Module 2- GEARS

Lecture 8 – SPUR GEAR DESIGN

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8.1 SURFACE DURABILITY BASIC CONCEPTS

Earlier various types of gear failures have been discussed in detail. Under contact conditions, gear teeth are subjected to Hertzian contact stresses and elasto-hydrodynamic lubrication. Excessive loading and lubrication breakdown can cause combinations of abrasion, pitting and scoring.

Fig. 8.1 Single tooth contact
Fig. 8.1 shows the contact of a gear and pinion tooth at the tip. The instantaneous contact point velocities of gear and pinion are vectorially denoted by $V_g$ and $V_p$.

a. The individual velocity can be resolved into two components normal and tangent to the tooth surface as $V_{pn}$ and $V_{pt}$ for the pinion and $V_{gn}$ and $V_{gt}$ for the gear respectively.

b. When the teeth do not separate or crush together, the components $V_{pn}$ and $V_{gn}$ normal to the surface must be the same.

c. Hence tangential velocity components of the surface are different.

d. The sliding velocity is the difference between $V_{pt}$ and $V_{gt}$.

![Gear tooth contact diagram]

**Fig. 8.2 Gear tooth contact position during operation.**

If the contact is at the pitch point $P$ in Fig. 8.2, the sliding velocity is zero; the tooth relative motion is of pure rolling.

a) At all the other contact points, the relative motion is one of pure rolling and sliding.
b) The sliding velocity is directly proportional to the distance between the pitch point and the point of contact.

c) The maximum sliding velocity occurs with contacts at the tooth tips.

d) Gear teeth with longer addenda have higher sliding velocities than gears with shorter addenda.

The relative sliding velocity reverses the direction as a pair of teeth roll through the pitch point. During approach, the sliding friction forces tend to compress the teeth and during recess friction forces tend to elongate the teeth to give smoother action.

8.2 SURFACE FAILURES

a. Gear teeth are also subjected to Hertz contact stresses and the lubrication is often elastohydrodynamic.

b. Excessive loading and lubrication breakdown results in various combinations of abrasion, pitting and scoring.

1. Abrasive wear is caused

   a) by the presence of foreign particles, in gears that are not enclosed,
   b) in enclosed gears that were assembled with abrasive particles present,
   c) in gears lubricated by an oil supply with inadequate filtration.

2. Scoring:

   a) It occurs at high speeds when adequate lubrication is not provided by the elastohydrodynamic action.
   b) Lack of lubrication causes high sliding friction. High tooth loading and high sliding velocities that produce a high rate of heat in the localized contact region causes welding and tearing of surfaces apart.
   c) Scoring can often be prevented by directing adequate flow of appropriate lubricant that maintains hydrodynamic lubrication.
d) Surface finish is also an important factor for scoring. Surface finish as fine as 0.5\(\mu\)m cla is desirable to avoid scoring.

3. Pitting or surface fatigue failure:
Complex stresses within the contact zone cause surface and subsurface fatigue failures. Pitting occurs only after a large number of repeated loading mainly in the vicinity of the pitch line where the oil film breaks down because of zero sliding velocity.

**8.3 SPUR GEAR – BUCKINGHAM CONTACT STRESS EQUATION**

Buckingham adapted the Hertz contact stress equation for a pair of gear teeth shown in Fig. 8.3. He treated a pair of gear teeth as two cylinders of radii equal to the radii of curvature of the mating involutes at the pitch point. From basic involute geometry, these radii are given as,

\[
R_1 = \frac{(d_1 \sin \varnothing)}{2} \quad \& \quad R_2 = \frac{(d_2 \sin \varnothing)}{2}
\]  

(8.1)

---

*Fig. 8.3 Forces acting at the tooth contact*
\[
\sigma_{H}^{'} = 0.564 \frac{F E}{b R}
\]  
(8.2)

Where \( F = \frac{F_t}{\cos \varnothing} \)  
(8.3)

\[
\frac{1}{E} = \frac{1 - \mu_i^2}{E_1} + \frac{1 - \mu_i^2}{E_2}
\]  
(8.4)

\[
\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{2}{d_1 \sin \phi} + \frac{2}{d_2 \sin \phi}
\]  
(8.5)

Substituting the value of \( F_t, E \) and \( R \) from equation (8.3), (8.4) & (8.5) into (8.2) we get

\[
\sigma_{H}^{'} = 0.564 \frac{F_t \left( \frac{2}{d_1 \sin \phi} + \frac{2}{d_2 \sin \phi} \right)}{b \cos \phi \left( \frac{1 - \mu_i^2}{E_1} + \frac{1 - \mu_i^2}{E_2} \right)}
\]  
(8.6)

It is seen from eqn. (8.6) that,

- Since contact area also increases with load, the contact stress increases only as the square root of load \( F_t \)
- Contact area increases with decrease of modulii of elasticity, \( E_1 \) and \( E_2 \).
- Larger gears have greater radii of curvature, hence lower stress.

