2.1 Losses in Prestress (Part I)

This section covers the following topics.

- Introduction
- Elastic Shortening

The relevant notations are explained first.

Notations

Geometric Properties

The commonly used geometric properties of a prestressed member are defined as follows.

A_c = Area of concrete section
= Net cross-sectional area of concrete excluding the area of prestressing steel.

A_p = Area of prestressing steel
= Total cross-sectional area of the tendons.

A = Area of prestressed member
= Gross cross-sectional area of prestressed member.
$= A_c + A_p$

A_t = Transformed area of prestressed member
= Area of the member when steel is substituted by an equivalent area of concrete.
$= A_c + mA_p$
$= A + (m – 1)A_p$

Here,

m = the modular ratio = E_p/E_c
E_c = short-term elastic modulus of concrete
E_p = elastic modulus of steel.

The following figure shows the commonly used areas of the prestressed members.

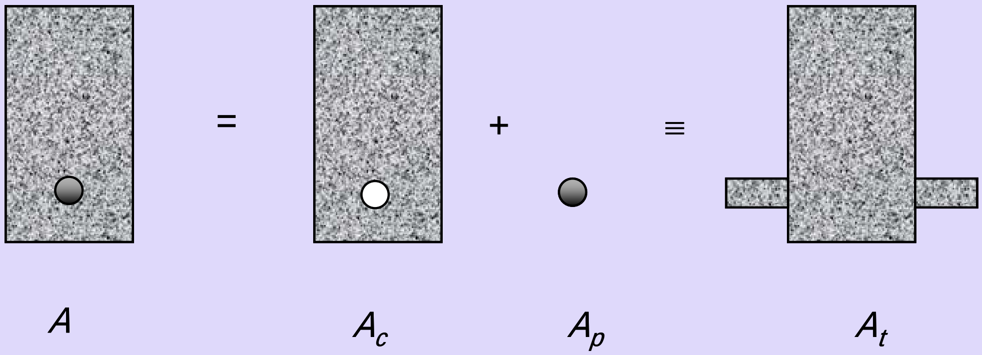


Figure 2-1.1 Areas for prestressed members

CGC = Centroid of concrete

= Centroid of the gross section. The CGC may lie outside the concrete (Figure 2-1.2).

CGS = Centroid of prestressing steel

= Centroid of the tendons. The CGS may lie outside the tendons or the concrete (Figure 2-1.2).

I = Moment of inertia of prestressed member

= Second moment of area of the gross section about the CGC.

I_t = Moment of inertia of transformed section

= Second moment of area of the transformed section about the centroid of the transformed section.

e = Eccentricity of CGS with respect to CGC

= Vertical distance between CGC and CGS. If CGS lies below CGC, e will be considered positive and vice versa (Figure 2-1.2).

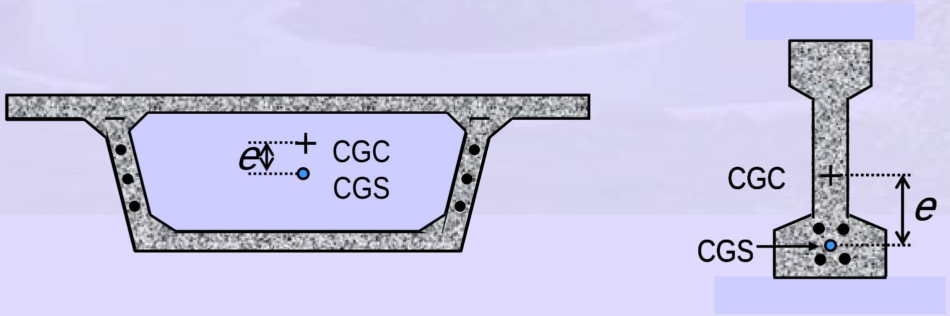


Figure 2-1.2 CGC, CGS and eccentricity of typical prestressed members

<u>Load Variables</u>

P_i = Initial prestressing force

= The force which is applied to the tendons by the jack.

P_0 = Prestressing force after immediate losses
= The reduced value of prestressing force after elastic shortening, anchorage slip and loss due to friction.

P_e = Effective prestressing force after time-dependent losses
= The final value of prestressing force after the occurrence of creep, shrinkage and relaxation.

2.1.1 Introduction

In prestressed concrete applications, the most important variable is the prestressing force. In the early days, it was observed that the prestressing force does not stay constant, but reduces with time. Even during prestressing of the tendons and the transfer of prestress to the concrete member, there is a drop of the prestressing force from the recorded value in the jack gauge. The various reductions of the prestressing force are termed as the losses in prestress.

