Module 5 : MODERN PHYSICS
Lecture 25 : Compton Effect

Objectives

In this course you will learn the following

- Scattering of radiation from an electron (Compton effect).
- Compton effect provides a direct confirmation of particle nature of radiation.
- Why photoelectric effect cannot be exhited by a free electron.

Compton Effect

Photoelectric effect provides evidence that energy is quantized. In order to establish the particle nature of radiation, it is necessary that photons must carry momentum. In 1922, Arthur Compton studied the scattering of x-rays of known frequency from graphite and looked at the recoil electrons and the scattered x-rays.

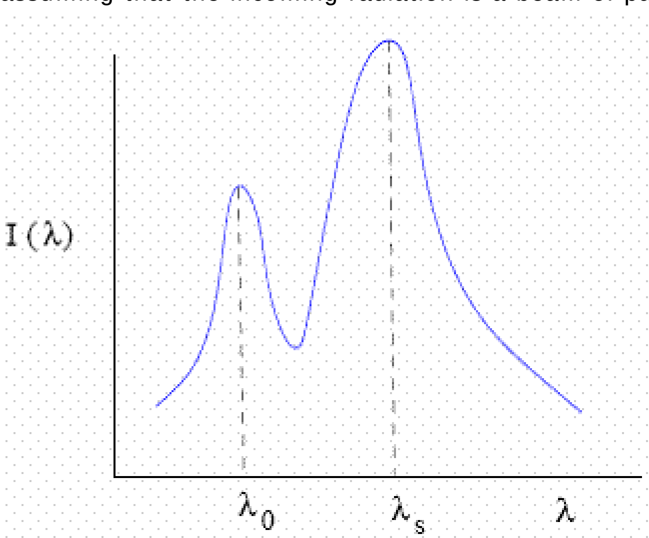
According to wave theory, when an electromagnetic wave of frequency ν_0 is incident on an atom, it would cause electrons to oscillate. The electrons would absorb energy from the wave and re-radiate electromagnetic wave of a frequency $\nu_s < \nu_0$. The frequency of scattered radiation would depend on the amount of energy absorbed from the wave, i.e. on the intensity of incident radiation and the duration of the exposure of electrons to the radiation and not on the frequency of the incident radiation.

Compton found that the wavelength of the scattered radiation does not depend on the intensity of incident radiation but it depends on the angle of scattering and the wavelength of the incident beam. The wavelength of the radiation scattered at an angle θ is given by

$$\lambda_s = \lambda_0 + \frac{h}{m_0 c}(1 - \cos\theta)$$

.where m_0 is the rest mass of the electron. The constant $h/m_0 c$ is known as the **Compton wavelength of the electron** and it has a value 0.0024 nm.

The spectrum of radiation at an angle θ consists of two peaks, one at λ_0 and the other at λ_s. Compton effect can be explained by assuming that the incoming radiation is a beam of particles with



- Energy $E = h\nu_0$
- Momentum $p = h\nu_0/c$

In arriving at the last relationship, we use the energy - momentum relation of the *special theory of relativity* , according to which,

