8 Per Unit Calculations

As stated earlier, transformers of various sizes, ratings, voltage ratios can be seen being used in a power system. The parameters of the equivalent circuits of these machines also vary over a large range. Also the comparison of these machines are made simple if all the parameters are normalized. If simple scaling of the parameters is done then one has to carry forward the scaling factors in the calculations. Expressing in percent basis is one example of scaling. However if the scaling is done on a logical basis one can have a simple representation of the parameters without the bother of the scaling factors. Also different units of measurement are in use in the different countries (FPS, CGS, MKS, etc.). These units also underwent several revisions over the years. If the transformer parameter can be freed from the units then the system becomes very simple. The ‘per unit’ system is developed keeping these aspects in mind. The parameters of the transformer are referred to some base values and thus get scaled. In the case of power system a common base value is adopted in view of different ratings of the equipments used. In the case of individual equipments, its own nominal parameters are used as base values. Some base parameters can be chosen as independent base values while some others become derived base parameters. Once the base values are identified the per unit values are calculated for any parameter by dividing the same by its base value. The units must be the same for both the parameters and their bases. Thus the per unit value is a unit-less dimensionless number. Let us choose nominal voltage and nominal current on the primary side of a transformer as the base values \( V_{base} \) and \( I_{base} \). Other base values like volt ampere \( S_{base} \), short circuit impedance \( Z_{base} \) can be
calculated from those values.

\[ P_{\text{base}}, Q_{\text{base}}, S_{\text{base}} = V_{\text{base}} \times I_{\text{base}} \]  
\[ R_{\text{base}}, X_{\text{base}}, Z_{\text{base}} = \frac{V_{\text{base}}}{I_{\text{base}}} \]  
\[ G_{\text{base}}, B_{\text{base}}, Y_{\text{base}} = \frac{I_{\text{base}}}{V_{\text{base}}} \]

Normally \( S_{\text{base}} \) and \( V_{\text{base}} \) are known from name plate details. Other base values can be derived from them.

\[ V_{\text{p.u}} = \frac{V(\text{volt})}{V_{\text{base}}(\text{volt})} \]
\[ I_{\text{p.u}} = \frac{I(\text{Amps})}{I_{\text{base}}(\text{amps})} = \frac{I(\text{amps})}{S_{\text{base}}/V_{\text{base}}} \]
\[ Z_{\text{p.u}} = \frac{Z(\text{ohm})}{Z_{\text{base}}(\text{ohm})} = Z(\text{ohm}) \times \frac{I_{\text{base}}}{V_{\text{base}}} = Z(\text{ohm}) \times \frac{S_{\text{base}}}{V_{\text{base}}^2} \]

Many times, when more transformers are involved in a circuit one is required to choose a common base value for all of them. Parameters of all the machines are expressed on this common base. This is a common problem encountered in the case of parallel operation of two or more transformers. The conversion of the base values naturally lead to change in the per unit values of their parameters. An impedance \( Z_{\text{p.u.old}} \) on the old base of \( S_{\text{base.old}} \) and \( V_{\text{base.old}} \) shall get modified on new base \( S_{\text{base.new}}, V_{\text{base.new}} \) as

\[ Z_{\text{p.u.new}} = (Z_{\text{p.u.old}} \times \frac{V_{\text{base.old}}^2}{S_{\text{base.old}}^2}) \times \frac{S_{\text{base.new}}}{V_{\text{base.new}}^2} \]

The term inside the bracket is nothing but the ohmic value of the impedance and this gets converted into the new per unit value by the new \( S_{\text{base}} \) and \( V_{\text{base}} \).

If all the equivalent circuit parameters are referred to the secondary side and per unit values of the new equivalent circuit parameters are computed with secondary voltage and
current as the base values, there is no change in the per unit values. This can be easily seen by,

\[ Z'_{\text{p.u.}} = Z'_{\text{ohm}} \frac{S'_{\text{base}}}{V'_{\text{base}}^2} \quad \text{but} \quad Z'_{\text{ohm}} = \frac{1}{a^2} Z_{\text{ohm}} \] (51)

Where \( a \) - is the turns ratio of primary to secondary

\( Z \) - impedance as seen by primary,

\( Z' \) - impedance as seen by secondary.

\( S'_{\text{base}} = S_{\text{base}} \) - as the transformer rating is unaltered.

\( V'_{\text{base}} = V_{\text{base}} \frac{1}{a} \)

From the above relationships it can be seen that \( Z'_{\text{p.u.}} = Z_{\text{p.u.}} \).

This becomes obvious if we realize that the mmf of the core for establishing a given flux is the same whether it is supplied through primary or the secondary. Also the active power and reactive power absorbed inside the transformer are not dependant on the winding connected to supply. This is further illustrated by taking the equivalent circuit of a transformer derived earlier and expressing the same in per unit form.

Thus the per unit values help in dispensing away the scaling constants. The veracity of the parameters can be readily checked. Comparison of the parameters of the machines with those of similar ones throw in useful information about the machines. Comparing the efficiencies of two transformers at any load one can say that the transformer with a higher p.u.resistance has higher copper losses without actually computing the same.

Application of per unit values for the calculation of voltage regulation, efficiency and load sharing of parallel connected transformers will be discussed later at appropriate places.