16.2 Derivation of Uniform Flow Equations

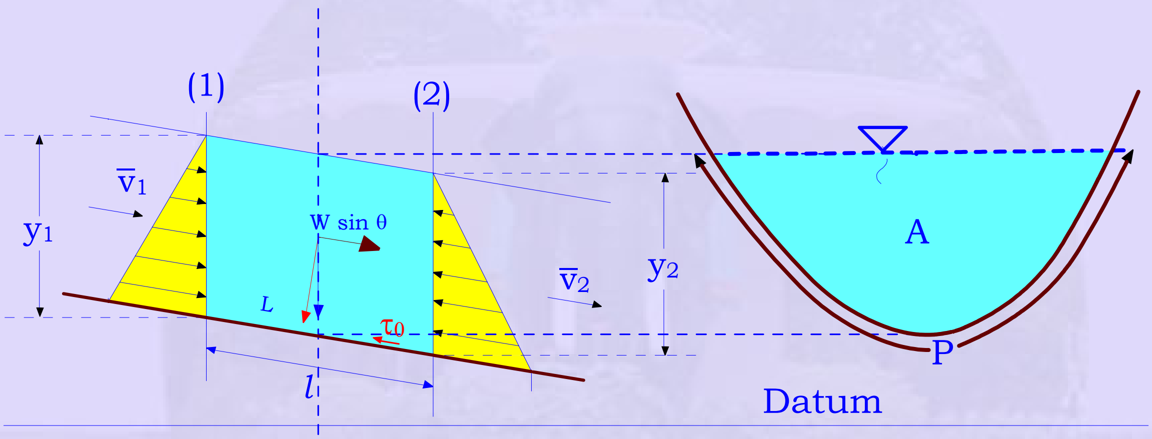
The mean velocity of a turbulent uniform open channel flow is obtained using the following concept.

Gravitational force = Shear force

The uniform flow equations are in the following format $\overline{V} = CR^{x} S^{y}$ in which x and y are components, and vary depending on uniform formula.

$$\overline{V} = \frac{1}{A}\int_0^y\int_0^b v \, dx \, dy$$

Momentum Equation:



$$\gamma \frac{Q}{g}\left(\beta_2 \overline{V_2} - \beta_1 \overline{V_1}\right) = P_1 \text{-} P_2 + W \sin\theta \text{ - } P_f$$

If $\overline{V_2} = \overline{V_1}$, $\beta_1 = \beta_2$, $P_1 = P_2$ then

$$W \sin\theta = P_f \qquad (1)$$

$P_f =$ shear force acting on boundary $=$ Shear stress * Area

$= \tau_o$ * Area

$= \tau_o\ PL$

P is the wetted perimeter, $\sin\theta = S_o$

Weight $W = \rho g\ AL$

$W \sin\theta = \rho g\ AL \sin\theta$

Substituting in equation (1)

$$\rho g\ AL\ S_o = \tau_o\ PL$$

$$\tau_o = \frac{\rho g\ AL\ S_o}{PL} = \gamma R S_o \qquad (2)$$

Note $\upsilon_* = \sqrt{\frac{\tau_o}{\rho}} = \sqrt{gRS_o}$ Critical shear velocity

$$\text{But } \tau_o = c_f \rho \frac{\overline{V}^2}{2} \qquad (3)$$

$$\gamma R S_o = c_f\ \rho \frac{\overline{V}^2}{2}$$

$$\text{or } \overline{V} = \left[\frac{2\gamma}{c_f \rho} R S_o\right]^{1/2}$$

$$\overline{V} = \sqrt{\frac{2g}{c_f}} \sqrt{RS_o}$$

If $\sqrt{\frac{2g}{c_f}} = C$ then

$$\overline{V} = C\sqrt{RS_o}.$$

This is known as Chezy equation. The coefficient C is either estimated or determined experimentally. C has dimension of $\left[L^{1/2} T^{-1/2}\right]$

2. Consider Darcy Weisbach equation for loss in pipe due to friction

$$h_f = f\frac{L}{d_o}\frac{V^2}{2g}$$

$$\therefore\ \overline{V}^2 = \frac{1}{f}\,\frac{h_f}{L}\,2g\,d_o, \qquad \left[\frac{h_f}{L} = S_o = S_f\right]$$

$$\overline{V}^2 = \frac{1}{f}\,4R*2g*S_f, \qquad \left[\frac{R}{P} = \frac{\frac{\pi d_0^2}{4}}{\pi D} = \frac{d_0}{4}\right]$$

$$\overline{V} = \sqrt{\frac{8gRS_f}{f}}$$

Comparing with Chezy equation:

$$C = \sqrt{\frac{8g}{f}}$$

$$\frac{C}{\sqrt{8g}} = \frac{1}{\sqrt{f}}$$

Manning formula is an emprical relation based on field observations and is given by

$$\overline{V} = \frac{1}{n}R^{2/3}S_o^{1/2}$$

in which $\overline{V}$ in m/s, R in m. Thus 'n' has dimensions of $\left[L^{-\frac{1}{3}}T\right]$

$\left[\text{If } R=15\text{ cm},\ n = 0.015,\ S_o = 0.0004,\ \text{then } V = 0.376\text{ m/s}\right]$

The hydraulic engineers use the n or C without bothering about dimension even though it is very important. The treatment here is only for channels with plane bed.