The losses are broadly classified into two groups, immediate and time-dependent. The immediate losses occur during prestressing of the tendons and the transfer of prestress to the concrete member. The time-dependent losses occur during the service life of the prestressed member. The losses due to elastic shortening of the member, friction at the tendon-concrete interface and slip of the anchorage are the immediate losses. The losses due to the shrinkage and creep of the concrete and relaxation of the steel are the time-dependent losses. The causes of the various losses in prestress are shown in the following chart.

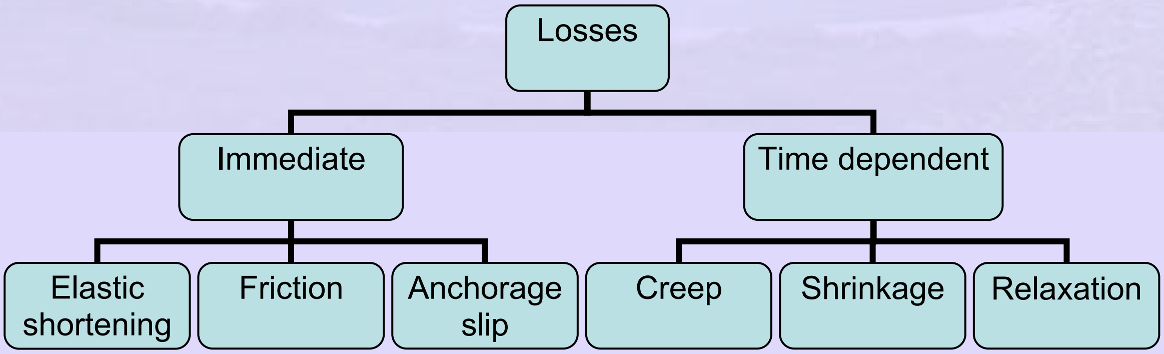


Figure 2-1.3 Causes of the various losses in prestress

2.1.2 Elastic Shortening

Pre-tensioned Members

When the tendons are cut and the prestressing force is transferred to the member, the concrete undergoes immediate shortening due to the prestress. The tendon also shortens by the same amount, which leads to the loss of prestress.

Post-tensioned Members

If there is only one tendon, there is no loss because the applied prestress is recorded after the elastic shortening of the member. For more than one tendon, if the tendons are stretched sequentially, there is loss in a tendon during subsequent stretching of the other tendons.

The elastic shortening loss is quantified by the drop in prestress (Δf_p) in a tendon due to the change in strain in the tendon ($\Delta\varepsilon_p$). It is assumed that the change in strain in the tendon is equal to the strain in concrete (ε_c) at the level of the tendon due to the prestressing force. This assumption is called **strain compatibility** between concrete and steel. The strain in concrete at the level of the tendon is calculated from the stress in concrete (f_c) at the same level due to the prestressing force. A linear elastic relationship is used to calculate the strain from the stress.

The quantification of the losses is explained below.

$$
\begin{aligned}
\Delta f_p &= E_p \Delta\varepsilon_p \\
&= E_p \varepsilon_c \\
&= E_p\left(\frac{f_c}{E_c}\right) \\
\Delta f_p &= m f_c
\end{aligned}
\quad \textbf{(2-1.1)}
$$

For simplicity, the loss in all the tendons can be calculated based on the stress in concrete at the level of CGS. This simplification cannot be used when tendons are stretched sequentially in a post-tensioned member. The calculation is illustrated for the following types of members separately.

- Pre-tensioned Axial Members
- Pre-tensioned Bending Members
- Post-tensioned Axial Members

- Post-tensioned Bending Members

Pre-tensioned Axial Members

The following figure shows the changes in length and the prestressing force due to elastic shortening of a pre-tensioned axial member.

