Lecture 2: Measurement of Quantities

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Key words: unit and dimension, fundamental units, power, pressure

Preamble

The material balance shows the weights and analysis of input and output materials and the calculated inputs and outputs of each of the important elements and compounds. This accounting serves as a check on plant data in that the various totals of inputs and output should be equal.

This lecture discusses the different ways of expression of measured quantities. It is felt that many a times the students have problems in unit and dimensions and to converting one unit to another.

Standard units of measurement

The adoption of standards has varied greatly as regards unit in different parts of the world. The following table lists some of the commonly used set of fundamental units from which all other units can be derived.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>FPS</th>
<th>Absolute units</th>
<th>SI</th>
<th>Engg. Units English</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>Pound (lb)</td>
<td>gram (g)</td>
<td>kg</td>
<td>kg</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Slug</td>
</tr>
<tr>
<td>Length</td>
<td>ft</td>
<td>cm</td>
<td>m</td>
<td>m</td>
</tr>
<tr>
<td>Time</td>
<td>sec</td>
<td>sec</td>
<td>sec</td>
<td>sec</td>
</tr>
<tr>
<td>Amount of substance</td>
<td>Lb. mole</td>
<td>g. mole</td>
<td>kg mole</td>
<td>mole</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Lb. mole</td>
</tr>
<tr>
<td>Temperature</td>
<td>°F</td>
<td>°C</td>
<td>°k(ork)</td>
<td>k</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>°F</td>
</tr>
</tbody>
</table>
FPS stands for feet, pound and second. It is a British system.

CGS stands for centimeter, gram and second.

MKS stands for meter, kilogram and second.

SI stands for the International System of Units. Extension of MKS system.

**Derived units and quantities**

Physical quantities which can be derived from other physical quantities are called derived quantities.

Derived units: The units of physical quantities which can be expressed in terms of fundamental units, for example, area, volume etc.

Let us derive

(i) **Unit of Force**

Force = mass x acceleration.

In SI unit force = \( \frac{kg}{m^2/s^2} \) = 1 Newton (N).

In CGS unit force = \( \frac{g}{cm^2/s^2} \) = 1 dyne.

In MKS unit force is expressed in Newton (N).

In FPS system force is \( \frac{Lb ft}{s^2} \) = 1 poundal.

(ii) **Energy** = \( mu^2 \)

we can substitute the units of m and u to derive the unit of energy.

In SI system \( E = kg \frac{m^2}{s^2} \) = 1 Joule.

In CGS system \( E = g \frac{cm^2}{s^2} \) = 1 erg.

1 joule = \( 10^7 \) ergs.

In FPS system \( E = Lb \ \text{ft}^2/s^2 \) = ft. poundal.

1 British thermal unit (Btu) = 778 ft. Lbf = 1054.2 Joule = 252 cal

1 kcal = 1000 cal. = 3.968 Btu

(iii) **Power** (watts or W).
1W = $\frac{1}{s^2}$ kg m$^2$/s = 1J/s.

1 horse power = 550 ft. lb/s = 746 W

1 kW = 1000 W = 3414 Btu/hr.

1 kW - hr = 860 kcal = 3414 Btu.

Composition of a mixture

It is expressed in terms of mole fraction or mass fraction. If mixture contains $n$, moles of component 1, $n_2$ moles of component 2, $n_3$ moles of component 3; then mole fraction of $i^{th}$ component is

$$x_i = \frac{n_i}{\sum n_i}$$

Similarly mass fraction of $i^{th}$ component in the mixture is given by

$$y_i = \frac{m_i}{\sum m_i}$$

Derivation of unit of universal constant ($R$)

$$R = \frac{P_0 V_0}{T_0} = \frac{1 \text{ atm} \times 22415 \text{ cm}^3}{\text{g mole} \times 273 \text{ ok}} = 82.10 \frac{\text{cm}^3 \text{ atm}}{\text{g mole ok}}$$

In CGS unit

$$P_0 = 1.013125 \times 10^6 \text{ dynes cm}^{-2}, \ T_0 = 273 \text{k, } V_0 = 22451 \frac{\text{cm}^3}{\text{g mole}}$$

$$R = \frac{1.013125 \times 10^6 \times 22415}{273} = 8.319 \times 10^7 \frac{\text{ergs}}{\text{g mole k}}$$

Since 1 J = $10^7$ ergs and 1 cal. = 4.184 J

$\therefore R = 1.988 \text{ cal/(g mole k)}$.

