FIBER OPTICS

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Lecture: 38

Introduction to

Cross Phase Modulation and Four-Wave Mixing
In our earlier discussions, we studied various outcomes of the $\chi^{(3)}$ non-linearity in an optical fiber. However, these discussions were limited to single channel propagation inside an optical fiber. In the following sections too, we shall study the same $\chi^{(3)}$ non-linearity, but in a multi-channel environment and investigate the types and the behaviours of the interactions that take place between different channels on the link.

In particular, we shall discuss two phenomena - Cross-Phase Modulation and Four-Wave Mixing. Both of these phenomena are the result of the presence of $\chi^{(3)}$ non-linearity term in the polarization of the material.

**CROSS-PHASE MODULATION**

Let us assume that the electric field has two frequency components as shown below:

$$E = E_1 e^{j\omega_1 t} + E_2 e^{j\omega_2 t} \quad (38.1)$$

If we substitute this into the expression for the polarization, we obtain the following expression for the non-linear polarization $P_{NL}$:

$$P_{NL} = P_{NL}(\omega_1) e^{j\omega_1 t} + P_{NL}(\omega_2) e^{j\omega_2 t} + P_{NL}(2\omega_1 - \omega_2) e^{j(2\omega_1 - \omega_2) t} + \ldots \quad (38.2)$$

In the above expression,

$$P_{NL}(\omega_1) = \chi^{(3)} \{ |E_1|^2 + 2|E_2|^2 \} E_1 \quad (38.3)$$

$$P_{NL}(\omega_2) = \chi^{(3)} \{ 2|E_1|^2 + |E_2|^2 \} E_2 \quad (38.4)$$

Due to the above non-linear terms, the change in the refractive index due to the $j^{th}$ frequency is given by:

$$\Delta n_j \approx n_2 \left\{ |E_j|^2 + 2|E_{3-j}|^2 \right\} \quad (38.5)$$

The parameter $j$, in this case, can take values from $\{1,2\}$. In the presence of a single channel (frequency), the second term in the above equation vanishes and the resulting expression is the Kerr non-linearity which we have already analysed earlier. The term $n_2$ is the non-linearity coefficient as discussed earlier. Thus, when there are two signals present simultaneously, the change in phase is not only dependent on the original signal but there would be contribution from the co-existing signal too. However, the most important thing to note is that, the non-linear effect of the co-existing signal is twice as strong compared to the signal itself. This is suggested by the presence of the enhancement factor of 2 in the above expression for change in refractive index.

If we recall our earlier discussions, the above change in refractive index led to a phenomenon known as the self-phase-modulation (SPM) when only one signal was
present. Since the above situation consists of two signals and the change in refractive index is related to the contribution from both the signals, the phase modulation produced, as a result, is known as the cross-phase-modulation (XPM). The phase change in the above case is given by:

\[ \phi_{NL} = \frac{\omega_j}{c} \Delta n_j z ; \quad j = 1, 2 \]  

\[(38.6)\]

In WDM type of systems where there is a large number of optical channels present simultaneously, if the power in each channel is sufficient enough, then there would be phase change brought about in one channel due to the presence of the other channels. Since the contribution of the co-existing channels is twice stronger than the self-channel, the cross-phase modulation would dominate over the self-phase modulation. In order to formulate an expression for XPM we need to solve the non-linear Schrodinger (NLS) equation as we did in case of SPM. Let us assume that the field distribution of the \( j \)th channel inside the optical fiber is given by:

\[ E_j = F_j(x, y)A_j(z)e^{-j\beta_{0j}z} \]  

\[(38.7)\]

If we follow a similar analysis to that in case of SPM, we obtain two distinct expressions describing the behaviour of the transverse field function \( F_j(x, y) \) and the envelope function \( A_j(z) \) with distance \( z \). The non-linear Schrodinger equation for the above situation can be written as:

\[ \frac{\partial A_j}{\partial z} + \beta_{1j} \frac{\partial A_j}{\partial t} - \frac{i}{2} \beta_{2j} \frac{\partial^2 A_j}{\partial t^2} + \frac{\alpha_j}{2} A_j = -j \frac{n_2 \omega_j}{c} \left\{ f_{jj} |A_j|^2 + 2f_{jk} |A_k|^2 \right\} A_j \]  

\[(38.8)\]

As seen earlier, the first term on the LHS denotes the rate of change of the envelope with distance, the second term determines the group velocity, the third term indicates the group velocity dispersion and the fourth term determines the attenuation on the optical fiber. The RHS of the NLS determines the non-linearity on the optical fiber. The coefficients \( f_{jj} \) and \( f_{jk} \) are known as the overlap integrals which determine the amount of overlap between the different fields corresponding to the different channels that co-exist in the optical fiber. The overlap integral is defined as:

\[ f_{jk} = \frac{\iint |F_j(x, y)|^4 |F_k(x, y)|^2 dxdy}{\iint |F_j(x, y)|^2 dxdy \iint |F_k(x, y)|^2 dxdy} \]  

\[(38.9)\]

