5.4 Strength of compression members in practice

The highly idealized straight form assumed for the struts considered so far cannot be achieved in practice. Members are never perfectly straight and they can never be loaded exactly at the centroid of the cross section. Deviations from the ideal elastic plastic behaviour defined by Fig. 5 are encountered due to strain hardening at high strains and the absence of clearly defined yield point in some steel. Moreover, residual stresses locked-in during the process of rolling also provide an added complexity.

Thus the three components, which contribute to a reduction in the actual strength of columns (compared with the predictions from the “ideal” column curve) are

(i) Initial imperfection or initial bow.

(ii) Eccentricity of application of loads.

(iii) Residual stresses locked into the cross section.

5.4.1 The effect of initial out-of-straightness

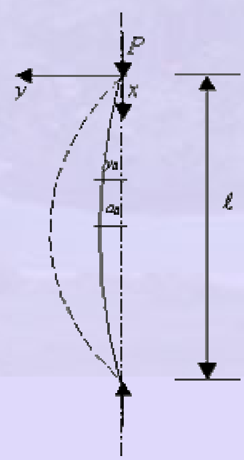

Fig 5.7 Pin -ended strut with initial imperfection

A pin-ended strut having an initial imperfection and acted upon by a gradually increasing axial load is shown in Fig 5.7. As soon as the load is applied, the member experiences a bending moment at every cross section, which in turn causes a bending deformation. For simplicity of calculations, it is usual to assume the initial shape of the column defined by

$$y_0 = a_0 \sin \frac{\pi x}{l} \qquad \textbf{(5.8)}$$

where a_o is the maximum imperfection at the centre, where x = l / 2. Other initial shapes are, of course, possible, but the half sine-wave assumed above corresponding to the lowest mode shape, represents the greatest influence on the actual behaviour, and hence is adequate.

Provided the material remains elastic, it is possible to show that the applied force, P, enhances the initial deflection at every point along the length of the column by a multiplier factor, given by

$$MF = \frac{1}{1 - \left(\frac{P}{P_{cr}}\right)} \qquad \textbf{(5.9)}$$

The deflection will tend to infinity, as P is tends to P_{cr} as shown by curve-A, in Fig. 5.8. However the column will fail at a lower load P_f when the deflection becomes large enough. The corresponding stress is denoted as f_f

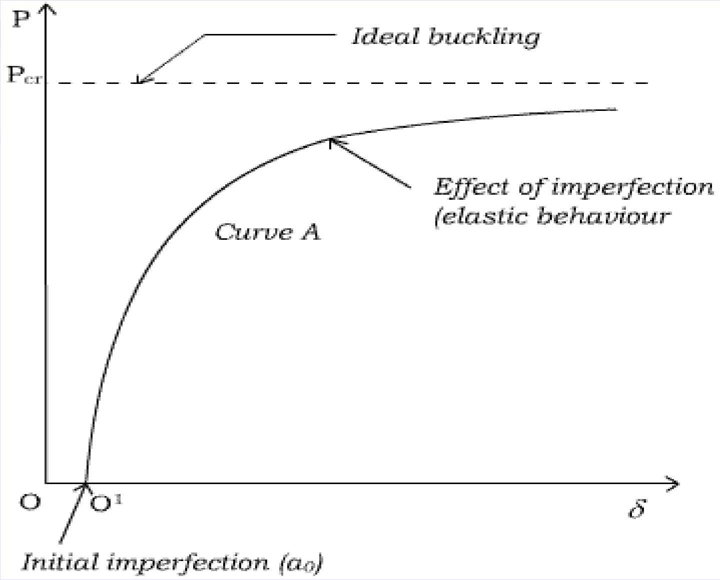

Fig 5.8 Theoretical and actual load deflection response of a strut with initial imperfection

If a large number of imperfect columns are tested to failure, and the data points representing the values of the mean stress at failure plotted against the slenderness (λ) values, the resulting lower bound curve would be similar to the curve shown in Fig. 5.9.