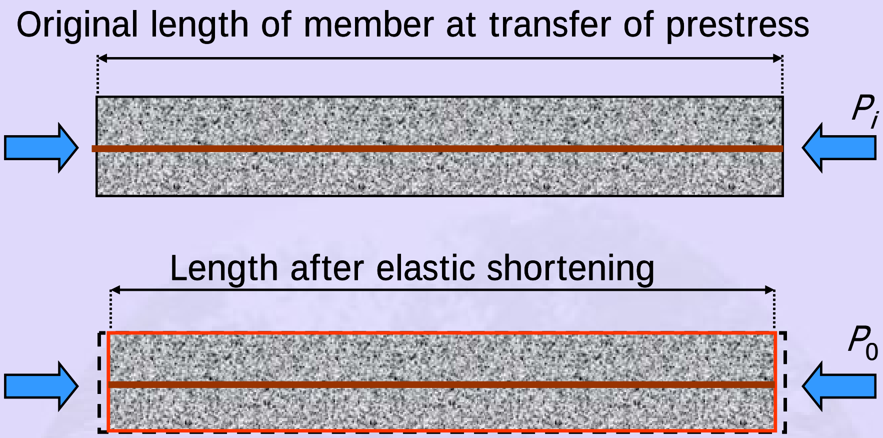


Figure 2-1.4 Elastic shortening of a pre-tensioned axial member

The loss can be calculated as per Eqn. (2-1.1) by expressing the stress in concrete in terms of the prestressing force and area of the section as follows.

$$\Delta f_p = mf_c$$
$$= m\left(\frac{P_0}{A_c}\right)$$
$$\Delta f_p = m\left(\frac{P_i}{A_t}\right) \approx m\left(\frac{P_i}{A}\right) \quad \textbf{(2-1.2)}$$

Note that the stress in concrete due to the prestressing force after immediate losses (P_0/A_c) can be equated to the stress in the transformed section due to the initial prestress (P_i/A_t). This is derived below. Further, the transformed area A_t of the prestressed member can be approximated to the gross area A.

The following figure shows that the strain in concrete due to elastic shortening (ε_c) is the difference between the initial strain in steel (ε_{pi}) and the residual strain in steel (ε_{p0}).

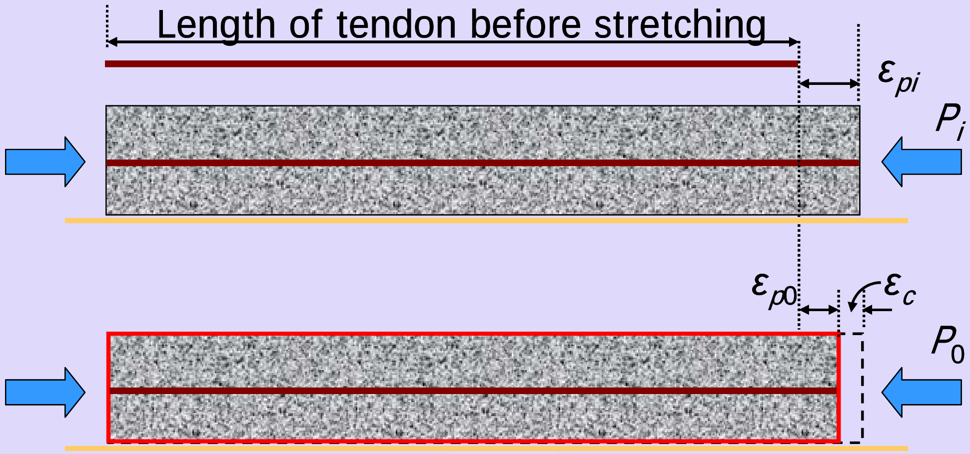


Figure 2-1.5 Strain variables in elastic shortening

The following equation relates the strain variables.

$$\varepsilon_c = \varepsilon_{pi} - \varepsilon_{p0} \tag{2-1.3}$$

The strains can be expressed in terms of the prestressing forces as follows.

$$\varepsilon_c = \frac{P_0}{A_c E_c} \tag{2-1.4}$$

$$\varepsilon_{pi} = \frac{P_i}{A_p E_p} \tag{2-1.5}$$

$$\varepsilon_{p0} = \frac{P_0}{A_p E_p} \tag{2-1.6}$$

Substituting the expressions of the strains in Eqn. (2-1.3)

$$\frac{P_0}{A_c E_c} = \frac{P_i}{A_p E_p} - \frac{P_0}{A_p E_p}$$

or, $$P_0\left(\frac{1}{A_c E_c} + \frac{1}{A_p E_p}\right) = \frac{P_i}{A_p E_p}$$

or, $$P_0\left(\frac{m}{A_c} + \frac{1}{A_p}\right) = \frac{P_i}{A_p}$$

or, $$\frac{P_0}{A_c} = \frac{P_i}{mA_p + A_c}$$

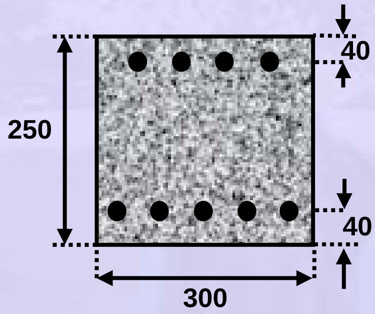
or $$\frac{P_0}{A_c} = \frac{P_i}{A_t} \tag{2-1.7}$$

Thus, the stress in concrete due to the prestressing force after immediate losses (P_0/A_c) can be equated to the stress in the transformed section due to the initial prestress (P_i/A_t).

