Lecture 28

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Key words: Heat transfer, conduction, convection, radiation, furnace, heat transfer coefficient

Heat transfer importance

There are several unit operations and unit processes which operate at high temperatures. Flow of heat is important to attain uniform temperature in the furnace chamber. Ideally the available heat must be utilized to raise the temperature of the reactants and products to the desired value, but some amount of heat is always lost to the surrounding. Loss of heat to the surrounding is loss of energy and one of the main objectives of an engineer is to minimize the heat losses. Fundamentals of heat transfer mechanisms are important to calculate the flow of heat and to design the most efficient flow path conforming to the process. Transfer of heat takes place by conduction, convection and radiation. In the following a brief account of these mechanisms is given. For the detailed description, number of books on heat transfer is available. Some references are given at the end of this lecture

Conduction

Conduction is the flow of heat through a body occurring without displacement of the particles which make up the body. Fourier’s law of heat conduction is the basic law which says that the rate of heat flow across a unit area (Q_x) at steady state is proportional to the temperature gradient perpendicular to the area. Heat flow in one dimension, i.e. in X is

\[ Q_x = -KA \frac{dT}{dx} \]  

1)
\( \frac{dT}{dx} \) is the temperature gradient and \( A \) is the area which is assumed to be invariant along the heat flow path. The constant \( K \) is thermal conductivity of the material through which heat is flowing. The thermal conductivity of the material indicates the relative ease or difficulty of the transfer of heat through the material. \( K \) can vary from about 0.01 \( \frac{W}{mK} \) for gases to 1000 \( \frac{W}{mK} \) for pure metals. Thermal conductivity depends, on temperature, bonding and structure of the material. Thermal conductivity for ceramic materials is lower than metals. Porosity in the material decreases \( K \). Thermal conductivity of the material varies with the temperature. The variation of thermal conductivity with temperature can be described by

\[
K = K_0 (1 + \beta T), \tag{2}
\]

where \( \beta \) is the temperature coefficient of thermal conductivity with the dimensions \(^\circ C^{-1} \), and \( K_0 \) is the thermal conductivity at \( 0^\circ C \).

It must be noted that conduction of heat through gases is usually negligibly small compared with heat flow by convection and radiation.

The general equation for heat conduction is solids at steady state without any heat source is

\[
\nabla^2 T = \left[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right] \tag{3}
\]

Equation 3 applies to steady-state conduction in systems without heat source. It is also termed as Laplace equation.

**Convection**

Heat transfer by convection results due to fluid motion. Fluid motion can be caused either by buoyancy force (due to density gradient) or by inertial force. The former is called “free” or “natural” convection and the later is “forced” convection heat transfer. In both modes of heat transfer velocity of the fluid governs the rate of heat flow. To quantify the heat transfer rate by convection, it is necessary to know the velocity of fluid. Differential approach can be used to determine the fluid velocity and the reader may refer to any text book on transport phenomena as given in the reference. However, in most engineering calculations involving heat flow between fluids and their confining surfaces, the following empirical relation has been found to be very useful:

\[
Q = h A (T_w - T_\infty) \tag{4}
\]

The relation is known as Newton’s law of cooling and can be used to express the overall effect of convection. In the equation 4 \( h \) is heat transfer coefficient and \((T_w - T_\infty)\) is the temperature difference.\(T_w\) is the wall temperature and \(T_\infty\) is temperature of the surrounding. For many engineering applications equation 4 is very useful. Equation 4 is consistent with equation 1 in which \( Q \) is shown to be proportional to surface area and temperature gradient. When the heat transfer between the fluid and the surface composed of a series heat path, we may define thermal resistance \( (R) \) due to convection.
\[ R = \frac{1}{hA} \]  \hspace{1cm} (5) \\
h is an empirically derived value from the heat transfer experiments. It is to be noted that the heat transfer coefficient is an empirically derived quantity and its numeral value depends in addition to physico-chemical-thermal variables on the design of experiments.

**Free Convection**

The experiments to determine heat transfer coefficient for natural heat transfer consists of heating a flat plate or a cylinder to a particular temperature and then cooling in air. In some experiments the orientation of plate or cylinder with reference to air cooling is also varied; the plate/cylinder is kept horizontal or vertical. In all these experiments heat transfer coefficient is shown to be a function of Prandtl (Pr) number and Grashof number (Gr). Heat transfer coefficient is related to Nusselt number (Nu).

\[ Nu = f^n (Pr \times Gr) \]  \hspace{1cm} (6) \\
\[ Nu = \frac{h_c D}{K} \]  \hspace{1cm} Nusselt number \\
\[ Pr = \frac{C_p \mu}{K} \]  \hspace{1cm} Prandtl number \\
\[ Gr = \frac{D^3 \rho^2 \beta g (\Delta T)}{\mu^2} \]  \hspace{1cm} Grashof number.

