. between the two loads where the shear force is zero (Fig. 1.1.1 of Lesson 1). Since the beam is in static equilibrium, any cross-section of the beam is also in static equilibrium. Considering the cross-section in the middle zone (Fig. 3.4.18) the three equations of equilibrium are the following (Figs. 3.4.19 and 20):

(i) Equilibrium of horizontal forces: $\Sigma H = 0$ gives $T = C$
(3.1)

(ii) Equilibrium of vertical shear forces: $\Sigma V = 0$
(3.2)

This equation gives an identity 0 = 0 as there is no shear in the middle third zone of the beam.

(iii) Equilibrium of moments: $\Sigma M = 0$,
(3.3)

This equation shows that the applied moment at the section is fully resisted by moment of the resisting couple $T\,a = C\,a$, where a is the operating lever arm between T and C (Figs. 3.4.19 and 20).

3.4.5 Computations of *C* and *T*

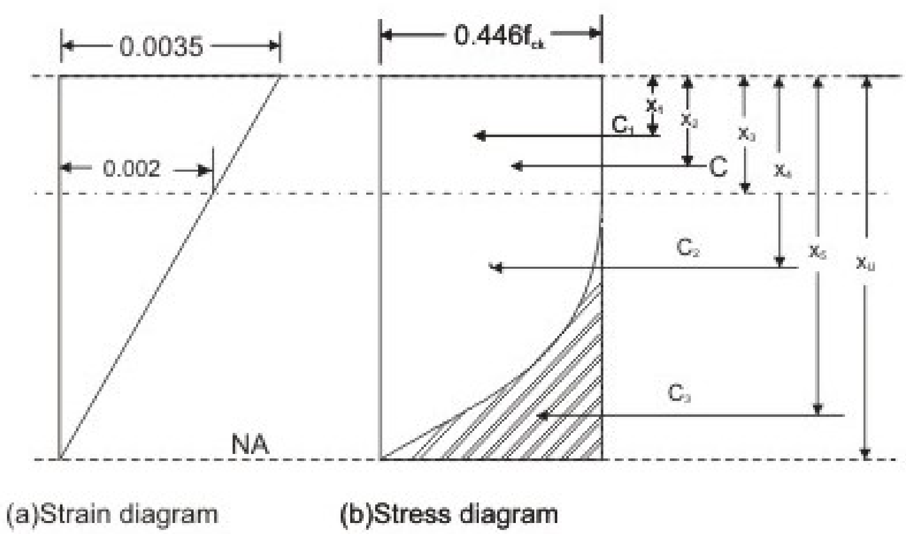

Fig. 3.4.21: Stress and strain diagrams above neutral axis

Figures 3.4.21a and b present the enlarged view of the compressive part of the strain and stress diagrams. The convex parabolic part of the stress block of Fig. 3.4.21b is made rectangular by dotted lines to facilitate the calculations adding another concave parabolic stress zone which is really non-existent as marked by hatch in Fig. 3.4.21b.

The different compressive forces C, C_1, C_2 and C_3 and distances x_1 to x_5 and x_u as marked in Fig. 3.4.21b are explained in the following:

C = Total compressive force of concrete = $C_1 + C_2$

C_1 = Compressive force of concrete due to the constant stress of 0.446 f_{ck} and up to a depth of x_3 from the top fibre

C_2 = Compressive force of concrete due to the convex parabolic stress block of values ranging from zero at the neutral axis to 0.446 f_{ck} at a distance of x_3 from the top fibre

C_3 = Compressive force of concrete due to the concave parabolic stress block (actually non-existent) of values ranging from 0.446 f_{ck} at the neutral axis to zero at a distance of x_3 from the top fibre

x_1 = Distance of the line of action of C_1 from the top compressive fibre

x_2 = Distance of the line of action of C (= $C_1 + C_2$) from the top compressive fibre

x_3 = Distance of the fibre from the top compressive fibre, where the strain = 0.002 and stress = 0.446 f_{ck}

x_4 = Distance of the line of action of C_2 from the top compressive fibre

x_5 = Distance of the line of action of C_3 from the top compressive fibre

x_u = Distance of the neutral axis from the top compressive fibre.

From the strain triangle of Fig. 3.4.21a, we have

$$\frac{x_u - x_3}{x_u} = \frac{0.002}{0.0035} = \frac{4}{7} = 0.57 \text{, giving}$$

$$x_3 = 0.43\, x_u \tag{3.4}$$

Since C_1 is due to the constant stress acting from the top to a distance of x_3, the distance x_1 of the line of action of C_1 is:

$$x_1 = 0.5\, x_3 = 0.215\ x_u$$
(3.5)

From Fig. 3.4.21a:

$$x_5 = x_3 + \frac{3}{4}\,(x_u - x_3) = 0.43\, x_u + 0.75(0.57\, x_u)$$

or $\quad x_5 = 0.86\, x_u$
(3.6)

The compressive force C_1 due to the rectangular stress block is:

$$C_1 = b\, x_3(0.446\, f_{ck}) = 0.191\, b\, x_u\, f_{ck}$$
(3.7)

The compressive force C_2 due to parabolic stress block is:

$$C_2 = b\, (x_u - x_3)\, \frac{2}{3}\, (0.446\, f_{ck}) = 0.17\, b\, x_u\, f_{ck}$$
(3.8)

Adding C_1 and C_2, we have

$$C = C_1 + C_2 = 0.361\, b\, x_u\, f_{ck} = 0.36\, b\, x_u\, f_{ck} \text{ (say)}$$
(3.9)

The non-existent compressive force C_3 due to parabolic (concave) stress block is:

$$C_3 = b\, (x_u - x_3)\, \frac{1}{3}\, (0.446\, f_{ck}) = 0.085\, b\, x_u\, f_{ck}$$
(3.10)

