9.1 Composite Sections

This section covers the following topics.
- Introduction
- Analysis of Composite Sections
- Design of Composite Sections
- Analysis for Horizontal Shear Transfer

9.1.1 Introduction

A composite section in context of prestressed concrete members refers to a section with a precast member and cast-in-place (CIP) concrete. There can be several types of innovative composite sections. A few types are sketched below.

![Examples of composite sections](image)

The following photos show the reinforcement for the slab of a box girder bridge deck with precast webs and bottom flange. The slab of the top flange is cast on a stay-in formwork. The reinforcement of the slab is required for the transverse bending of the slab. The reinforcement at the top of the web is required for the horizontal shear transfer.
The advantages of composite construction are as follows.

1) Savings in form work
2) Fast-track construction
3) Easy to connect the members and achieve continuity.

Figure 9-1.2  Box girder bridge deck with precast webs and bottom flange and CIP slab; Top: Aerial view, Bottom: Close-up view (Courtesy: José Turmo)
The prestressing of composite sections can be done in stages. The precast member can be first pre-tensioned or post-tensioned at the casting site. After the cast-in-place (cast-in-situ) concrete achieves strength, the section is further post-tensioned. The grades of concrete for the precast member and the cast-in-place portion may be different. In such a case, a transformed section is used to analyze the composite section.

### 9.1.2 Analysis of Composite Sections

The analysis of a composite section depends upon the type of composite section, the stages of prestressing, the type of construction and the loads. The type of construction refers to whether the precast member is **propped** or **unpropped** during the casting of the CIP portion. If the precast member is supported by props along its length during the casting, it is considered to be propped. Else, if the precast member is supported only at the ends during the casting, it is considered to be unpropped.

The following diagrams are for a composite section with precast web and cast-in-place flange. The web is prestressed before the flange is cast. At transfer and after casting of the flange (before the section behaves like a composite section), the following are the stress profiles for the precast web.

![Stress profiles for the precast web](image)

**Figure 9-1.3** Stress profiles for the precast web

Here,

- $P_0$ = Prestress at transfer after short term losses
- $P_e$ = Effective prestress during casting of flange after long term losses
- $M_{SW}$ = Moment due to self weight of the precast web
- $M_{CIP}$ = Moment due to weight of the CIP flange.

At transfer, the loads acting on the precast web are $P_0$ and $M_{SW}$. By the time the flange is cast, the prestress reduces to $P_e$ due to long term losses. In addition to $P_e$ and $M_{SW}$,
the web also carries $M_{CIP}$. The width of the flange is calculated based on the concept of effective flange width as per Clause 23.1.2, IS:456 - 2000.

At service (after the section behaves like a composite section) the following are the stress profiles for the full depth of the composite section.

![Figure 9-1.4 Stress profiles for the composite section](image)

Here, $M_{LL}$ is the moment due to live load. If the precast web is unpropped during casting of the flange, the section does not behave like a composite section to carry the prestress and self weight. Hence, the stress profile due to $P_e + M_{SW} + M_{CIP}$ is terminated at the top of the precast web. If the precast web is propped during casting and hardening of the flange, the section behaves like a composite section to carry the prestress and self weight after the props are removed. The stress profile is extended up to the top of the flange. When the member is placed in service, the full section carries $M_{LL}$.

From the analyses at transfer and under service loads, the stresses at the extreme fibres of the section for the various stages of loading are evaluated. These stresses are compared with the respective allowable stresses.

Stress in precast web at transfer

$$f = -\frac{P_0}{A} \pm \frac{P_e c}{l} \pm \frac{M_{SW}c}{l}$$ \hspace{1cm} (9-1.1)

Stress in precast web after casting of flange

$$f = -\frac{P_e}{A} \pm \frac{P_e c}{l} \pm \frac{(M_{SW} + M_{CIP})c}{l}$$ \hspace{1cm} (9-1.2)
Stress in precast web at service

(a) For unpropped construction

\[
f = \frac{P_e}{A} \pm \frac{P_e c}{l} \pm \frac{(M_{SW} + M_{CIP})c}{l} \pm \frac{M_{LL}c'}{l'}
\]  
(9-1.3a)

(b) For propped construction

\[
f = \frac{P_e}{A} \pm \frac{P_e c}{l} \pm \frac{M_{SW}c}{l} \pm \frac{(M_{CIP} + M_{LL})c'}{l'}
\]  
(9-1.3b)

Here,

\[A\] = area of the precast web
\[c\] = distance of edge from CGC of precast web
\[c'\] = distance of edge from CGC of composite section
\[e\] = eccentricity of CGS
\[l\] = moment of inertia of the precast web
\[l'\] = moment of inertia of the composite section.

