Flywheel

A flywheel is an inertial energy-storage device. It absorbs mechanical energy and serves as a reservoir, storing energy during the period when the supply of energy is more than the requirement and releases it during the period when the requirement of energy is more than the supply.

Flywheels-Function need and Operation

The main function of a flywheel is to smoothen out variations in the speed of a shaft caused by torque fluctuations. If the source of the driving torque or load torque is fluctuating in nature, then a flywheel is usually called for. Many machines have load patterns that cause the torque time function to vary over the cycle. Internal combustion engines with one or two cylinders are a typical example. Piston compressors, punch presses, rock crushers etc. are the other systems that have flywheel.

Flywheel absorbs mechanical energy by increasing its angular velocity and delivers the stored energy by decreasing its velocity.

![Figure 3.3.1](image-url)
Design Approach

There are two stages to the design of a flywheel.

First, the amount of energy required for the desired degree of smoothening must be found and the (mass) moment of inertia needed to absorb that energy determined.

Then flywheel geometry must be defined that caters the required moment of inertia in a reasonably sized package and is safe against failure at the designed speeds of operation.

Design Parameters

Flywheel inertia (size) needed directly depends upon the acceptable changes in the speed.

Speed fluctuation

The change in the shaft speed during a cycle is called the speed fluctuation and is equal to $\omega_{\text{max}} - \omega_{\text{min}}$

$$F_l = \omega_{\text{max}} - \omega_{\text{min}}$$

We can normalize this to a dimensionless ratio by dividing it by the average or nominal shaft speed ($\omega_{\text{ave}}$).

$$C_f = \frac{\omega_{\text{max}} - \omega_{\text{min}}}{\omega_{\text{ave}}}$$

Where $\omega_{\text{avg}}$ is nominal angular velocity

Co-efficient of speed fluctuation

The above ratio is termed as coefficient of speed fluctuation $C_f$ and it is defined as

$$C_f = \frac{\omega_{\text{max}} - \omega_{\text{min}}}{\omega}$$
Where \( \omega \) is nominal angular velocity, and \( \omega_{\text{ave}} \) the average or mean shaft speed desired. This coefficient is a design parameter to be chosen by the designer.

The smaller this chosen value, the larger the flywheel have to be and more the cost and weight to be added to the system. However the smaller this value more smoother the operation of the device.

It is typically set to a value between 0.01 to 0.05 for precision machinery and as high as 0.20 for applications like crusher hammering machinery.

**Design Equation**

The kinetic energy \( E_K \) in a rotating system

\[
E_K = \frac{1}{2} I \left( \omega^2 \right)
\]

Hence the change in kinetic energy of a system can be given as,

\[
E_K = E_2 - E_1
\]

\[
\omega_{\text{avg}} = \frac{\omega_{\text{max}} + \omega_{\text{min}}}{2}
\]

\[
E_K = \frac{1}{2} I_s \left( 2 \omega_{\text{avg}} \right) \left( C_f \omega_{\text{avg}} \right)
\]

\[
E_2 - E_1 = C_f I \omega^2
\]

\[
I_s = \frac{E_k}{C_f \omega_{\text{avg}}^2}
\]

Thus the mass moment of inertia \( I_m \) needed in the entire rotating system in order to obtain selected coefficient of speed fluctuation is determined using the relation.
\[ E_K = \frac{1}{2} I_s \left( 2 \omega_{avg} \right) \left( C_f \omega_{avg} \right) \]

\[ I_s = \frac{E_k}{C_f \omega_{avg}^2} \]

The above equation can be used to obtain appropriate flywheel inertia \( I_m \) corresponding to the known energy change \( E_k \) for a specific value coefficient of speed fluctuation \( C_f \).

**Torque Variation and Energy**

The required change in kinetic energy \( E_k \) is obtained from the known torque time relation or curve by integrating it for one cycle.

\[
\int_{\theta \at \omega_{min}}^{\theta \at \omega_{max}} \left( T_1 - T_{avg} \right) d\theta = E_K
\]

Computing the kinetic energy \( E_k \) needed is illustrated in the following example.