For \( j=k \), the above overlap integral reduces to the reciprocal of the effective area \( A_{eff} \). That is:

\[ f_{jj} = \frac{\iint |F_j(x, y)|^4 dxdy}{\left( \iint |F_j(x, y)|^2 dxdy \right)^2} = \frac{1}{A_{eff}} \]  

\[(38.10)\]

The physical interpretation of the overlap integral is that, it indicates how effectively different fields interact in a multi-channel environment. The effectiveness of this interaction is based on the amount of overlap between the fields. If the fields are widely separated in frequency, the overlap is very less resulting in very low or even no interaction between the fields. Hence, the NLS for channel 1 can be written as:
\[
\frac{\partial A_1}{\partial z} + \frac{1}{v_{g1}} \frac{\partial A_1}{\partial t} - \frac{j\beta_{21}}{2} \frac{\partial^2 A_1}{\partial t^2} + \frac{\alpha_1}{2} A_1 = -j\gamma_1 \left( |A_1|^2 + 2|A_2|^2 \right) A_1 \tag{38.11}
\]

Where,
\[
\gamma_1 = \frac{n_2 \omega_1}{c A_{\text{eff}}} \tag{38.12}
\]

The term \(v_{g1}\) is the group velocity of the signal on channel 1. If we compare equation (38.11) with that in case of SPM, we find the introduction of an additional non-linear term on the RHS. Depending on the individual values of \(A_1\) and \(A_2\) one of them dominates the non-linearity term. Replacing \(j=1\) by \(j=2\) in equation (38.8) we obtain an identical equation for channel 2.

The above situation can, now, be visualised as follows: let us assume the simultaneous propagation of two optical pulses through an optical fiber with different speeds. Due to the difference in speeds, the pulse with the higher speed slides ahead of the pulse with the lower speed. During sliding, the overlap between the two pulses takes place and we witness the phenomenon of XPM. But once the faster pulse ‘walks off’ (fully overtakes) the slower pulse, the overlap between the pulses becomes negligible and the XPM phenomenon vanishes. The “walk-off” phenomena can be inferred to be related to the difference in two group velocities of the two pulses. If the group velocities are very close to each other, the overlap between the pulses would be higher and the XPM phenomenon would be significantly large whereas for widely separated group velocities the overlap between the pulses would be very short and the XPM would not be too significant. So, in this case too, there would be a situation almost similar to self-phase modulation, the only difference being that the phase modulation would be affected by the presence of the other signals too. If we want to weaken the XPM on the optical fiber, we must reduce the overlap interval between the two pulses; we must choose a fiber which has a large group velocity variation as a function of frequency. In other words, if we consider an optical fiber which has a large dispersion, the walk-off time would be very small and as a result the non-linear interaction between the two pulses would be very small. On the other hand, if we want the XPM to be significantly large, we must choose a dispersion flattened kind of optical fiber in which all the frequencies travel with almost equal group velocities and the non-linear interaction between the pulses is thus enhanced. In other words, a dispersion flattened fiber would show higher XPM than a dispersive optical fiber. So, to avoid the occurrence of XPM, some dispersion on the optical fiber is highly desired. Especially, in a WDM system where each channel is affected by the remaining channels, the use of a dispersive optical fiber is desirable to reduce the strength of the non-linear interactions between the different channels.

The reader may now wonder that the initial discussions on optical fibers suggested the reduction of dispersion on optical fibers to reduce pulse-broadening and inferred that fibers with low dispersion are better whereas the above discussion suggests that some amount of dispersion is desired in an optical fiber for better performance. However, it must be noted that dispersive fibers are useful only in the presence of non-linear effects which occur when the optical power in the pulse is considerable high. At low optical intensities, the inference on reduction of dispersion still holds. XPM results in the similar type of pulse
evolutions as that in case of SPM, the only difference being that the pulse-broadening around the centre frequency may not be symmetric in case of XPM.

**FOUR-WAVE MIXING**

The four-wave mixing phenomenon in the optical domain is analogous to the inter-channel mixing or the inter-modulation products in the electronic systems. When an electronic amplifier goes into saturation, application of two distinct frequencies results in the generation of new frequencies which may be either the sum or the difference between the applied frequencies. This happens due to the non-linearity in the amplifier performance. Exactly similar situation occurs inside an optical fiber in the presence of the third order non-linear susceptibility $\chi^{(3)}$.

