Module 2 - GEARS

Lecture 2 – INVOLUTE SPUR GEARS

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2.1 INTRODUCTION:

The function of a gear is to work smoothly while transmitting motion or torque. For this the angular velocity ratio at all times should remain constant. This aspect is explained here using various gear terminology that are peculiar to gears. Understanding of the definition of these terminologies helps to grasp the functioning of gears and the design of gears.

Fig.2.1 Portion of involute spur gear
2.2. NOMENCLATURE OF INVOLUTE SPUR GEARS

Refer to the Figs.2.1 and 2.2 which show a portion of a pair of involute gears in mesh.

- **Pitch surface**: The surface of the imaginary rolling cylinder (cone, etc.) that replaces the toothed gear.
- **Pitch surface**: The surface of the imaginary rolling cylinder (cone, etc.) that replaces the toothed gear.
- **Pitch circle**: A normal section of the pitch surface.
- **Addendum circle**: A circle bounding the ends of the teeth, in a normal section of the gear.
- **Dedendum circle or Root circle**: The circle bounding the spaces between the teeth, in a normal section of the gear.
- **Addendum**: The radial distance between the pitch circle and the addendum circle.
- **Dedendum**: The radial distance between the pitch circle and the root circle.
- **Clearance**: The difference between the Dedendum of one gear and the addendum of the mating gear.
- **Face of a tooth**: That part of the tooth surface lying outside the pitch surface.
- **Flank of a tooth**: The part of the tooth surface lying inside the pitch surface.
- **Top land**: The top surface of a gear tooth.
- **Bottom land**: The bottom surface of the tooth space.
- **Circular thickness (tooth thickness)**: The thickness of the tooth measured on the pitch circle. It is the length of an arc and not the length of a straight line.
- **Tooth space**: The space between successive teeth.
- **Width of space**: The distance between adjacent teeth measured on the pitch circle.
- **Backlash**: The difference between the tooth thickness of one gear and the tooth space of the mating gear.
Circular pitch $p$: The width of a tooth and a space, measured on the pitch circle. It is equal to the pitch circumference divided by the number of teeth. If,

$$p = \frac{\pi d}{z} \quad (2.1)$$

- $p$ - circular pitch
- $P$ - diametral pitch
- $z$ - number of teeth
- $d$ - pitch diameter

Diametral pitch $P$: The number of teeth of a gear per unit pitch diameter. The diametral pitch is hence the number of teeth divided by the pitch diameter.

$$P = \frac{z}{d} \quad (2.2)$$
- The product of the diametral pitch and the circular pitch equals $\pi$.

$$p \cdot P = \pi \quad (2.3)$$

- The effect of diametral pitch on the size of the gear tooth is shown in Fig. 2.3

![Fig.2.3 Variation of tooth size with diametral pitch](image)

- Actual tooth size for various diametral pitches is shown in Fig.2.4. The diametral pitches are standardized and these values are given Table 2.1.

<table>
<thead>
<tr>
<th>Table 2.1 Standard diametral pitches</th>
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<tr>
<td>1</td>
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<td>4</td>
</tr>
<tr>
<td>16</td>
</tr>
<tr>
<td>72</td>
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In metric system, module is used instead of diametral pitch. It is nothing but the inverse of diametral pitch. The standard modules for which cutters are readily available in the market are given in Table 2.2

- Module \( m \): Pitch diameter divided by number of teeth. The pitch diameter is usually specified in millimeters.
- \( m = \frac{d}{z} \) \hspace{1cm} (2.4)
• **Table 2.2 Standard modules in mm**

<table>
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<tr>
<th></th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
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<td>2.25</td>
<td>2.5</td>
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<td>4.5</td>
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<td>6.0</td>
<td>6.5</td>
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</tr>
</tbody>
</table>

Further increase is in terms of 5 mm

• Fillet Radius: The small radius that connects the profile of a tooth to the root circle.
• Crowning: Grinding of tooth edges to prevent edge loading is known as crowning. This is shown in Fig. 2.5.

*Fig. 2.5 Crowning of the tooth to overcome edge loading*
Figs. 2.6 and 2.7 illustrate various terminologies one comes across during meshing of gear teeth.

**Fig.2.6 Meshing of two pairs of gear teeth which explains the various terminologies used in involute gearing in detail**

From the Fig.2.6 and 2.7:

- **Pinion**: The smallest of any pair of mating gears.
- **Gear**: The largest of the pair is called simply the gear.
- **Velocity ratio** $i$: The ratio of rotational speed of the driving gear $n_1$ (or input gear) to the rotational speed of the driven gear $n_2$ (or output gear). If $z_1$ & $z_2$, $d_1$ & $d_2$, $r_1$ & $r_2$, and $\omega_1$ & $\omega_2$ are corresponding number of teeth, pitch diameters, radii and angular velocities of pinion and the gear, then

$$i = \frac{n_1}{n_2} = \frac{\omega_1}{\omega_2} = \frac{z_2}{z_1} = \frac{d_2}{d_1} = \frac{r_2}{r_1} \quad (2.5)$$

- **Base circle**: An imaginary circle used in involute gearing to generate the involutes that form the tooth profiles.
Fig. 2.7 Meshing of two pairs of gear teeth which explains the various terminologies used in involute gearing.