**Torque Time Relation without Flywheel**

A typical torque time relation for example of a mechanical punching press without a fly wheel is shown in the figure.

In the absence of fly wheel surplus or positive energy is avalible initially and intermedialty and enery absorption or negative energy during punching and stripping operations. A large magitidue of speed fluctuation can be noted. To smoothen out the speed fluctuation fly wheel is to be added and the fly wheel energy needed is computed as illustrated below.
Torque

Area +20 073
Area +15 388
Area +20 073
Area +9 356
Area -9 202
Area -26 105
Area -9 202
Area -6 032
Shaft angle time t
Average
rms

Figure 3.3.2
Accumulation of Energy pulses under a Torque-Time curve

<table>
<thead>
<tr>
<th>From</th>
<th>Δ Area=ΔE</th>
<th>Accumulated sum =E</th>
<th>Min &amp; max</th>
</tr>
</thead>
<tbody>
<tr>
<td>A to B</td>
<td>+20 073</td>
<td>+20 073</td>
<td>( \omega_{\text{min}} )@B</td>
</tr>
<tr>
<td>B to C</td>
<td>-26 105</td>
<td>-6 032</td>
<td>( \omega_{\text{max}} )@C</td>
</tr>
<tr>
<td>C to D</td>
<td>+15 388</td>
<td>+9 356</td>
<td></td>
</tr>
<tr>
<td>D to A</td>
<td>-9 202</td>
<td>+154</td>
<td></td>
</tr>
</tbody>
</table>

Total Energy = \( E \@\omega_{\text{min}} - E \@\omega_{\text{min}} \)
=\((-6 032)-(+20 073)\) = 26 105 Nmm²

Figure 3.3.3
Geometry of Flywheel

The geometry of a flywheel may be as simple as a cylindrical disc of solid material, or may be of spoked construction like conventional wheels with a hub and rim connected by spokes or arms. Small flywheels are solid discs of hollow circular cross section. As the energy requirements and size of the flywheel increase, the geometry changes to a disc of central hub and peripheral rim connected by webs and to hollow wheels with multiple arms.
The latter arrangement is a more efficient of material especially for large flywheels, as it concentrates the bulk of its mass in the rim which is at the largest radius. Mass at largest radius contributes much more since the mass moment of inertia is proportional to $mr^2$. 
For a solid disc geometry with inside radius $r_i$ and outside radius $r_o$, the mass moment of inertia $I_m$ is

$$I_m = mk^2 = \frac{m}{2} (r_o^2 + r_i^2)$$

The mass of a hollow circular disc of constant thickness $t$ is

$$m = \frac{W}{g} = \pi \frac{\gamma}{g} (r_o^2 - r_i^2) t$$

Combining the two equations we can write

$$I_m = \frac{\pi \gamma}{2g} \left( \frac{4}{r_o^4} - \frac{4}{r_i^4} \right) t$$

Where $\gamma$ is material’s weight density.

The equation is better solved by geometric proportions i.e. by assuming inside to outside radius ratio and radius to thickness ratio.

**Stresses in Flywheel**

Flywheel being a rotating disc, centrifugal stresses acts upon its distributed mass and attempts to pull it apart. Its effect is similar to those caused by an internally pressurized cylinder.

$$\sigma_t = \frac{\gamma}{g} \omega^2 \left( \frac{3 + \nu}{8} \right) \left( \frac{r_i^2 + r_o^2}{3 + \nu} - \frac{1 + 3\nu}{3 + \nu} r_i^2 \right)$$

$$\sigma_r = \frac{\gamma}{g} \omega^2 \left( \frac{3 + \nu}{8} \right) \left( \frac{r_i^2 + r_o^2}{3 + \nu} - \frac{r_i^2 r_o^2}{r^2} - r_i^2 \right)$$

$\gamma = \text{material weight density}$, $\omega = \text{angular velocity in rad/sec}$. $\nu = \text{Poisson’s ratio}$, is the radius to a point of interest, $r_i$ and $r_o$ are inside and outside radii of the solid disc flywheel.