The following problem illustrates the calculation of loss due to elastic shortening in an idealised pre-tensioned railway sleeper.

Example 2-1.1

A prestressed concrete sleeper produced by pre-tensioning method has a rectangular cross-section of 300mm × 250 mm (*b* × *h*). It is prestressed with 9 numbers of straight 7mm diameter wires at 0.8 times the ultimate strength of 1570 N/mm^2. Estimate the percentage loss of stress due to elastic shortening of concrete. Consider *m* = 6.



Solution

a) Approximate solution considering gross section

The sectional properties are calculated as follows.

Area of a single wire, $A_w = \pi/4 \times 7^2$

$= 38.48 \text{ mm}^2$

Area of total prestressing steel, $A_p = 9 \times 38.48$

$= 346.32 \text{ mm}^2$

Area of concrete section, $A = 300 \times 250$

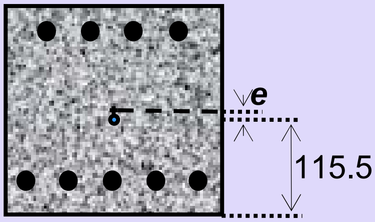
$= 75 \times 10^3 \text{ mm}^2$

Moment of inertia of section, $I = 300 \times 250^3/12$

$= 3.91 \times 10^8 \text{ mm}^4$

Distance of centroid of steel area (CGS) from the soffit,

$$\bar{y} = \frac{4\times38.48\times(250-40)+5\times38.48\times40}{9\times38.48}$$
$$= 115.5 \text{ mm}$$



Prestressing force, P_i = 0.8 × 1570 × 346.32 N

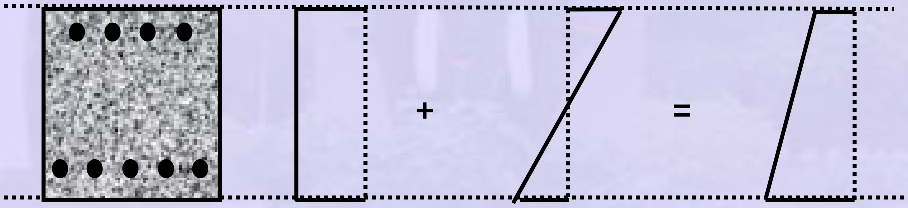
= 435 kN

Eccentricity of prestressing force,

e = (250/2) – 115.5

= 9.5 mm

The stress diagrams due to P_i are shown.



$$-\frac{P_i}{A} \qquad \pm\frac{P_i.e}{I}y \qquad -\frac{P_i}{A}\pm\frac{P_i.e}{I}y$$

Since the wires are distributed above and below the CGC, the losses are calculated for the top and bottom wires separately.

Stress at level of top wires ($y = y_t$ = 125 – 40)

$$\begin{aligned}(f_c)_t &= -\frac{P_i}{A}+\frac{P_i.e}{I}y_t\\ &= -\frac{435\times10^3}{75\times10^3}+\frac{435\times10^3\times9.5}{3.91\times10^8}\times(125-40)\\ &= -5.8+0.9\\ &= -4.9 \text{ N/mm}^2\end{aligned}$$

Stress at level of bottom wires ($y = y_b = 125 – 40$),

$$
\begin{aligned}
(f_c)_b &= -\frac{P_i}{A} - \frac{P_i.e}{I}y_b \\
&= -\frac{435\times10^3}{75\times10^3} - \frac{435\times10^3\times9.5}{3.91\times10^8}\times(125-40) \\
&= -5.8-0.9 \\
&= -6.7\ \text{N/mm}^2
\end{aligned}
$$

Loss of prestress in top wires (in terms of force) $= mf_cA_p$

$= 6 \times 4.9 \times (4 \times 38.48)$

$= 4525.25$ N

Loss of prestress in bottom wires $= 6 \times 6.7 \times (5 \times 38.48)$

$= 7734.48$ N

Total loss of prestress $= 4525 + 7735$

$= 12259.73$ N

≈ 12.3 kN

Percentage loss $= (12.3 / 435) \times 100\%$

$= 2.83\%$

b) Accurate solution considering transformed section.