From the analysis for ultimate strength, the ultimate moment capacity is calculated. This is compared with the demand under factored loads. The analysis at ultimate is simplified by the following assumptions.

1) The small strain discontinuity at the interface of the precast and CIP portions is ignored.
2) The stress discontinuity at the interface is also ignored.
3) If the CIP portion is of low grade concrete, the weaker CIP concrete is used for calculating the stress block.

The strain and stress diagrams and the force couples at ultimate are shown below.
The variables in the above figure are explained.

- $b_f = \text{breadth of the flange}$
- $b_w = \text{breadth of the web}$
- $D_f = \text{depth of the flange}$
- $d = \text{depth of the centroid of prestressing steel (CGS)}$
- $A_p = \text{area of the prestressing steel}$
- $\Delta \varepsilon_p = \text{strain difference for the prestressing steel}$
- $x_u = \text{depth of the neutral axis at ultimate}$
- $\varepsilon_{pu} = \text{strain in prestressing steel at the level of CGS at ultimate}$
- $f_{pu} = \text{stress in prestressing steel at ultimate}$
- $f_{ck} = \text{characteristic compressive strength of the weaker concrete}$
- $C_{uw} = \text{resultant compression in the web (includes portion of flange above precast web)}$
- $C_{uf} = \text{resultant compression in the outstanding portion of flange}$
- $T_{uw} = \text{portion of tension in steel balancing } C_{uw}$
- $T_{uf} = \text{portion of tension balancing } C_{uf}$

The expressions of the forces are as follows.

- $C_{uw} = 0.36f_{ck}x_u b_w \quad (9-1.4)$
- $C_{uf} = 0.447f_{ck}(b_f - b_w)D_f \quad (9-1.5)$
- $T_{uw} = A_{pw}f_{pu} \quad (9-1.6)$
- $T_{uf} = A_{pf}f_{pu} \quad (9-1.7)$

The symbols for the areas of steel are as follows.
\[ A_{pf} = \text{part of } A_p \text{ that balances compression in the outstanding flanges} \]
\[ A_{pw} = \text{part of } A_p \text{ that balances compression in the web} \]

The equilibrium equations are given below. These equations are explained in Section 3.5, Analysis of Members under Flexure (Part IV). The ultimate moment capacity \((M_{uR})\) is calculated from the second equation.

\[
\sum F = 0 \\
\Rightarrow (A_{pw} + A_{pf})f_{pu} = 0.36 f_{ck} x_u b_w + 0.447 f_{ck} (b_t - b_w)D_t \tag{9-1.8}
\]
\[
M_{uR} = A_{pw}f_{pu} (d - 0.42x_u) + A_{pf}f_{pu} (d - 0.5D_t) \tag{9-1.9}
\]

### 9.2.3 Design of Composite Sections

The design is based on satisfying the allowable stresses under service loads and at transfer. The section is then analysed for ultimate loads to satisfy the limit state of collapse. The member is also checked to satisfy the criteria of limit states of serviceability, such as deflection and crack width (for Type 3 members only). Before the calculation of the initial prestressing force \((P_0)\) and the eccentricity of the CGS \((e)\) at the critical section, the type of composite section and the stages of prestressing need to be decided. Subsequently, a trial and error procedure is adopted for the design.

The following steps explain the design of a composite section with precast web and cast-in-place flange. The precast web is prestressed before the casting of the flange. The member is considered to be Type 1 member.

**Step 1. Compute \(e\).**

With a trial section of the web, the CGS can be located at the maximum eccentricity \((e_{max})\). The maximum eccentricity is calculated based on zero stress at the top of the precast web. This gives an economical solution. The following stress profile is used to determine \(e_{max}\).
Here, \( e_{max} = k_b + \frac{M_{sw}}{P_0} \)

- **CGC** = Centroid of the precast web
- \( k_b \) = Distance of the bottom kern of the precast web from CGC
- \( M_{sw} \) = Moment due to self weight of the precast web.
- \( P_0 \) = A trial prestressing force at transfer.

**Step 2.** Compute equivalent moment for the precast web.

A moment acting on the composite section is transformed to an equivalent moment for the precast web. This is done to compute the stresses in the precast web in terms of the properties of the precast web itself and not of the composite section.

For a moment \( M_c \) which acts after the section behaves like a composite section, the stresses in the extreme fibres of the precast web are determined from the following stress profile.

Here, \( CGC' = \) centroid of the composite section
\( c_i' \) = Distance of the top of the precast web from the CGC’
\( c_i'' \) = Distance of the top of the composite section from the CGC’.
\( c_b' \) = Distance of the bottom of the precast web (or composite section) from the CGC’
\( I' \) = moment of inertia for the composite section.