Analogous to a thick cylinder under internal pressure the tangential and radial stress in a solid disc flywheel as a function of its radius $r$ is given by:
The point of most interest is the inside radius where the stress is a maximum. What causes failure in a flywheel is typically the tangential stress at that point from where fracture originated and upon fracture fragments can explode resulting extremely dangerous consequences. Since the forces causing the stresses are a function of the rotational speed also, instead of checking for stresses, the maximum speed at which the stresses reach the critical value can be determined and safe operating speed can be calculated or specified based on a safety factor. Generally some means to preclude its operation beyond this speed is desirable, for example like a governor. Consequently

\[ F.O.S \ (N) = N_{os} = \frac{\omega}{\omega_{yield}} \]
WORKED OUT EXAMPLE 1

A 2.2 kw, 960 rpm motor powers the cam driven ram of a press through a gearing of 6:1 ratio. The rated capacity of the press is 20 kN and has a stroke of 200 mm. Assuming that the cam driven ram is capable of delivering the rated load at a constant velocity during the last 15% of a constant velocity stroke. Design a suitable flywheel that can maintain a coefficient of Speed fluctuation of 0.02. Assume that the maximum diameter of the flywheel is not to exceed 0.6m.

Work done by the press=

\[ U = 20 \times 10^3 \times 0.2 \times 0.15 \]
\[ = 600 \text{Nm} \]

Energy absorbed= work done= 600 Nm

Mean torque on the shaft:

\[ \frac{2.2 \times 10^3}{2 \times \pi \times \frac{960}{60}} = 21.88 \text{Nm} \]

Energy supplied= work done per cycle

\[ = 2\pi \times 21.88 \times 6 \]
\[ = 825 \text{Nm} \]

Thus the mechanical efficiency of the system is =

\[ \eta = \frac{600}{825} = 0.727 = 72\% \]

Therefore the fluctuation in energy is =

\[ E_k = \text{Energy absorbed} - \text{Energy supplied} \]
\[ 600 - 825 \times 0.075 (21.88 \times 6 \times \pi \times 0.15) \]
\[ 538.125 \text{Nm} \]
\[ I = \frac{E_k}{C_f \left( \omega_{\text{avg}} \right)^2} = \frac{538.125}{0.02 \left( 2 \pi \times \frac{960}{60} \right)^2} \]
\[ = 2.6622 \text{ kg m}^2 \]
\[ I = \frac{\pi \cdot r}{2 \cdot g} \left( r_o^2 - r_i^2 \right), t \]

Assuming \( \frac{r_i}{r_o} = 0.8 \)
\[ 2.6622 = \frac{\pi}{2} \left( \frac{78500}{9.86} \right) \left( 0.30^4 - 0.24^4 \right) t \]
\[ = 59.805 t \]
\[ \therefore \ t = \frac{2.6622}{59.805} = 0.0445 \]

or

45 mm

\[ \sigma_t = \frac{r}{g} \omega^2 \left( \frac{3 + \gamma}{8} \right) \left( r_i^2 + r_o^2 - r^2 \right) + \frac{1 + 3 \gamma}{3 + \gamma} \]
\[ \sigma_t = \frac{78500}{9.81} \omega^2 \left( \frac{3 + 0.3}{8} \right) \left( 0.24^2 + 0.3^2 - \frac{1.9}{3.3} \times 0.24^2 \right) \]
\[ \sigma_t = 0.543 \times \left( 2 \pi \times \frac{960}{60} \right)^2 \]
\[ = 55667 \text{ N/m}^2 \]
\[ = 0.556 \text{ MPa} \]

or if \( \sigma_t = 150 \text{ MPa} \)
\[ 150 \times 10^6 = 7961.4 \omega^2 \left( 0.4125 \right) \left( 0.0376 \right) \left( 0.090 \right) \left( 0.0331 \right) \]
\[ = 0.548 \omega^2 \]
\[ \omega = 16544 \text{ rad/sec}^2 \]

\[ N_{OS} = \frac{\omega}{32 \pi} \]
\[ = 164.65 \]