- **Pitch point**: The point of tangency of the pitch circles of a pair of mating gears.
- **Common tangent**: The line tangent to the pitch circle at the pitch point.
- **Line of action**: A line normal to a pair of mating tooth profiles at their point of contact.
- **Path of contact**: The path traced by the contact point of a pair of tooth profiles.
- **Pressure angle $\phi$**: The angle between the common normal at the point of tooth contact and the common tangent to the pitch circles.
- **Pressure angle** is also the angle between the line of action and the common tangent.
2.3 Gear meshing

Having known various terminologies during meshing of gears, how the contact points traverses is illustrated by flash in Fig. 2.8. It can be seen clearly during operation the contact point demarked by red point is established on the left hand side of the axis joining the centre of the gears and moves to the right and vanishes. New contact is established again at the left side. It moves along the straight line known as line of action which is tangent to both the base circles. The angle made by it with the common tangent to the pitch circles at the pitch point is known as the pressure angle. It can be noted that the line of action passes through the pitch point which is the point of intersection of the line connecting the centres of the gears with the common tangent.

If the portion of the gear exists below the base circle, then it results in interference and leads to undercutting of the tooth. In Fig. 2.9 the shaded portion of the teeth are below the base circles. They are going to cause interference. In Fig.2.10 portion of the pinion tooth below the gear tooth surface is seen. In practice this cannot happen unless the part of the
2.4 Gear tooth Interference

![Diagram of gear meshing showing interference](image)

**Fig. 2.9 Illustration of interference in gear meshing**

![Diagram of gears in mesh](image)

**Fig. 2.10 Gears in mesh showing portion of the tooth of pinion digging into the gear tooth on the left**
A gear tooth is relieved of material causing this interference. This is achieved by the harder pinion tooth removing away the portion of the gear tooth to avoid interference. Note that the tooth portion below the base circle is not having involute profile. Such a situation will arise when a gear with a certain number of teeth mates with pinion having number of teeth below a critical value.

### 2.5 Methods of elimination of Gear tooth Interference

In certain spur designs if interference exists, it can be overcome by:

- (a) removing the cross hatched tooth tips i.e., using stub teeth
- (b) increasing the number of teeth on the mating pinion.
- (c) increasing the pressure angle
- (d) tooth profile modification or profile shifting
- (e) increasing the centre distance as illustrated in Fig.2.11.

For a given gear, the interference can also be eliminated by increasing the centre distance.

![Fig.2.11 Method of elimination of interference in spur gears](image-url)
2.6 Minimum No. of Teeth on pinion to avoid interference for a given gear

- Referring to Fig. 2.10, the involute profile doesn’t exist beyond base circle. When the pinion rotates clockwise, first and last point of contacts are e and g where the line of action is tangent to the base circles.
- Any part of the pinion tooth face extending beyond a circle through g interferes with gear flank as shown at i.
- The interference limits the permissible length of addendum. As the diameter of the pinion is reduced, the permissible addendum of larger gear becomes smaller.
- Let the addendum height be k times the module i.e., km. From the Fig.2.12 the maximum gear addendum circle radius is:

\[
AE = r_2 + km = \sqrt{AG^2 + GE^2} \tag{2.6}
\]

\[\begin{align*}
AG &= r_2 \cos \phi \\
GE &= (r_1 + r_2) \sin \phi
\end{align*}\]

Substituting in equation (2.6) and simplifying

\[
r_2 + km = \sqrt{r_2^2 \cos^2 \phi + (r_1 + r_2)^2 \sin^2 \phi} \tag{2.7}
\]

Substituting \(r_1 = m z_1\) and \(r_2 = m z_2\) in equation (2.7) and rearranging
\[ z_1^2 + 2z_1 z_2 = \frac{4k(z_2 + k)}{\sin^2 \varphi} \quad (2.8) \]

For a rack and pinion, \( z_2 = \infty \) and the equation \( (2.8) \) reduces to
\[ Z_1 = \frac{2k}{\sin^2 \varphi} \quad (2.9) \]

For full depth gears (i.e., \( k = 1 \)) engaging with rack, minimum teeth on the pinion to avoid interference is
\[ z_1 = 31.9 = 32 \text{ for } 14.5^0 \text{ pressure angle} \]
\[ z_1 = 17.097 = 17 \text{ for } 20^0 \text{ pressure angle} \]
\[ z_1 = 13.657 = 14 \text{ for } 22.5^0 \text{ pressure angle} \]
nrounded to integer value.

The equation \( (2.8) \) indicates that the minimum number of teeth on pinion permissible and it depends on the gear ratio and pressure angle.

From the practical consideration it is observed that rack gear generation and hobbing process for lower value than the one given earlier, a little undercutting takes place and the strength of the gear is not affected. Hence, corresponding minimum number of teeth are 27, 14 and 12 for 14.50, 200, and 22.50 instead of 32, 17 & 14.

### 2.7 Line of Action

In order to have smooth continuous rotation, the arc of action should be greater than the circular pitch or line of action or path of contact should be equal to or greater than base pitch.

- \( GE = (r_2 + r_1) \sin \varphi \geq p \cos \varphi \quad (2.10) \)
- From which \( p \leq (r_2 + r_1) \tan \varphi \quad (2.11) \)
- And \( z_1 + z_2 \geq \frac{2\pi}{\tan \varphi} \quad (2.12) \)
- The line of action should be at least 1.4 times the circular pitch for continuous action.
2.8 Contact ratio

Referring to the Fig. 2.13, the theoretical length of line of action of any pair of true involute gears

- \( L_a = \frac{A_1 B + A_1 B_1}{2} \)

\[
L_a = \sqrt{(r_1 + a_1)^2 - r_1^2 \cos^2 \phi} + \sqrt{(r_2 + a_2)^2 - r_2^2 \cos^2 \phi} - (r_1 + r_2) \sin \phi \tag{2.13}
\]

Contact ratio: is defined as the maximum number of teeth in contact at any time. Higher the contact ratio smoother will be gear operation.

\[
\text{Contact ratio} = \frac{L_a}{p \cos \alpha} = \frac{L_a}{\pi m \cos \alpha} \tag{2.14}
\]