Assignment #8 - Solutions

1. As derived in the lectures, the optimal level is $v_k = \dfrac{\int_{m_{k-1}}^{m_k} m f_M(m)dm}{\int_{m_{k-1}}^{m_k} f_M(m)dm}$.

   Ans b

2. As derived in the lectures, the optimal boundary point m_k is $\dfrac{v_k + v_{k+1}}{2}$.

   Ans a

3. Given the Gaussian sample distribution $f_M(m) = \dfrac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{m^2}{2\sigma^2}}$ for $-\infty < m < \infty$. The optimal quantization level corresponding to the interval $[0, \infty)$ can be found as follows
   $\int_0^\infty \dfrac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{m^2}{2\sigma^2}}\ dm = \dfrac{1}{2}\int_{-\infty}^\infty \dfrac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{m^2}{2\sigma^2}}\ dm = \dfrac{1}{2} \times 1 = \dfrac{1}{2}$. Further,
   $\int_0^\infty \dfrac{m}{\sqrt{2\pi\sigma^2}} e^{-\frac{m^2}{2\sigma^2}}\ dm = \int_0^\infty \dfrac{m}{\sigma^2} \times \sigma^2 \dfrac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{m^2}{2\sigma^2}}\ dm = -\dfrac{\sigma^2}{\sqrt{2\pi\sigma^2}} e^{-\frac{m^2}{2\sigma^2}}\Big|_0^\infty = \dfrac{\sigma^2}{\sqrt{2\pi\sigma^2}} = \dfrac{\sigma}{\sqrt{2\pi}}$.
   Therefore, the optimal level is $\dfrac{\frac{\sigma}{\sqrt{2\pi}}}{\frac{1}{2}} = \sqrt{\dfrac{2}{\pi}}\sigma$.

   Ans a

4. Given the exponential sample distribution $f_M(m) = \lambda e^{-\lambda m}, 0 \leq m < \infty$. The optimal quantization level corresponding to the interval $[0, \infty)$ can be found as follows :
   $\int_0^\infty \lambda e^{-\lambda m}\ dm = -e^{-\lambda m}\Big|_0^\infty = 1$. Also,
   $\int_0^\infty \lambda m e^{-\lambda m}\ dm = \underbrace{-me^{-\lambda m}\Big|_0^\infty}_{0} + \int_0^\infty e^{-\lambda m}\ dm = -\dfrac{e^{-\lambda m}}{\lambda}\Big|_0^\infty = \dfrac{1}{\lambda}$.

   Hence, the optimal quantization level is $\dfrac{1}{\lambda}$.

   Ans d

5. 64 quantization levels $\implies$ resolution $R = \log_2(64) = 6$ bits. Given $m_{max} = 100$. Hence, from the formula derived in lectures, quantization noise power σ_q^2 is,
   $-10\log_{10} 3 + 20\log_{10}(m_{max}) - 6R = -4.77 + 40 - 36 = -0.77\ dB$.

   Ans d

6. Given a μ-law quantizer with $\mu = 10$. The quantization characteristic is $v = \dfrac{log_2(1+\mu|m|)}{log_2(1+\mu)}$.

   For $m = 0.1, v = \dfrac{log_2(1+10\times 0.1)}{log_2(1+10)} = 0.2891$

For $m = 0.9, v = \dfrac{log_2(1 + 10 \times 0.9)}{log_2(1 + 10)} = 0.9603$

Hence, smallest 10% amplitudes are mapped to the interval $[0, 0.2891]$, while largest 10% are mapped to interval $[0.9603, 1]$ of length $= 0.0397$. Since, the output is uniformly quantized, assuming step size of Δ, the ratio of the number of quantization intervals corresponding to the smallest 10% signal amplitudes to that of the largest 10% signal amplitudes is, $\frac{\frac{0.2891}{\Delta}}{\frac{0.0397}{\Delta}} = 7.28$.

Ans c

7. Given sinusoidal modulating signal $m(t) = A\ cos(2\pi f_m t)$. Let the step size of the DM be Δ. For no slope overload distortion, we must have, $max\left|\dfrac{d}{dt}m(t)\right| \leq \dfrac{\Delta}{T} = \Delta f_s \implies$ $max| - A2\pi f_m sin(2\pi f_m t)| \leq \Delta f_s \implies A \leq \dfrac{\Delta f_s}{2\pi f_m}$.

   Therefore, maximum power of signal $= \dfrac{A^2}{2} \leq \dfrac{\Delta^2 f_s^2}{8\pi^2 f_m^2}$ and output quantization noise power is $\frac{\Delta^2}{3}$. Hence, maximum output signal to noise power ratio is, $\dfrac{\frac{\Delta^2 f_s^2}{8\pi^2 f_m^2}}{\frac{\Delta^2}{3}} = \dfrac{3f_s^2}{8\pi^2 f_m^2}$.

   Ans b

8. Since a superhet is employed, $f_{LO} > f_c$. Given, the broadcast band frequency range from 540 to 1600 KHz and IF = 455 KHz, the range of tuning required for f_{LO} is $540 + 455$ KHz to $1600 + 455$ KHz i.e. 995 KHz to 2055 KHz. Hence, ratio of highest to lowest frequencies is $2055/995 \approx 2.07$.

   Ans a

9. The bit-rate of the T_1 TDM system is 1.544 Mbps as described in the lectures..

   Ans d

10. Given $m_i(t)$, $1 \leq i \leq 5$, sampled at their Nyquist rates. Nyquist rate of $m_1(t)$ is 12.8 KHz and rates of rest of the signals are 3.2 KHz. Hence, total number of samples per second is $12.8 + 3.2 \times 4 = 25.6 \times 10^3$. Since the number of quantization levels = 2048, number of bits per sample is $\log_2(2048) = 11$. Hence, total bit rate is $25.6 \times 10^3 \times 11 = 281600$ i.e., 281.6 Kbps. Hence, bandwidth required for transmission is 140.8 KHz.

    Ans b