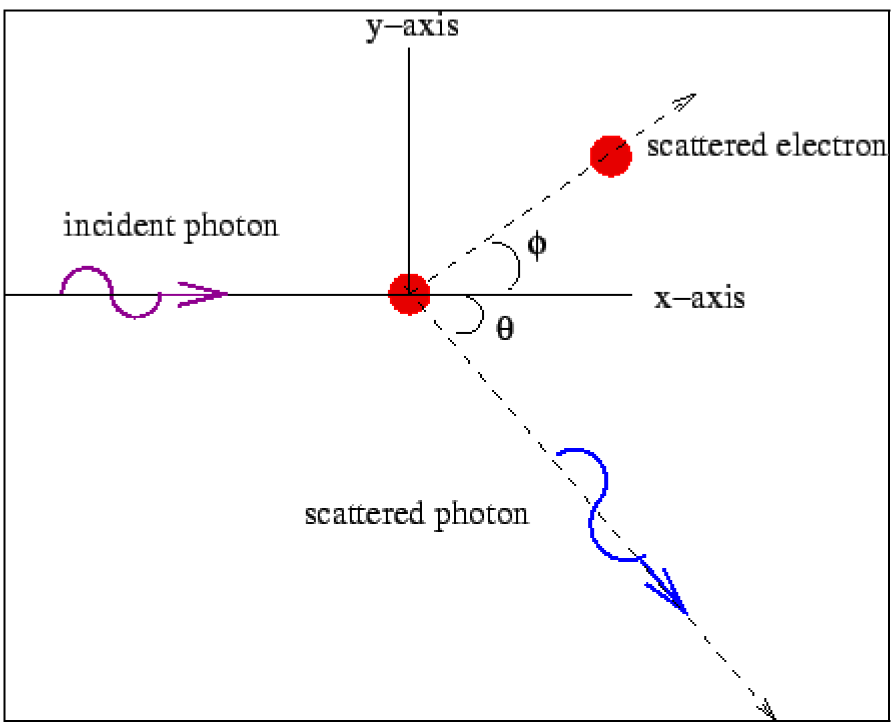
$$E^2 = m^2c^4 + p^2c^2$$

where m is the rest mass of a particle. Since photons are massless ($m = 0$), we get $E = pc$.

Compton's observation is consistent with what we expect if photons, considered as particles, collide with electrons in an **elastic collision**.

Derivation of Compton's Formula

Consider a photon of energy $h\nu_0$ and momentum $p_i = h\nu_0/c$ colliding *elastically* with an electron at rest. Let the direction of incoming photon be along the x-axis. After scattering, the photon moves along a direction making an angle θ with the x-axis while the scattered electron moves making an angle ϕ. Let the magnitude of the momentum of the scattered electron be p_e while that of the scattered photon be p_f.



[See the animation](#)

Conservation of Momentum

x-direction :

$$p_i = p_f \cos\theta + p_e \cos\phi \quad (1)$$

y-direction :

$$0 = -p_f \sin\theta + p_e \sin\phi \quad (2)$$

From Eqns. **(1)** and **(2)**, we get

$$p_e^2 = p_e^2(\cos^2\phi + \sin^2\phi) = (p_i - p_f\cos\theta)^2 + p_f^2\sin^2\theta$$

$$= p_i^2 + p_f^2 - 2p_ip_f\cos\theta \qquad (3)$$

Conservation of Energy : (relativistic effect)

If the rest mass of the electron is taken to be m_0, the initial energy is m_0c^2 and the final energy is $\sqrt{m_0^2c^4 + p_e^2c^2}$. Thus

$$h\nu_0 + m_0c^2 = h\nu + \sqrt{m_0^2c^4 + p_e^2c^2} \quad (4)$$

From Eqn. (4), we get, on squaring,

$$m_0^2c^4 + p_e^2c^2 = (h\nu_0 - h\nu + m_0c^2)^2$$

$$= (h\nu_0 - h\nu)^2 + m_0^2c^4 + 2m_0c^2(h\nu_0 - h\nu)$$

Thus,

$$p_e^2c^2 = (h\nu_0 - h\nu)^2 + 2m_0c^2(h\nu_0 - h\nu)$$

On substituting expression (3) for p_e in the above equation, we get

$$p_i^2c^2 + p_f^2c^2 - 2p_ip_f\cos\theta c^2 = (h\nu_0 - h\nu)^2 + 2m_0c^2(h\nu_0 - h\nu)$$