Transformed area of top steel,

$A_1 = (6 – 1)\ 4 \times 38.48$

$= 769.6\ \text{mm}^2$

Transformed area of bottom steel,

$A_2 = (6 – 1)\ 5 \times 38.48$

$= 962.0\ \text{mm}^2$

Total area of transformed section,

$A_T = A + A_1 + A_2$

$= 75000.0 + 769.6 + 962.0$

$= 76731.6\ \text{mm}^2$

Centroid of the section (CGC)

$$\bar{y} = \frac{A \times 125 + A_1 \times (250 - 40) + A_2 \times 40}{A}$$

$$= 124.8 \text{ mm from soffit of beam}$$

Moment of inertia of transformed section,

$$I_T = I_g + A(0.2)^2 + A_1(210 – 124.8)^2 + A_2(124.8 – 40)^2$$
$$= 4.02 \times 10^8 \text{mm}^4$$

Eccentricity of prestressing force,

$$e = 124.8 – 115.5$$
$$= 9.3 \text{ mm}$$

Stress at the level of bottom wires,

$$(f_c)_b = -\frac{435 \times 10^3}{76.73 \times 10^3} - \frac{(435 \times 10^3 \times 9.3)84.8}{4.02 \times 10^8}$$
$$= -5.67 - 0.85$$
$$= -6.52 \text{ N/mm}^2$$

Stress at the level of top wires,

$$(f_c)_t = -\frac{435 \times 10^3}{76.73 \times 10^3} + \frac{(435 \times 10^3 \times 9.3)85.2}{4.02 \times 10^8}$$
$$= -5.67 + 0.86$$
$$= -4.81 \text{N/mm}^2$$

Loss of prestress in top wires = 6 × 4.81 × (4 × 38.48)

= 4442 N

Loss of prestress in bottom wires = 6 × 6.52 × (5 × 38.48)

= 7527 N

Total loss = 4442 + 7527

= 11969 N

≈ 12 kN

Percentage loss = (12 / 435) × 100%

= 2.75 %

It can be observed that the accurate and approximate solutions are close. Hence, the simpler calculations based on A and I is acceptable.

Pre-tensioned Bending Members

The following figure shows the changes in length and the prestressing force due to elastic shortening of a pre-tensioned bending member.

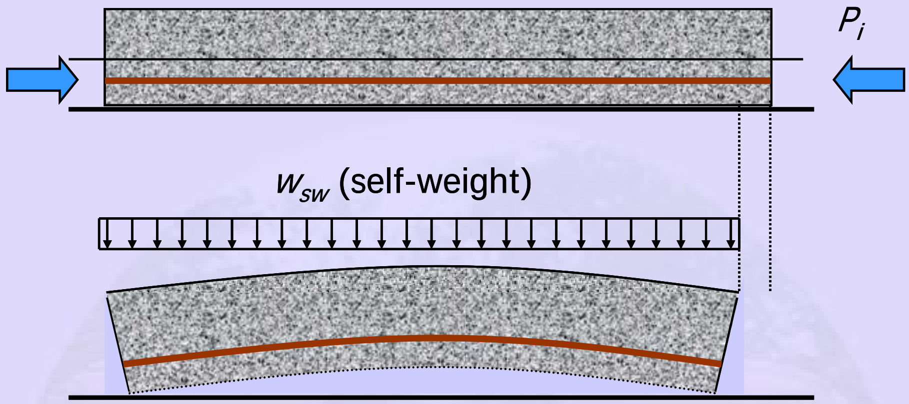


Figure 2-1.6 Elastic shortening of a pre-tensioned bending member

Due to the effect of self-weight, the stress in concrete varies along length (Figure 2-1.6). The loss can be calculated by Eqn. (2-1.1) with a suitable evaluation of the stress in concrete. To have a conservative estimate of the loss, the maximum stress at the level of CGS at the mid-span is considered.

$$f_c = -\frac{P_i}{A} - \frac{P_i e.e}{I} + \frac{M_{sw} e}{I} \tag{2-1.8}$$

Here, M_{sw} is the moment at mid-span due to self-weight. Precise result using A_t and I_t in place of A and I, respectively, is not computationally warranted. In the above expression, the eccentricity of the CGS (e) was assumed to be constant.