D is a characteristic linear dimension, \( C_p \) specific heat, \( \rho \) is density, and \( g \) is acceleration due to gravity.

The following relation is proposed to calculate heat transfer coefficient for natural convection

\[ Nu = \alpha (Pr \times Gr)^{0.25} \]  \hspace{1cm} (7) \\
The numeral value of \( \alpha \) depends on the orientation of the plate/cylinder with reference to the cooling air. This will be illustrated in the exercises.

**Forced convection**

In forced convection heat transfer, heat transfer coefficient depends on Reynolds’s and Prandtl number. The correlations are of the form

\[ \frac{h_c D}{K} = \alpha \left( \frac{D G}{\mu} \right)^m \left( \frac{C_p \mu}{K} \right)^n \]  \hspace{1cm} (8) \\
\( G \) is mass flux(\( \text{kg/m}^2\text{s} \)), \( m \) and \( n \) are exponents The following correlation is useful for turbulent flow through the pipes,

\[ \frac{h_c D}{K} = 0.023 \left( \frac{D G}{\mu} \right)^{0.8} \left( \frac{C_p \mu}{K} \right)^{0.4} \]  \hspace{1cm} (9)
Alternatively

\[
\frac{h_c D}{K} = 0.026 \left( \frac{D V \mu}{\rho} \right)^{0.8} \left( \frac{C_p \mu}{K} \right)^{0.33}
\]

In correlation 9, Reynolds’s number is calculated by mass flux. In the absence of a relation between a physical property of the fluid and temperature, the fluid properties may be evaluated at the average temperature of the fluid.

It is to be noted that there are several empirical correlations available for the convective heat transfer coefficients. The reader must make sure that the experimental conditions of a correlation match with the conditions of the problem before use of a correlation.

Radiation

Heat transfer by radiation occurs in the form of electro–magnetic waves of various wave lengths. We are concerned primarily with flow of heat through gas filled spaces, and specifically with rate of heat exchange between two surfaces.

Stefan’s Boltzmann law states that rate of radiation of heat from a surface is proportional to the fourth power of the absolute temperature. Radiation is the principle mechanisms of heat transfer in high temperature fuel fired furnaces.

The emissive power of a black body \(E_b\) is according to Stefan-Boltzmann

\[
E_b = \sigma T^4
\]

Here \(\sigma\) is Stefan-Boltzmann constant and its value is \(0.173 \times 10^{-8} \text{ Btu} \cdot \text{hr}/\text{ft}^2\cdot\text{R}^{-4}\) in FPS systems and \(5.67 \times 10^{-8} \text{ watt}/\text{m}^2 \cdot \text{K}^{-4}\) in SI system. \(T\) is expressed in Rankin \(\left(^\circ\text{F} + 460\right)\) in FPS and in Kelvin \(\left({}^\circ\text{C} + 273\right)\) in CGS systems.

Most real bodies are not black, but they are selective absorbers and emitters. We define emissivity \(\epsilon\)

\[
\epsilon = \frac{\text{emissive power of a body}}{\text{emissive power of a black body}} = \frac{E}{E_b}
\]

Thus emissive power of any body

\[
E = \sigma \epsilon T^4
\]

Non black surfaces do not absorb the entire radiant energy incident on them. At thermal equilibrium absorptivity \(\alpha\) of any surface equals emissivity \(\epsilon\).

Surfaces with emissivities nearly unity are good absorbers and hence poor reflectors of incident radiation. Thus, most highly polished, unoxidized metal surfaces are good reflectors of thermal radiations with total emissivities less than 0.1. A roughened or an oxidized surface has correspondingly
higher emissivities for thermal radiation. Nonmetallic surfaces have emissivities and absorptivities above 0.8. An emissivity value of about 0.85 can be taken for furnace refractory and bricks at high temperatures. The slagging reduces the emissivity value to 0.6 to 0.7.

Consider rate of heat exchange between any two surfaces \( A_1 \) and \( A_2 \) at temperature \( T_1 \) and \( T_2 \) respectively. The surface \( A_1 \) is emitting radiation at the rate \( (0.01T_1)^4 \) and a certain fraction of this radiation is absorbed by the surface \( A_2 \). After a while the surface \( A_2 \) is heated and begins to emit radiation at the rate of \( (0.01T_2)^4 \); the fraction of which is absorbed by \( A_1 \). Net heat exchange between \( A_1 \) and \( A_2 \) can be expressed as heat loss of \( A_1 \) to \( A_2 \) as

\[
Q = 5.67 FA \left[ \left( \frac{T_1}{100} \right)^4 - \left( \frac{T_2}{100} \right)^4 \right]
\]  

(14)

\( F \) is a view factor which takes into account

I. Geometric relationship of the two surfaces. Physically \( F \) can be visualized as the fraction of total radiation that is intercepted by the other,

II. The emissivity and absorptivity of surface

III. Geometric relationship between two surface and or third surface e.q. a refractory surface.

The calculation of radiant heat exchange between any two surfaces involves the determination of view factor \( F \). The following procedure may be of help to evaluate \( F \).