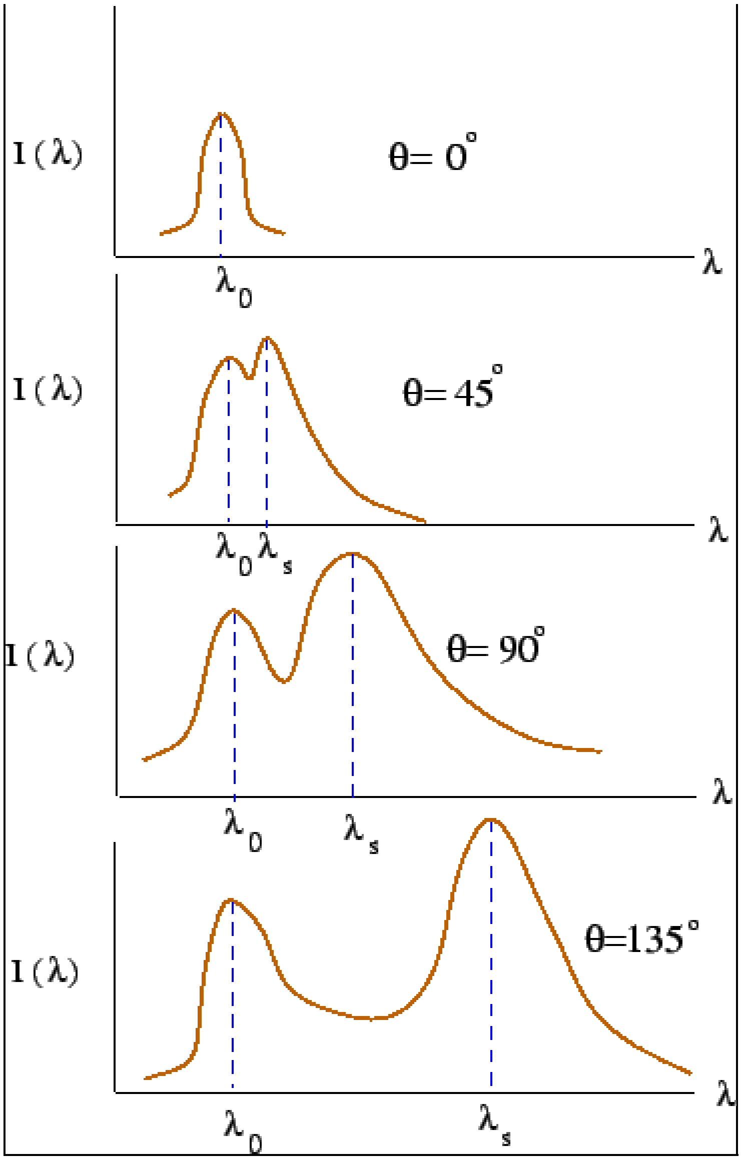
Recalling $p_i = h\nu_0/c$ and $p_f = h\nu/c$ and on simplification, we get

$$h\nu\nu_0(1 - \cos\theta) = m_0c^2(\nu_0 - \nu)$$

Using $\lambda_s = c/\nu$, we get Compton's formula

$$\boxed{\lambda_s - \lambda_0 = \frac{h}{m_0c}(1 - \cos\theta)}$$

$h/m_0c \equiv \lambda_c$ is known as the **Compton Wavelength** of an electron.



Exercise 1

Show that the angle ϕ by which the electron is scattered is related to the scattering angle θ of the photon by

$$\cot\frac{\theta}{2} = \left(1 + \frac{\lambda_c}{\lambda_0}\right)\tan\phi$$

Exercise 2

Is Compton effect easier to observe with I.R., visible, UV or X-rays ? In a Compton scattering experiment the scattered electron moves in the same direction as that of the incident photon. In which direction does the photon scatter ?

(Answer : X-rays, 180°.)

Exercise 3

A 200 MeV photon strikes a stationary proton (rest mass 931 MeV) and is back scattered. Find the kinetic energy of the proton after the scattering.

(Ans. 60 MeV)

Reason for the unshifted peak in the spectrum

When a photon strikes an atom (say carbon atom in a graphite crystal), it may scatter from a loosely bound electron, which is essentially free. In this case there is a measurable shift in the wavelength of the scattered photon. It is also likely that the photon scatters from an electron that is tightly bound to an atom. In such a case, the mass appearing in Compton's formula must be replaced by the mass of the carbon atom itself, which is approximately 20,000 times heavier than an electron. The maximum wavelength shift of the photon for scattering from a free electron is twice the Compton wavelength of an electron, i.e. 4.8×10^{-3} nm. In case of scattering from the carbon atom, the maximum wavelength shift is approximately 2×10^{-7} nm, which is very small. Thus we find an intensity maximum at an wavelength which is essentially equal to that of the incident wavelength.

A free electron cannot absorb a photon and increase its energy as doing so would violate energy-momentum conservation.

Consider a free electron at rest which absorbs a photon of energy $h\nu$ (and momentum $h\nu/c$). The final energy of the electron would be $h\nu + m_0c^2$. According to relativistic principle, if the momentum of the electron is p, the total energy is given by $\sqrt{p^2c^2 + m_0^2c^4}$. When the electron absorbs the incident photon, the momentum of the photon would be transferred to the electron. Since the electron was initially at rest (i.e. with zero momentum), its final momentum is $p = h\nu/c$. Thus we have

$$pc + m_0c^2 = \sqrt{p^2c^2 + m_0^2c^4}$$

which simplifies to $2m_0pc^3 = 0$, which is not possible.