For a large member, the calculation of the loss can be refined by evaluating the strain in concrete at the level of the CGS accurately from the definition of strain. This is demonstrated later for post-tensioned bending members.

Post-tensioned Axial Members

For more than one tendon, if the tendons are stretched sequentially, there is loss in a tendon during subsequent stretching of the other tendons. The loss in each tendon can be calculated in progressive sequence. Else, an approximation can be used to calculate the losses.

The loss in the first tendon is evaluated precisely and half of that value is used as an average loss for all the tendons.

$$\begin{aligned} \Delta f_p &= \frac{1}{2}\Delta f_{p1} \\ &= \frac{1}{2} m f_{c1} \\ &= \frac{1}{2} m \sum_{j=2}^{n} \frac{P_{i,j}}{A} \end{aligned} \tag{2-1.9}$$

Here,

$P_{i,j}$ = initial prestressing force in tendon j

n = number of tendons

The eccentricity of individual tendon is neglected.

Post-tensioned Bending Members

The calculation of loss for tendons stretched sequentially, is similar to post-tensioned axial members. For curved profiles, the eccentricity of the CGS and hence, the stress in concrete at the level of CGS vary along the length. An average stress in concrete can be considered.

For a parabolic tendon, the average stress ($f_{c,avg}$) is given by the following equation.

$$f_{c,avg} = f_{c1} + \frac{2}{3}\left(f_{c2} - f_{c1}\right) \tag{2-1.10}$$

Here,

f_{c1} = stress in concrete at the end of the member

f_{c2} = stress in concrete at the mid-span of the member.

A more rigorous analysis of the loss can be done by evaluating the strain in concrete at the level of the CGS accurately from the definition of strain. This is demonstrated for a beam with two parabolic tendons post-tensioned sequentially. In Figure 2-1.7, Tendon B is stretched after Tendon A. The loss in Tendon A due to elastic shortening during tensioning of Tendon B is given as follows.

$$\begin{aligned} \Delta f_p &= E_p \varepsilon_c \\ &= E_p\left[\varepsilon_{c1} + \varepsilon_{c2}\right] \end{aligned} \tag{2-1.11}$$

Here, ε_c is the strain at the level of Tendon A. The component of ε_c due to pure compression is represented as ε_{c1}. The component of ε_c due to bending is represented as ε_{c2}. The two components are calculated as follows.

$$
\begin{aligned}
\varepsilon_{c1} &= \frac{P_B}{AE_c} \\
\varepsilon_{c2} &= \frac{\delta L}{L} \\
&= \frac{1}{L}\int_0^L \frac{P_B.e_B(x).e_A(x)}{IE_c}dx \\
&= \frac{P_B}{E_c LI}\int_0^L e_B(x).e_A(x)\,dx
\end{aligned}
\qquad \textbf{(2-1.12)}
$$

Here,

A = cross-sectional area of beam

P_B = prestressing force in Tendon B

E_c = modulus of concrete

L = length of beam

$e_A(x)$, $e_B(x)$ = eccentricities of Tendons A and B, respectively, at distance x from left end

I = moment of inertia of beam

δL = change in length of beam

The variations of the eccentricities of the tendons can be expressed as follows.

$$e_A(x) = e_{A1} + 4\Delta e_A \frac{x}{L}\left(1-\frac{x}{L}\right) \qquad \textbf{(2-1.13)}$$

$$e_B(x) = e_{B1} + 4\Delta e_B \frac{x}{L}\left(1-\frac{x}{L}\right) \qquad \textbf{(2-1.14)}$$

Where, $\Delta e_A = e_{A2} - e_{A1}$

$\Delta e_B = e_{B2} - e_{B1}$

e_{A1}, e_{A2} = eccentricities of Tendon A at 1 (end) and 2 (centre), respectively.

e_{B1}, e_{B2} = eccentricities of Tendon B at 1 and 2, respectively.

Substituting the expressions of the eccentricities in Eqn. (2-1.12), the second component of the strain is given as follows.

$$\frac{P_B}{E_c I} = \left[\frac{1}{5} e_{A1} e_{B1} + \frac{2}{15} \left(e_{A1} e_{B2} + e_{A2} e_{B1} \right) + \frac{8}{15} e_{A2} e_{B2} \right] \qquad \textbf{(2-1.15)}$$