Equation (8.6) can be rewritten by combining terms relating to the elastic properties of the material into single factor \( C_p \) given by:

\[
C_p = 0.564 \left( \frac{1}{1 - \mu_i^2} + \frac{1}{1 - \mu_i^2} \right)^{1 \over \sqrt{E_1 + E_2}}
\]  
(8.7)

The \( C_p \) values are given in Table 8.1.
Combining terms relating to tooth shape into second factor, I, known as the geometry factor:

\[ I = \frac{\sin \phi \cos \phi}{2} \frac{i}{i+1} \]  

(8.8)

Where the speed ratio \( i = \frac{d_2}{d_1} \)

The simplified contact stress equation is:

\[ \sigma_H' = C_p \sqrt{\frac{F_t}{b d_i I}} \]  

(8.9)

In this equation \( F_t \) is considered as static since the Hertz equation is derived for static loads. Rearranging the terms,

\[ F_t = b d_i I \left( \frac{\sigma_H}{C_p} \right)^2 \]  

(8.10)

If we substitute \( \sigma_H' \) by the permissible stress \( [\sigma_H] \) for the material, then what we get is the tooth surface strength of the pinion \( F_{ts} \).

\[ F_{ts} = b d_i I \left( \frac{[\sigma_H]}{C_p} \right)^2 \]  

(8.11)
\[ F_{ts} > F_d \]  \hspace{1cm} (8.12)

For safe operation of the gear from surface fatigue considerations, \( F_d \) which is the Buckingham dynamic load on gear tooth should be less than the tooth surface strength of the gear. This approach gives quick results for preliminary design. This is the Buckingham design approach for wear strength.

### 8.4 CONTACT STRESS AGMA

Introducing the factors \( K_v \), \( K_o \) and \( K_m \) used in the bending fatigue analysis into the contact stress equation, the dynamic contact stress is obtained as \( \sigma_H \):

\[ \sigma_H = C_p \sqrt{\frac{F_t}{b d_1 I}} K_v K_o K_m \]  \hspace{1cm} (8.13)

\( K_v \) = Velocity or dynamic factor, indicates the severity of impact on successive pairs of teeth during engagement. This is a function of pitch line velocity and manufacturing accuracy. It is given by equation (8.14), (8.15) and (8.16).

\[ K_v = \frac{6 + V}{6} \]  \hspace{1cm} (8.14)

Equation (8.14) is used for cut or milled teeth or for gears not carefully generated.

\[ K_v = \frac{50 + (200 V^{0.5})}{50} \]  \hspace{1cm} (8.15)

Equation (8.15) is used for hobbed and shaped gears.

\[ K_v = \left[ \frac{78 + (200 V^{0.5})}{78} \right]^{0.5} \]  \hspace{1cm} (8.16)

Equation (8.16) is used for high-precision shaved or ground teeth.

\( K_o \) = Overload factor which reflects the degree of non-uniformity of driving and load torques. It is given in Table 8.2.
Table 8.2 - Overload factor $K_o$

<table>
<thead>
<tr>
<th>Source of power</th>
<th>Uniform</th>
<th>Moderate Shock</th>
<th>Heavy Shock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform</td>
<td>1.00</td>
<td>1.25</td>
<td>1.75</td>
</tr>
<tr>
<td>Light shock</td>
<td>1.25</td>
<td>1.50</td>
<td>2.00</td>
</tr>
<tr>
<td>Medium shock</td>
<td>1.50</td>
<td>1.75</td>
<td>2.25</td>
</tr>
</tbody>
</table>

$K_m =$ Load distribution factor which accounts for non uniform spread of the load across the face width. It depends on the accuracy of mounting, bearings, shaft deflection and accuracy of gears. Values are given in Table 8.3.