The reason why an electron bound to an atom can absorb a photon (as in Compton effect) is that the electron can share some of the resulting momentum with the ion which has a much larger mass

Example 11

A photon of wavelength 6000 nm collides with an electron at rest. After scattering, the wavelength of the scattered photon is found to change by exactly one Compton wavelength. Calculate (i) the angle by which the photon is scattered, (ii) the angle by which the electron is scattered and (iii) the change in the energy of the electron due to scattering.

Solution :

- Since the change in wavelength is one Compton wavelength, $(1 - \cos\theta) = 1$, i.e. $\theta = 90^\circ$. Thus the photon is scattered at right angles to the incident direction.

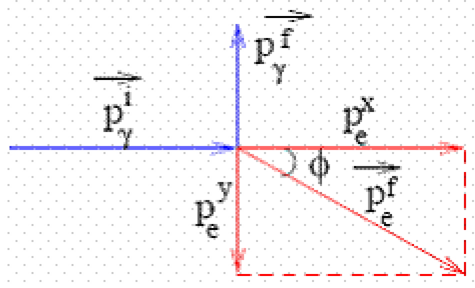
- Initial momentum of the photon is

$$\frac{h\nu_0}{c} = \frac{h}{\lambda_0} = \frac{6.63 \times 10^{-34}}{6 \times 10^{-12}} = 1.105 \times 10^{-22} \text{ kgm/s along } \hat{\imath}$$

The final momentum of the photon is

$$\frac{h\nu}{c} = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{8.4 \times 10^{-12}} = 7.9 \times 10^{-23} \text{ kgm/s along } \hat{\jmath}$$

Thus the final momentum of the electron is $1.105 \times 10^{-22}\hat{\imath} - 7.9 \times 10^{-23}\hat{\jmath}$. The angle that the final direction of electron makes with the x-axis is $\phi = \tan^{-1}(-7.9/11.05) = 35.6°$.



The change in the energy of the electron is negative of the change in the energy of the photon which is $(hc/\lambda_0 - hc/\lambda) = 9.47 \times 10^{-15}\ J = 59.2$ keV.

Exercise 4

A photon of wavelength 6000 nm scatters from an electron at rest. The electron recoils with an energy of 60 keV. Calculate the energy of the scattered photon and the angle through it is scattered.

(Ans. 147 keV, $91.3°$)

Example 12

A photon of frequency ν_0 scatters from an electron at rest and moves in a direction making an angle of 60 ^{o} with the incident direction. If the frequency of scattered photon is half that of incident photon, calculate the frequency of the incident photon.

Solution

Since frequency is halved, wavelength is doubled. Thus change in wavelength $\lambda - \lambda_0$ is equal to λ_0.

Using Compton's formula

$$\lambda_0 = 2.4 \times 10^{-12}(1 - \cos 60°) = 1.2 \times 10^{-12}\ \text{m}$$

The corresponding frequency is $\nu_0 = c/\lambda_0 = 2.5 \times 10^{20}$ Hz., which is a gamma ray photon.

Exercise 5

A photon scatters from a proton, initially at rest. After the collision, the proton is found to scatter at an angle of 30 ^{o} with the original direction of the incident photon with a kinetic energy of 100 MeV. Find **(i) the initial energy of the photon** and **(ii) the angle through it is scattered**

[**Hints :** The rest mass of proton is 938 MeV. Total energy of a relativistic particle is $\sqrt{p^2c^2 + m^2c^4}$.

Use these to determine momentum of the scattered proton. Use momentum and energy conservation. Answer (i) 329 MeV (ii) $104°$.]

Exercise 6

Find the smallest energy that a photon can have in order to be able to transfer half of its energy to an electron at rest (rest mass of an electron is 0.5 Mev)

(Ans. 0.256 Mev)

Exercise 7

A photon has the same wavelength as the Compton wavelength of an electron. What is the energy of the photon in eV ?

(Ans. 0.51 MeV)

Recap

In this lecture you will learn the following

- Compton effect is a process in which x-rays collide with electrons and are scattered.
- Unlike the prediction of classical wave theory, the wavelength of the scattered radiation does not depend on the intensity of radiation but depends on the scattering angle and the wavelength of the incident beam.
- Compton effect can be explained by considering radiation to be of particulate nature.
- Compton effect is best exhibited with short wavelength radiation like x-rays.
- A free electron cannot absorb a photon because it is not possible to simultaneously satisfy energy-momentum conservation.