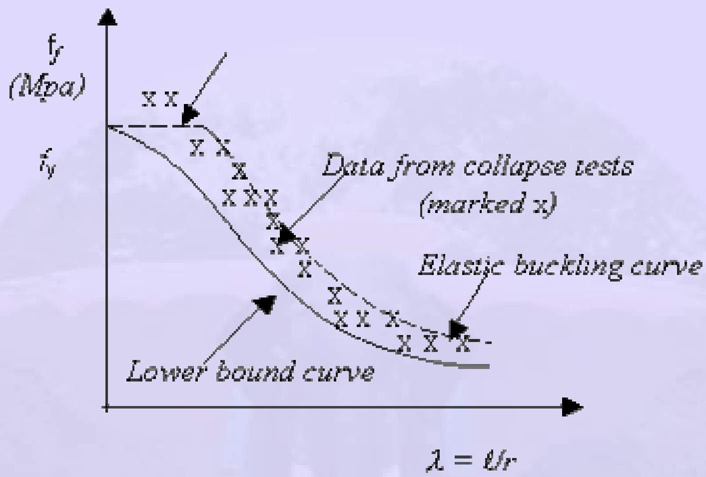

Fig 5.9 Strength curves for strut with initial imperfection

For very stocky members, the initial out of straightness – which is more of a function of length than of cross sectional dimensions – has a very negligible effect and the failure is at plastic squash load. For a very slender member, the lower bound curve is close to the elastic critical stress (f_{cr}) curve. At intermediate values of slenderness the effect of initial out of straightness is very marked and the lower bound curve is significantly below the f_y line and f_{cr} line.

5.4.2 The effect of eccentricity of applied loading

As has already been pointed out, it is impossible to ensure that the load is applied at the exact centroid of the column. Fig. 5.10 shows a straight column with a small eccentricity (e) in the applied loading. The applied load (P) induces a bending moment (P.e) at every cross section. This would cause the column to deflect laterally, in a manner similar to the initially deformed member discussed previously. Once again the greatest compressive stress will occur at the concave face of the column at a section midway along its length. The load-deflection response for purely elastic and elastic-plastic behaviour is similar to those described in Fig. 5.8 except that the deflection is zero at zero load.

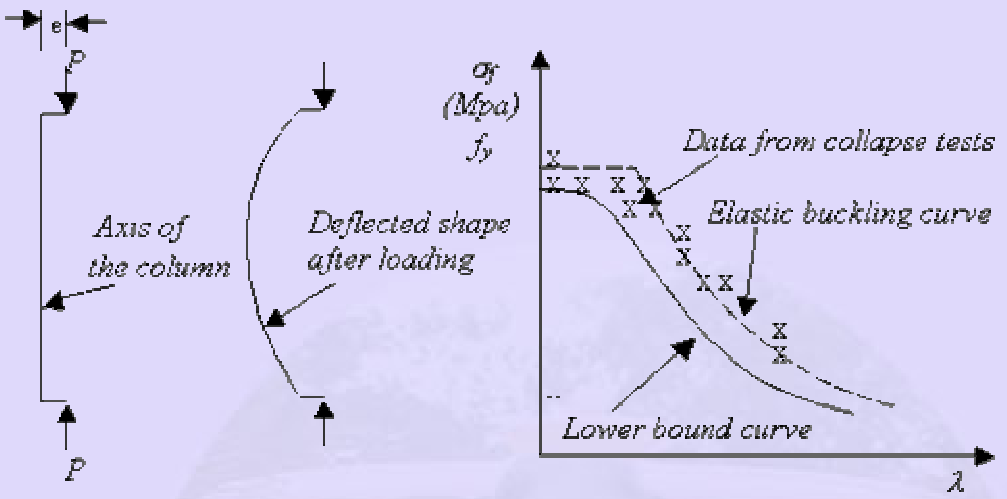

Fig 5.10 Strength curves for *eccentrically loaded columns*

The form of the lower bound strength curve obtained by allowing for eccentricity is shown in Fig. 5.10. The only difference between this curve and that given in Fig. 5.9 is that the load carrying capacity is reduced (for stocky members) even for low values of λ .

5.4.3 The effect of residual stress

As a consequence of the differential heating and cooling in the rolling and forming processes, there will always be inherent residual stresses. A simple explanation for this phenomenon follows. Consider a billet during the rolling process when it is shaped into an I section. As the hot billet shown in Fig. 5.11(a) is passed successively through a series of rollers, the shapes shown in 5.11(b), (*c*) and (d) are gradually obtained. The outstands (b-b) cool off earlier, before the thicker inner elements (a-a) cool down.