- First \( F_1 \) is calculated by considering the geometrical relationship between two surfaces. \( F_1 \) is the value of \( F \) in equation 14 when the two surfaces \( A_1 \) and \( A_2 \) are black and are in direct heat exchange

- Next the effect of the third reflecting surface for example refractory on heat exchange may be considered and \( F_C \) can be determined. \( F_C \) is a view factor which takes into account both radiation and reflection, but assumes \( A_1 \) and \( A_2 \) are black surfaces

- Next the emissivity value of the surface can be included to determine \( F \)

**Radiation Coefficient**

For a number of purposes where heat transfer occurs by a combination of mechanisms such as radiation + convection, it is convenient to deal with heat transfer coefficient for radiation. An example is the heat loss from the walls of the furnace. In this case heat losses occur both by convection and radiation to the surrounding. It is convenient to define a radiation heat transfer coefficient similar to convective heat transfer. Radiation heat transfer coefficient \( (h_r) \) is defined as

\[
Q_0 = h_r A_1 (T_1 - T_2)
\]  

(15)

By equations 14 and 15 we get for small temperature difference between \( T_1 \) and \( T_2 \),
\[ h_r = 0.2268 \times 10^{-8} F \left( \frac{T}{100} \right)^3 \]  

When both convection and radiation are involved in transfer of heat, for example heat loss from the external wall of the furnace. Total heat loss can be evaluated from

\[ Q_{\text{total}} = (h_r + h_c) A \Delta T. \]  

Thermal resistance by radiation \( R_r = \frac{1}{h_r A} \). If heat flow path involves the circuit of conduction to a surface and convection and radiation from the surface, with the overall temperature difference \( \Delta T \)

\[ Q = \frac{\Delta T}{\frac{1}{K A} + \frac{1}{(h_c + h_r) A}} \]  

Illustration on heat transfer coefficient-1

A round duct 0.6 m diameter carries 100 m³/min (25° & 1 atm), of preheated air at about 600 K the inside surface temp of the duct is 500 K. Estimate the heat loss/ running m

(b) Estimates the drop-in air temp/30 running m.

Thermal properties of air

<table>
<thead>
<tr>
<th>( \rho ) (kg/m³)</th>
<th>500 K</th>
<th>600 K</th>
<th>mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K ) (W/m°C)</td>
<td>40.76 \times 10^{-3}</td>
<td>46.9 \times 10^{-3}</td>
<td>43.8 \times 10^{-3}</td>
</tr>
<tr>
<td>( \nu ) (m²/s)</td>
<td>38.8 \times 10^{-6}</td>
<td>52.7 \times 10^{-6}</td>
<td>46 \times 10^{-6}</td>
</tr>
</tbody>
</table>

Prandtl number = 0.74

\[ h_c = 4.39 \frac{w}{m^2 K} \]

Use of 10 given \( h_c = 5.07 \frac{w}{m^2 K} \)

\[ 4.39 \times \Pi D \times (327 - 227). = \text{Heat loss/ meter} \]

827 W/m

The heat loss 827 W is loss of sensible heat of air

\[ m C_p \Delta T = 827 \frac{w}{m} \]

\[ \Delta T = 0.45 \degree C/m \]

For 30 m temp drop would be 14°C
Illustration 2

A large vertical plate 4 m high is maintained at 130°C and exposed to atmospheric air at 25°C. Calculate the heat transfer coefficient if the plate is 10 m wide.

First we find average temperature $\approx 350K$.

Properties are

$\beta = \frac{1}{350} = 2.857 \times 10^{-3} , \quad K = 0.03003 \frac{w}{m^\circ C}$

$\nu = 20.76 \times 10^{-6} \frac{m^2}{s0} \quad Pr = 0.0697$

$Gr \times Pr = 4.36 \times 10^{11}$ using equation 7 in which $\alpha = 0.59$ and D as height of plate.

$Nu = 812.59$

$h = 6.1 \frac{w}{m^2^\circ C}$

Another equation for heat transfer coefficient is

$Nu = 0.1 \ (Gr . Pr)^{1/3} \ 19$

The difference between equation 7 and 19 is in the value of pre exponent and exponent value.

$Nu = 758$

From this value of Nusselt number

$h = 5.7 \frac{w}{m^2^\circ C}$

$h$ is 0.93 times smaller than calculated by equation 7.

References
2) R. Schuhmann: Metallurgical Engineering, Volume 1 Engineering Principles
3) D.R.Poirier and G.H.Geiger: Transport phenomena in materials processing