If we consider the non-linear polarization $P_{NL}$ due to the third order susceptibility $\chi^{(3)}$ in the presence of three simultaneous signals $E_1$, $E_2$ and $E_3$, we can write:

$$P_{NL} = \varepsilon_0 \chi^{(3)} : E_1 E_2 E_3$$  \hspace{1cm} (38.13)

Due to inter-modulation, the three frequencies in the above expression produce a fourth frequency $\omega_4$ given by:

$$\omega_4 = \omega_1 \pm \omega_2 \pm \omega_3$$  \hspace{1cm} (38.14)

The wave number corresponding to this fourth frequency is given by:

$$K_4 = K_1 \pm K_2 \pm K_3$$  \hspace{1cm} (38.15)

So when three distinct frequencies are launched into a non-linear material, due to the non-linear interactions between the three signals, new frequencies get generated, in addition to the original frequencies, which are given by equation (38.14). The different frequency components which lie in the same band as the three original frequencies can be generated by considering the following possible combinations (using equation (38.14)):

$$(\omega_1 + \omega_2 - \omega_3), (\omega_1 + \omega_3 - \omega_2), (\omega_3 + \omega_2 - \omega_1), (2\omega_1 - \omega_2), (2\omega_1 - \omega_3), (2\omega_2 - \omega_1), (2\omega_2 - \omega_3), (2\omega_3 - \omega_1), (2\omega_3 - \omega_2)$$

The above phenomenon of generation of new frequencies through non-linear interactions between the parent signals is known as Four-Wave Mixing. This phenomenon in the electronic systems is analogous to inter-modulation. Generation of these new frequencies affects the propagation of optical signal inside the optical fiber. To understand better, let us consider a WDM system with three equally-spaced channels at $\omega_1$, $\omega_2$, $\omega_3$ as shown in the following figure:

![Figure 38.1: 3-Channel WDM system](image)
From the above figure, we can write:

\[ \omega_2 = \omega_1 + 2\Delta \omega ; \omega_3 = \omega_1 + \Delta \omega \] \hspace{1cm} (38.16)

Therefore one of the possible frequencies generated due to four-wave mixing is given by:

\[ \omega_4 = \omega_1 + \omega_2 - \omega_3 = \omega_1 + \Delta \omega = \omega_3 \] \hspace{1cm} (38.17)

Equation (38.17) shows an interesting result. We see that, when \( \omega_1 \) and \( \omega_2 \) propagate simultaneously on the optical fiber, they have a tendency to generate the frequency \( \omega_3 \) which, itself, is one of the WDM channel frequencies. That means, due to the non-linear interactions, \( \omega_1 \) and \( \omega_2 \) would couple power into the frequency \( \omega_3 \) which would lead to cross-talk between the channels. In the above situation we are dealing only with a 3-channel WDM system. In practice, there are large number of WDM channels propagating simultaneously and according to the above observation, there would be cross-talk among all the WDM channels due to non-linearity and four-wave mixing.

One obvious way of avoiding the above ‘cross-talk’ situation is to use WDM channels which are not equally spaced. So, from the viewpoint of non-linearity, equi-spaced WDM channels are undesirable as it leads to cross-talk among the channels.

Another intuitive solution can be derived from the relative group velocities between different channels. That is, if the walk-off time between two channels is considerably reduced, there would not be much interaction among the channels and the above cross-talk would be very weak. This requirement boils down to the same requirement of a dispersive optical fiber as in the preceding discussion.

Thus we see that, in case of a single channel transmission in linear optics, it was highly desirable if the dispersion on the optical fiber could be made zero. However, in view of the above non-linear effects which cannot be ignored, making dispersion zero is an undesired proposition. So in modern optical communication systems, a small residual dispersion on the optical fiber is, actually, necessary so that the non-linear effects leading to cross-talk, SPM, XPM and four-wave mixing do not build up.

On one hand where we argue that four-wave mixing is undesired, on the other hand the same phenomenon can be put to intelligent use in the construction of high throughput non-blocking type of optical networks. For a better understanding of the above proposition, let us consider a situation in which we input two frequency channels \( f_1 \) and \( f_2 \) to a four-wave mixer as shown below:

\[ f_3 = \frac{2f_1 - f_2}{2f_2 - f_1} \hspace{1cm} \text{OR} \hspace{1cm} \frac{2f_1 - f_2}{2f_2 - f_1} \]

**Figure 38.2:** Four-Wave Mixing for frequency switching
The output of the mixer may be either \(2f_1-f_2\) or \(2f_2-f_1\) which is due to the inherent non-linear interactions of the four-wave mixer. If the frequency \(f_2\) (or \(f_1\)) is so chosen that the one of the output frequencies matches with the frequency of interest, then the data riding on \(f_1\) can be made to ride on this new output frequency \(f_3\). Thus the channel frequency of the data can be switched to a new frequency with the help of the four-wave mixing phenomenon which, indeed, acts as a frequency conversion mechanism. This notion can be of great help in congested optical networks to increase the throughput of the network by switching and routing the blocked links onto other frequencies which are free. Thus a non-blocking type network can be realized.

In a similar way there are various other non-linear effects inside an optical fiber and the reader is encouraged to pursue further reading to get familiarized with these effects and their characteristics.