The following quantities are defined as the ratios of the properties of the precast web and composite section.

\[ m_t = \frac{l}{l'} \frac{c_t}{c_t'} \]
\[ m_b = \frac{l}{l'} \frac{c_b}{c_b'} \]

Then the stresses in the extreme fibres of the precast web can be expressed in terms of \( m_t \) and \( m_b \) as follows.

\[ f_t = \frac{m_t M_c c_t}{l} = \frac{m_t M_c}{Ak_b} \]
\[ f_b = \frac{m_b M_c c_b}{l} = \frac{m_b M_c}{Ak_t} \] (9-1.10)

Here,

\( A = \text{Area of the precast web} \)
\( k_b = \text{Distance of the bottom kern of the precast web from CGC} \)
\( k_t = \text{Distance of the top kern of the precast web from CGC} \)

The quantities \( m_t, M_c \) and \( m_b, M_c \) are the equivalent moments. Thus, the stresses in the precast web due to \( M_c \) are expressed in terms of the properties of the precast web itself.

**Step 3. Compute \( P_e \)**

Let \( M_P \) be the moment acting on the precast web prior to the section behaving like a composite section. After \( M_c \) is applied on the composite section, the total moment for the precast web is \( M_P + m_b M_c \).
The stress at the bottom for Type 1 member due to service loads is zero.

Therefore,

\[
\frac{P_e}{A} - \frac{P_e e}{Ak_t} + \frac{M_p + m_c M_c}{Ak_t} = 0
\]

or,

\[
P_e = \frac{(M_p + m_c M_c)}{e + k_t}
\] (9-1.12)

Note that the prestressing force is acting only on the precast web and hence, \(e\) is the eccentricity of the CGS from the CGC of the precast web.

**Step 4.** Estimate \(P_0\) as follows.

a) 90% of the initial applied prestress \((P)\) for pre-tensioned members.

b) Equal to \(P_i\) for post-tensioned members.

The value of \(P_i\) is estimated as follows.

\[
P_i = A_p(0.8f_{pk})
\] (9-1.13)

\[
A_p = \frac{P_e}{0.7f_{pk}}
\] (9-1.14)

Revise \(e\), the location of CGS, as given in Step 1 based on the new value of \(P_0\).

\[
e_{max} = k_b + \frac{M_{sw}}{P_0}
\] (9-1.15)

**Step 5.** Check for the compressive stresses in the precast web.

At transfer, the stress at the bottom is given as follows.

\[
f_b = \frac{P_0}{A} - \frac{P_e e}{Ak_t} + \frac{M_{sw}}{Ak_t}
\] (9-1.16)

The stress \(f_b\) should be limited to \(f_{cc,all}\), where \(f_{cc,all}\) is the allowable compressive stress in concrete at transfer (available from Figure 8 of **IS:1343 - 1980**).

At service,

\[
f_t = \frac{P_e}{A} - \frac{P_e e + (M_p + m_c M_c)}{Ak_b}
\] (9-1.17)
The stress $f_t$ should be limited to $f_{cc,all}$, where $f_{cc,all}$ is the allowable compressive stress in concrete under service loads (available from Figure 7 of IS:1343-1980). If the stress conditions are not satisfied, increase $A$.

**Step 6.** Check for the compressive stress in the CIP flange.

$$ f_t' = \frac{M_{c,t}''}{I'} $$  \hspace{1cm} (9-1.18)

The stress $f_t'$ should be limited to $f_{cc,all}$, where $f_{cc,all}$ is the allowable compressive stress in concrete under service loads.

### 9.1.4 Analysis for Horizontal Shear Transfer

With increase in the load, the bottom face of the CIP portion tends to slip horizontally and move upwards with respect to the top face of the precast portion. To prevent this and to develop the composite action, shear connectors in the form of shear friction reinforcement is provided.

The required shear friction reinforcement (per metre span) is calculated as follows.

$$ A_{sv} = \frac{1000b_v\tau_h}{0.87f_y\mu} $$  \hspace{1cm} (9-1.19)

The minimum requirements of shear friction reinforcement and spacing are similar to that for shear reinforcement in the web.

In the previous equation,

- $A_{sv}$ = area of shear friction reinforcement in mm$^2$/m
- $b_v$ = width of the interface of precast and CIP portions
- $\tau_h$ = horizontal shear stress at the interface in N/mm$^2$
- $f_y$ = yield stress in N/mm$^2$
- $\mu$ = coefficient of friction
  - $= 1.0$ for intentionally roughened interface with normal weight concrete
The shear reinforcement in the web can be extended and anchored in the CIP portion to act as shear friction reinforcement, as shown below.