In SI units.

$$P_0 = 1.013125 \times 10^5 \text{ pascals, } T_0 = 273 \text{k, and } V_0 = 0.022451 \frac{\text{m}^3}{\text{g mole}}$$

$$R = 8.314 \frac{\text{J}}{\text{g mole k}}$$

Similarly in MKS system $R = 8.314 \frac{\text{J}}{\text{g mole k}}$. 
Current = Ampere (A). In solids current consists of electron flow. In electrolyte solutions, most of the current flow by motion of conic species for example cu⁺ or Na⁺.

1 coulomb is unit of charge: flow of 1 A/s.

SI unit of electrical potential is volt. Volt is the potential in which the charge of 1 coulomb experiences a force of 1 Newton.

SI unit of resistance is ohm. Ohm is defined as the resistance which permits flow of 1 A current under an imposed electrical potential difference of 1 V.

Some basic equations of electrical flow are:

\[ V = R \cdot I \]

\[ P = I \cdot V = I^2 \cdot R \quad P = \text{power} \]

\[ t = \text{time} \]

\[ W = t \cdot P = I^2 \cdot R \cdot t. \quad W = \text{energy measured in joules and power in watts} \]

A Faraday is one mole of electrons.

1 faraday = 96500 coulomb. One faraday will discharge one gram equivalent of ions.

The liberation of one g equivalent of any metal consumes 96500 coulombs of electricity.

How many gram moles of Al³⁺ ions could be discharged in one minute by 1.9 X 10⁴ A current, if no loss of current occur.

In one minute a current of 1.9 X 10⁴ A will carry 1.9 X 10⁴ X 60 coulombs of electricity.

Gram moles of Al deposited = \( \frac{1.14 \times 10^6}{3 \times 96500} \) = 3.94

**Concentration of solids in slurry:**

Many metallurgical processes have feed and/or product streams that consist of mixtures of solids and liquids. These mixtures are called slurries.

The relationship between wt % solid (%x) and specific gravity of solid phase (\( P_s \)) and that of slurry (\( P_m \)) when water is used as a medium can be obtained:

Volume of slurry = Volume of solid x Volume of water. Consider 1 kg slurry with %x as solids weight percent, then

\[ \frac{1}{\rho_m} = \frac{%x}{100 \rho_s} + \frac{(100 - %x)}{100 \rho_w} \]
\[ \rho_w = \text{density of water} \]

\[ \rho_s = \text{density of solid} \]

\[ \rho_m = \text{density of mixture (solid + water)} \]

\[ \text{(Wt percent solid)} \times x = \frac{100 \rho_s (\rho_m - 1000)}{\rho_m (\rho_s - 1000)} \quad (1) \]

Volume \% slurry = \% x \frac{\rho_m}{\rho_s}

Mass flow rate of dry solid in slurry (M)

\[ = \frac{\text{volumetric flow rate} \times \text{slurry density} \times \% x}{100} \]

\[ M = \times \frac{F \rho_m \% x}{100} \text{ kg/hr} \quad (2) \]

F is volume flow rate in m3/hr.

By 1 and 2

\[ M = \frac{F \rho_s (\rho_m - 1000)}{(\rho_s - 1000)} \quad (3) \]

Conclusion

In this lecture the units and dimensions of physical quantities are derived from fundamental unit of mass, length, temperature and time. Suitable examples are given to illustrate the derivation of units.

Reference:

Schuhmann” Metallurgical engineering principles