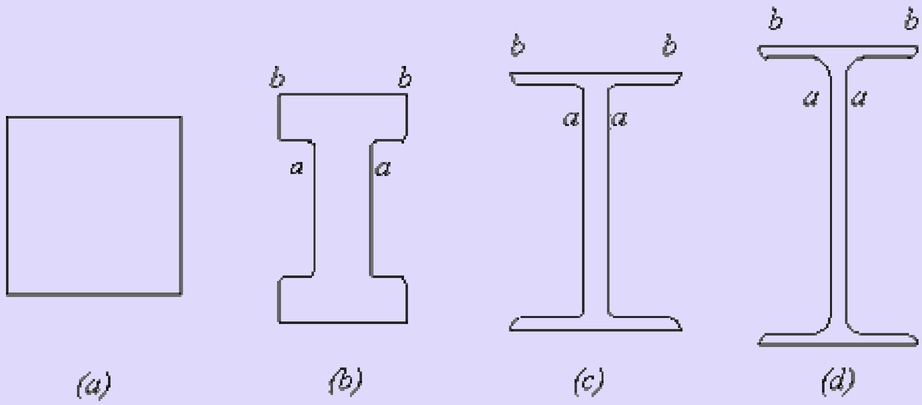

Fig 5.11 Various stages of rolling a steel grider

As one part of the cross section (b-b) cools off, it tends to shrink first but continues to remain an integral part of the rest of the cross section. Eventually the thicker element (a) also cools off and shrinks. As these elements remain composite with the edge elements, the differential shrinkage induces compression at the outer edges (b). But as the cross section is in equilibrium – these stresses have to be balanced by tensile stresses at inner location (*a*). These stress called residual stresses, can sometimes be very high and reach upto yield stress.

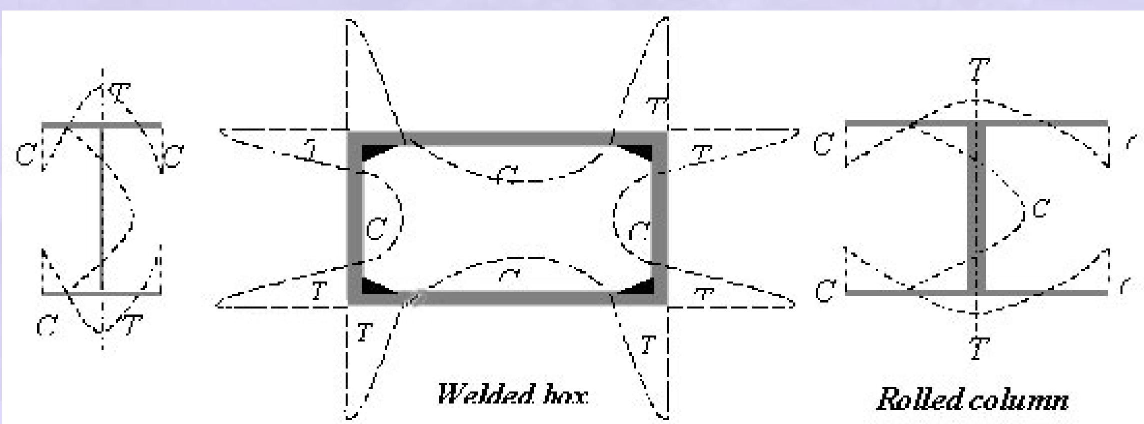

Fig. 5.12 Distribution of residual stresses

Consider a short compression member (called a “stub column”, having a residual stress distribution as shown in Fig. 5.12. When this cross section is subjected to an applied uniform compressive stress (f_a) the stress distribution across the cross section can be obtained by superposing the applied stress over the residual stress f_r, provided the total stress nowhere reaches yield, the section continues to deform elastically. Under incremental loading, the flange tips will yield first when $[(f_a + f_r) = f_y]$. Under further loading, yielding will spread inwards and eventually the web will also yield. When $f_a = f_y$, the entire section will have yielded and the column will get squashed.

Only in a very stocky column (i.e. one with a very low slenderness) the residual stress causes premature yielding in the manner just described. The mean stress at failure will be f_y, i.e. failure load is not affected by the residual stress. A very slender strut will fail by buckling, i.e. $f_{cr} << f_y$. For struts having intermediate slenderness, the premature yielding at the tips reduces the effective bending stiffness of the column; in this case, the column will buckle elastically at a load below the elastic critical load and the plastic squash load. The column strength curve will thus be as shown in Fig. 5.13.

