Geometry of the deformation zone

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1. Geometry of the deformation zone

1.1 Deformation zone geometry parameter:
In the analysis of metal forming process, friction and shear deformation (redundant deformation) of material play a notable role. Both factors contribute towards raising the forming load. Both of these factors are basically governed by the geometry of the deforming zone - shape, size of the zone of the material which gets deformed. The deformation zone is usually in the form of a converging channel. In axi-symmetric extrusion, for example, it is in the form of a truncated cone. The shape of deformation geometry is known to affect the forming load as well as the final properties of the formed products.

The deformation zone geometry is defined by a parameter, $\Delta$, which is defined as the ratio of the mean height or thickness to the mean length of the deformation zone.

$$\Delta = \frac{H}{L}$$  \hspace{1cm} (1.1)

This parameter is related to another parameter called reduction, $r$. It is defined for plane strain deformation as:

$$r = 1 - \frac{h_f}{h_i}$$  \hspace{1cm} (1.2)

For axi-symmetric deformation it is defined as:

$$r = 1 - \frac{\frac{dR^2}{dt}}{\frac{dR^2}{dt}}$$  \hspace{1cm} (1.3)

The effect of $\Delta$ on the deformation pressure was considered briefly in module 2. In this lecture, we will consider effects of deformation zone geometry on other aspects of forming.

Recall that plane strain deformation refers to the condition where the strain of the material in one of the three principal axes is zero.
We can write down the expressions for $\Delta$ for both plane strain and axi-symmetric deformations as followed:

Consider plane strain extrusion in which a strip of initial thickness $h_i$ gets reduced to a final thickness of $h_f$. Then the average height of deformation zone is given as: \( \frac{h_i + h_f}{2} \). Similarly, the length of the deformation zone, from the geometry of the die, can be written as: \( \frac{h_i - h_f}{2 \sin \alpha} \).

Therefore, \( \Delta = \frac{h_i + h_f}{h_i - h_f} \sin \alpha \) \hspace{1cm} 1.4

Introducing: \( r = 1 - \frac{h_f}{h_i} \), above, we get:

\[ \Delta = \frac{2-r}{r} \sin \alpha \] \hspace{1cm} 1.5 \hspace{0.5cm} (for plans strain deformation such as extrusion)

Similarly, for axisymmetric extrusion or drawing, we take the diameters $d_i$ and $d_f$, instead of the thickness. Therefore, we have:

\[ r = 1 - \frac{d_f^2}{d_i^2} \] \hspace{1cm} 1.6

Substituting 3 in 1, we get:

\[ \Delta = \left(1 + \sqrt{1 - r}\right)^2 \sin \alpha/r \] \hspace{1cm} 1.7

For strip rolling we can write down the deformation zone geometry factor as:

\[ \Delta = \frac{2-r}{r} \sqrt{\frac{h_i}{rR}} \] \hspace{1cm} 1.8

Note that the parameter $\Delta$ decreases as $r$ increases. Similarly, $\Delta$ increases as the die angle $\alpha$ increases.

**1.2 Effect of $\Delta$ on friction:**
It has been established that the ratio of frictional work to total work done in a forming process is inversely proportional to $\sin \alpha$. That is:
We know that $\Delta$ is proportional to $\sin \alpha$. Therefore, we can conclude that the frictional work is inversely proportional to $\Delta$. A larger deformation zone geometry has lower friction and vice versa. In other words, if the height of deformation zone is larger, there is lower friction. With lower $\Delta$, contribution of friction to the total work done in forming is larger.

**1.3 Redundant work factor:**

In order to account for the redundant shear deformation during forming, we can define the redundant work factor $\phi$ as:

$$\phi = \frac{\varepsilon_r - \varepsilon_h}{\varepsilon_h} \quad \text{-------------1.10}$$

where $\varepsilon_r$ is redundant strain and $\varepsilon_h$ is homogeneous strain. In wire drawing, for example, the factor is defined as:

$$\phi = \frac{\varepsilon^*}{\varepsilon} \quad \text{-------------1.11}$$

where $\varepsilon^*$ is the increased strain of a material subjected to redundant deformation, which otherwise would have undergone a yield strain of $\varepsilon$.

One can easily determine $\varepsilon^*$ from the flow curve. Drawing the flow curve for the drawn wire and the annealed material, then shifting the flow curve of the drawn material to the right so that it merges with the flow curve for annealed metal, and obtaining the corresponding strain from the shifted curve. This gives $\varepsilon^*$. See diagram below.

In general, the parameter $\phi$ is a function of die angle as well as reduction. Therefore, it could be related to $\Delta$ as:

$$\phi = A + B\Delta \quad \text{-------------1.12}$$

$\phi$ increases with deformation geometry parameter, linearly.
As seen from the diagram above, the redundant work factor increases with deformation zone geometry $\Delta$. We also understand that as the die angle increases redundant deformation also increases – contribution of redundant work towards the total work of deformation is larger. Redundant strain is the shear strain of the material as a result of the changing geometry of the flow. Redundant deformation is found to be non-uniform in the deformation zone. Because of this non-uniform distribution, hardness within the deformation zone is found to vary between center and surface. The variation of hardness is expressed by a factor called inhomogeneity factor (IF).

IF is defined as:  
$$IF = \frac{\text{Hardness on surface} - \text{Hardness at center}}{\text{Hardness at center}}$$

IF is found to increase with increase in $\Delta$. Further, IF is also found to increase with increasing die angle, $\alpha$ and decreasing reduction. Lower the reduction, higher the inhomogeneity factor. Inhomogeneity also introduces texture in the structure of the formed material. See diagram.
Inhomogeneity in the form of variation in hardness between center and surface is found to be larger for lower reductions in rolling. Lower reductions result in larger friction as well.

Yet another effect of the deformation zone geometry and die angle on is the density changes within the cross-section of the formed material. Larger $\Delta$ introduces high level of tensile stress – a kind of hydrostatic tensile stress at the center of the material. This causes center of the material to develop cracks and voids, which finally result in center cracks, chevron cracks in the drawn or extruded products. Larger die angle is also found to reduce density of the material at center compared to the surface. Such density variations are enhanced by the presence of inclusions such as oxides.

With larger values of $\Delta$, residual stresses are induced in the material during forming. With large $\Delta$ the surface of the material is subjected to high tensile stress while the center is subjected to high compression. High $\Delta$ condition in rolling could cause the center of the material to split, causing allegatoring. Larger reductions introduce residual tensile stress on the surface, while smaller reductions could introduce surface compressive stress.