Table 8.3 Load distribution factor $K_m$

<table>
<thead>
<tr>
<th>Characteristics of Support</th>
<th>0 - 50</th>
<th>150</th>
<th>225</th>
<th>400 up</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accurate mountings, small bearing clearances, minimum deflection, precision gears</td>
<td>1.3</td>
<td>1.4</td>
<td>1.5</td>
<td>1.8</td>
</tr>
<tr>
<td>Less rigid mountings, less accurate gears, contact across the full face</td>
<td>1.6</td>
<td>1.7</td>
<td>1.8</td>
<td>2.2</td>
</tr>
<tr>
<td>Accuracy and mounting such that less than full-face contact exists</td>
<td>Over 2.2</td>
<td>Over 2.2</td>
<td>Over 2.2</td>
<td>Over 2.2</td>
</tr>
</tbody>
</table>

8.5 SURFACE FATIGUE STRENGTH (AGMA)

Surface fatigue strength of the material is given by,

$$\sigma_{sf} = \sigma_{sf}' K_L K_r K_T$$  (8.17)

Where

$\sigma_{sf}' =$ surface fatigue strength of the material given in Table 8.4
KL = Life factor given in Fig. 8.4
KR = Reliability factor, given in Table 8.5

### Table 8.4 Surface fatigue strength $\sigma_{sf}$ (MPa) for metallic spur gears ($10^7$ cycle life 99% reliability and temperature < $120^\circ$ C)

<table>
<thead>
<tr>
<th>Material</th>
<th>$\sigma_{sf}$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>2.8 ( Bhn ) – 69 MPa</td>
</tr>
<tr>
<td>Nodular iron</td>
<td>0.95 [ 2.8 (Bhn ) – 69 MPa ]</td>
</tr>
<tr>
<td>Cast iron, grade 20</td>
<td>379</td>
</tr>
<tr>
<td>Cast iron, grade 30</td>
<td>482</td>
</tr>
<tr>
<td>Cast iron, grade 40</td>
<td>551</td>
</tr>
<tr>
<td>Tin Bronze, AGMA 2C (11% Sn)</td>
<td>207</td>
</tr>
</tbody>
</table>

**Fig. 8.4 Life Factor $K_L$**
### Table 8.5 Reliability factor $K_R$

<table>
<thead>
<tr>
<th>Reliability (%)</th>
<th>$K_R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>1.25</td>
</tr>
<tr>
<td>99</td>
<td>1.00</td>
</tr>
<tr>
<td>99.9</td>
<td>0.80</td>
</tr>
</tbody>
</table>

$K_T$ = temperature factor,
- $= 1$ for $T \leq 120^\circ C$ based on lubricant temperature.
- $< 1$ for $T > 120^\circ C$ based on AGMA standards.

Allowable surface fatigue stress for design is given by

$$[\sigma_H] = \sigma_{sf} / s \quad (8.18)$$

Factor of safety $s = 1.1$ to $1.5$

Hence Design equation is:

$$\sigma_H \leq [\sigma_H] \quad (8.19)$$

### 8.6 GEAR MATERIALS

Gears are commonly made of cast iron, steel, bronze, phenolic resins, acetal, nylon or other plastics. The selection of material depends on the type of loading and speed of operation, wear life, reliability and application. Cast iron is the least expensive. ASTM / AGMA grade 20 is widely used. Grades 30, 40, 50, 60 are progressively stronger and more expensive. CI gears have greater surface fatigue strength than bending fatigue strength. Better damping properties enable them to run quietly than steel.
Nodular cast iron gears have higher bending strength together with good surface durability. These gears are now a days used in automobile cam shafts. A good combination is often a steel pinion mated against cast iron gear. Steel finds many applications since it combines both high strength and low cost. Plain carbon and alloy steel usage is quite common.

Through hardened plain carbon steel with 0.35 - 0.6% C are used when gears need hardness more than 250 to 350 Bhn. These gears need grinding to overcome heat treatment distortion. When compactness, high impact strength and durability are needed as in automotive and mobile applications, alloy steels are used. These gears are surface or case-hardened by flame hardening, induction hardening, nitriding or case carburizing processes. Steels such as En 353, En36, En24, 17CrNiMo6 widely used for gears.

Bronzes are used when corrosion resistance, low friction and wear under high sliding velocity is needed as in worm-gear applications. AGMA recommends Tin bronzes containing small % of Ni, Pb or Zn. The hardness may range from 70 to 85Bhn. Non metallic gears made of phenolic resin, acetal, nylon and other plastics are used for light load lubrication free quiet operation at reasonable cost. Mating gear in many such applications is made with steel. In order to accommodate high thermal expansion, plastic gears must have higher backlash and undergo stringent prototype testing.