Notice the difference between the buckling strength and the plastic squash load is most pronounced when

$\lambda = l / r = \pi (E / f_y)^{1/2}$

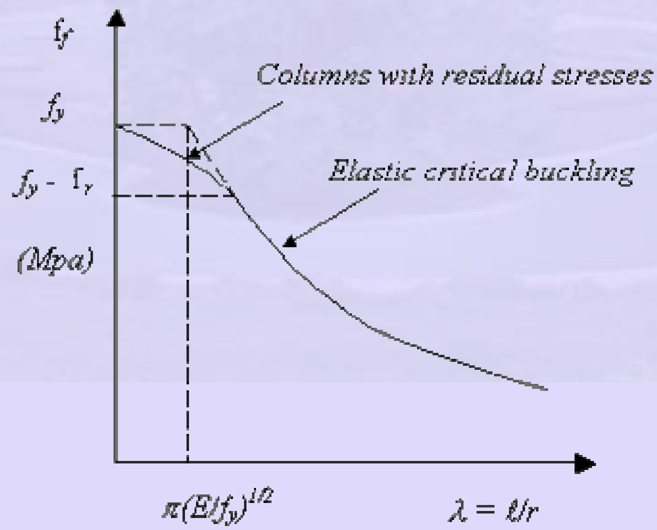

Fig 5.13 Strength curve for columns having residual stress

5.4.4 The effect of strain-hardening and the absence of clearly defined yield point

If the material of the column shows strain hardering after an yield platean, the onset of first yield will not be affected, but the collapse load may be increased. Designers tend to ignore the effect of strain hardening which in fact provides an additional margin of safety.

High strength steels generally have stress-strain curves without a clear yield point. At stresses above the limit of proportionality (f_p), the material behaviour is non linear and on unloading and reloading the material is linear-elastic. Most high strength structural steels have an ultimate stress beyond which the curve becomes more or less horizontal. Some steels do not have a plastic plateau and exhibit strain-hardening throughout the inelastic range. In such cases, the yield stress is generally taken as the 0.2% proof stress, for purposes of computation.

5.4.5 The effect of all features taken together

In practice, a loaded column may experience most, if not all, of the effects listed above i.e. out of straightness, eccentricity of loading, residual stresses and lack of clearly defined yield point and strain hardening occurring simultaneously. Only strain hardening tends to raise the column strengths, particularly at low slenderness values. All other effects lower the column strength values for all or part of the slenderness ratio range.

When all the effects are put together, the resulting column strength curve is generally of the form shown in Fig. 5.14. The beneficial effect of strain hardening at low slenderness values is generally more than adequate to provide compensation for any loss of strength due to small, accidental eccentricities in loading. Although the column strength can exceed the value obtained from the yield strength (f_y), for purposes of structural design, the column strength curve is generally considered as having a cut off at f_y, to avoid large plastic compressive deformation. Since it is impossible to quantify the variations in geometric imperfections, accidental eccentricity, residual stresses and material properties, it is impossible to calculate with certainty, the greatest reduction in strength they might produce in practice. Thus for design purposes, it may be impossible to draw a true lower bound column strength curve. A commonly employed method is to construct a curve on the basis of specified survival probability. (For example, over 98% of the columns to which the column curve relates, can be expected - on a statistical basis – to survive at applied loads equal to those given by the curve). All design codes provide column curves based on this philosophy. Thus a lower band curve (Fig 5.14) or a family of such curves is used in design.

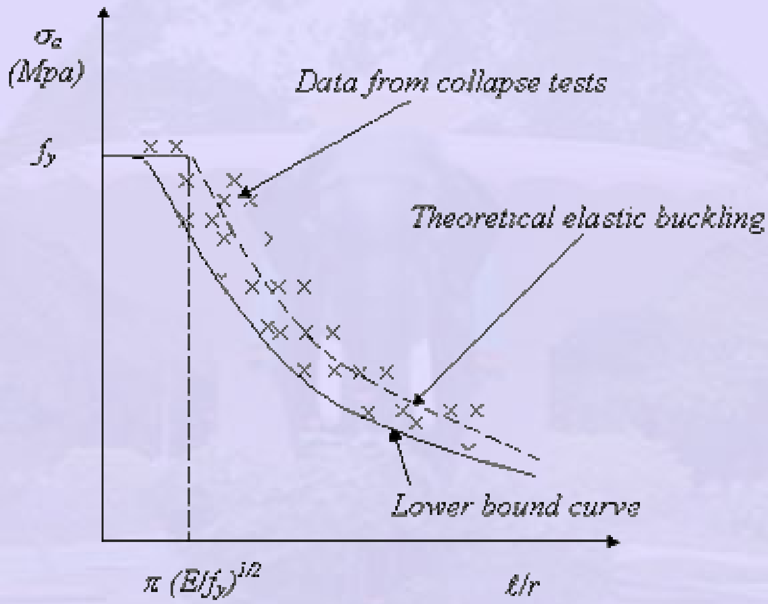

Fig. 5.14 Column strength curves for struts used in practice