Now, we can get x_4 by taking moment of C_2 and C_3 about the top fibre as follows:

$$C_2(x_4) + C_3\,(x_5) = (C_2 + C_3)\, (x_3 + \frac{x_u - x_3}{2})$$

which gives $x_4 = 0.64\, x_u$
(3.11)

Similarly, x_2 is obtained by taking moment of C_1 and C_2 about the top fibre as follows:

$$C_1(x_1) + C_2(x_4) = C(x_2)$$

which gives $x_2 = 0.4153\, x_u$

or $x_2 = 0.42\, x_u$ (say).
(3.12)

Thus, the required parameters of the stress block (Fig. 3.4.19) are

$$C = 0.36\, b\, x_u\, f_{ck} \qquad (3.9)$$

$$x_2 = 0.42\, x_u \qquad (3.12)$$

and lever arm $= (d - x_2) = (d - 0.42\, x_u)$
(3.13)

The tensile force T is obtained by multiplying the design stress of steel with the area of steel. Thus,

$$T = \left(\frac{f_y}{1.15}\right) A_{st} = 0.87\, f_y\, A_{st}$$
(3.14)

3.4.6 Practice Questions and Problems with Answers

Q.1: How do the beams and slabs primarily carry the transverse loads ?

A.1: The beams and slabs carry the transverse loads primarily by bending.

Q.2: Name three different types of reinforced concrete beams and their specific applications.

A.2: They are:

(i) Singly reinforced and doubly reinforced rectangular beams - used in resisting negative moments in intermediate spans of continuous beams over the supports or elsewhere in slab-beam monolithic constructions, and positive moments in midspan of isolated or intermediate spans of beams with inverted slab (monolithic) constructions and lintels.

(ii) Singly reinforced and doubly reinforced *T*-beams - used in resisting positive moments in isolated or intermediate spans (midspan) in slab-beam monolithic constructions and negative moments over the support for continuous spans with inverted slab (monolithic) constructions.

(iii) Singly reinforced and doubly reinforced *L*-beams - Same as (ii) above except that these are for end spans instead of intermediate spans.

Q.3: Name four parameters which determine the effective widths of *T* and *L*-beams.

A.3: The four parameters are:

(i) isolated or continuous beams,
(ii) the distance between points of zero moments in the beam,
(iii) the breadth of the web,
(iv) the thickness of the flange.

Q.4: Differentiate between one-way and two-way slabs.

A.4: One-way slab spans in one direction and two-way slab spans in both the directions. Slabs whose ratio of longer span (l_y) to shorter span (l_x) is more than 2 are called one-way. Slabs of this ratio up to 2 are called two-way slabs.

Q.5: State and explain the significance of the six assumptions of design of flexural members employing limit state of collapse.

A.5: Sec. 3.4.2 gives the full answer.

Q.6: Draw a cross-section of singly reinforced rectangular beam and show the strain and stress diagrams.

A.6: Fig. 3.4.19.

Q.7: Write the three equations of equilibrium needed to design the reinforced concrete beams.

A.7: Vide sec. 3.4.4 and Eqs. 3.1 to 3.

Q.8: Write the final expression of the total compressive force *C* and tensile force *T* for a rectangular reinforced concrete beam in terms of the designing parameters.

A.8: Eq. 3.9 for *C* and Eq. 3.14 for *T*.

3.4.7 References

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3.4.8 Test 4 with Solutions

Maximum Marks = 50, Maximum Time = 30 minutes

Answer all questions.

TQ.1: Tick the correct answer: (4 x 5 = 20 marks)

(i) Beams and slabs carry the transverse loads primarily by

(a) truss action
(b) balance of shear action
(c) bending
(d) slab-beam interaction

A.TQ.1: (i): (c)

(ii) The ratio of longer span (l_y) to shorter span (l_x) of a two-way slab is

(a) up to 2
(b) more than 2
(c) equal to 1
(d) more than 1

A.TQ.1: (ii): (a)

(iii) An inverted *T*-beam is considered as a rectangular beam for the design

(a) over the intermediate support of a continuous beam where the bending moment is negative

(b) at the midspan of a continuous beam where the bending moment is positive

(c) at the point of zero bending moment

(d) over the support of a simply supported beam

A.TQ.1: (iii): (b)

(iv) The maximum strain in the tension reinforcement in the section at failure shall be

(a) more than $f_y/(1.15\ E_s) + 0.002$

(b) equal to 0.0035

(c) more than $f_y/E_s + 0.002$

(d) less than $f_y/(1.15\ E_s) + 0.002$

A.TQ.1: (iv): (d)

TQ.2: Draw a cross-section of singly reinforced rectangular beam and show the strain and stress diagrams.
(10)

A.TQ.2: Fig. 3.4.19

TQ.3: Name four parameters which determine the effective widths of *T* and *L*-beams. (6)

A.TQ.3: The four parameters are:

(i) isolated or continuous beams,
(ii) the distance between points of zero moments in the beam,
(iii) the breadth of the web,
(iv) the thickness of the flange.

TQ.4: Derive the final expressions of the total compressive force *C* and tensile force *T* for a rectangular reinforced concrete beam in terms of the designing parameters.
(10 + 4 = 14)

A.TQ.4: Section 3.4.5 is the full answer.

3.4.9 Summary of this Lesson

Lesson 4 illustrates the primary load carrying principle in a slab-beam structural system subjected to transverse loadings. It also mentions three different types of singly and doubly reinforced beams normally used in construction. Various assumptions made in the design of these beams employing limit state of collapse are explained. The stress and strain diagrams of a singly reinforced rectangular beam are explained to write down the three equations of equilibrium. Finally, the computations of the total compressive and tensile forces are illustrated.