![Shear reinforcement used for shear transfer](image)

**Figure 9-1.8** Shear reinforcement used for shear transfer

The following example shows the analysis of a composite beam.

**Example 9-1.1**

The mid-span section of a composite beam is shown in the figure. The precast web 300 mm × 920 mm (depth) is post-tensioned with an initial force \(P_0\) of 2450 kN. The effective prestress \(P_e\) is estimated as 2150 kN. Moment due to the self weight of the precast web \(M_{SW}\) is 270 kNm at mid-span.

After the web is erected in place, the top slab of 150 mm × 920 mm (width) is casted (unpropped) producing a moment \(M_{CIP}\) of 135 kNm. After the slab concrete has hardened, the composite section is to carry a maximum live load moment \(M_{LL}\) of 720 kNm.

**Compute stresses in the section at various stages.**
Solution

1) Calculation of geometric properties.

Precast web

\[ A = 2.76 \times 10^5 \text{ mm}^2 \]
\[ I = 1.95 \times 10^{10} \text{ mm}^2 \]
Distance of CGC from bottom = 460 mm.

Composite section

\[ A' = 4.14 \times 10^5 \text{ mm}^2 \]
\[ I' = 4.62 \times 10^{10} \text{ mm}^2 \]
Distance of CGC' from bottom = 638 mm.

2) Calculation of stresses in web at transfer

\[
f = -\frac{P_0}{A} \pm \frac{P_0 ec}{I} \pm \frac{M_{swc}}{I}
\]
\[
= -\frac{2450 \times 10^3}{2.76 \times 10^5} \pm \frac{2450 \times 10^3 \times 260 \times 460}{1.95 \times 10^{10}} \pm \frac{270 \times 10^6 \times 460}{1.95 \times 10^{10}}
\]
\[
= -0.22 \text{ N/mm}^2
\]
\[
= -17.54 \text{ N/mm}^2
\]
3) Calculation of stresses in web after long term losses

\[ f = \frac{P_e}{A} \pm \frac{P_{e,c}}{I} \pm \frac{M_{SW,c}}{I} \]

\[ = \frac{2150 \times 10^3}{2.76 \times 10^5} \pm \frac{2150 \times 10^3 \times 260 \times 460}{1.95 \times 10^{10}} \pm \frac{270 \times 10^6 \times 460}{1.95 \times 10^{10}} \]

\[ = -0.97 \text{ N/mm}^2 \]

\[ = -14.61 \text{ N/mm}^2 \]

4) Calculation of stresses in web after casting of flange

\[ f = \frac{P_e}{A} \pm \frac{P_{e,c}}{I} \pm \frac{(M_{SW} + M_{CIP})c}{I} \]

\[ = \frac{2150 \times 10^3}{2.76 \times 10^5} \pm \frac{2150 \times 10^3 \times 260 \times 460}{1.95 \times 10^{10}} \pm \frac{(270+135) \times 10^6 \times 460}{1.95 \times 10^{10}} \]

\[ = -4.16 \text{ N/mm}^2 \]

\[ = -11.42 \text{ N/mm}^2 \]

5) Calculation of stresses in the composite section at service

Stress due to \( M_{LL} \)

At top fibre

\[ f_t' = \frac{M_{LL}c_{t'}}{I'} \]

\[ = \frac{750 \times 10^3 \times 432}{4.62 \times 10^{10}} \]

\[ = -7.01 \text{ N/mm}^2 \]

At bottom fibre

\[ f_b = \frac{M_{LL}c_{b'}}{I'} \]

\[ = \frac{750 \times 10^3 \times 638}{4.62 \times 10^{10}} \]

\[ = 10.36 \text{ N/mm}^2 \]

At top fibre of precast web, the stress due to \( M_{LL} \) is calculated from proportionality of triangles.

\[ f_t = -\frac{7.01 \times 282}{432} \]

\[ = -4.57 \text{ N/mm}^2 \]
Total stress in precast web

At top fibre

\[ f_t = -4.16 - 4.57 = -8.73 \text{ N/mm}^2 \]

At bottom fibre

\[ f_b = -11.42 + 10.36 = -1.06 \text{ N/mm}^2 \]

Total stress in CIP slab

The total stress is due to \( M_{LL} \) only.

At top fibre

\[ f_b' = -4.57 \text{ N/mm}^2 \]

At bottom fibre

\[ f_t' = -7.01 \text{ N/mm}^2 \]

Stress profiles

At transfer  After casting  At service

After losses  Due to \( M_{LL} \)