Assignment #7 - Solutions

1. From the Nyquist sampling theorem, the minimum required sampling rate to avoid aliasing is $2f_m$, where f_m is the maximum frequency of the message signal.

   Ans b

2. The spectrum of the resulting sampled signal is $f_s \sum_{n=-\infty}^{\infty} M(f - nf_s)$, as described in the lectures.

   Ans a

3. Consider $5sinc\,(2t)$. Its spectrum corresponds to a pulse of height $\frac{5}{2}$ from -1 Hz to 1 Hz. Hence, the spectrum $5sinc\,(2t)\cos(10\pi t)$ has two pulses of height $\frac{5}{4}$ each from 4 Hz to 6 Hz and -6 Hz to -4 Hz. As can be seen, f_m is 6 Hz. Hence the minimum sampling frequency required to avoid distortion using conventional low pass sampling Nyquist criterion is $2\times6 =$ 12 Hz.

   Ans a

4. Given the signal $m(t) = sinc^2(5t)$. Consider $sinc\,(5t)$. The corresponding spectrum is $\frac{1}{5}p_5(f)$. Hence the spectrum corresponding to $sinc^2(5t)$ is given by the convolution $\frac{1}{5}p_5(f) * \frac{1}{5}p_5(f)$ which is $\frac{1}{5}\left(1 - \frac{|f|}{5}\right), |f| \leq 5$ and 0 otherwise. The maximum frequency component is seen to be 5 Hz. Hence the minimum sampling frequency required is 10 Hz.

   Ans d

5. From the previous problem it is seen that the spectrum of $sinc^2(5t)$ is given by the triangular pulse $M(f) = \frac{1}{5}\left(1 - \frac{|f|}{5}\right), |f| \leq 5$ and 0 otherwise. If this is sampled at f_s=5 Hz, the resulting spectrum is $5 \sum_{n=-\infty}^{\infty} M(f - 5n)$ which can be seen to be equal to 1 for all f. Thus, when it is sampled with an LPF with cutoff frequency 10 Hz, the resulting spectrum is a pulse of height 1 between -10 Hz to 10 Hz i.e. $p_{20}(f)$. The corresponding time domain signal is $20sinc\,(20t)$.

   Ans a

6. Given that a wagon wheel is rotating clockwise at a speed of 2400 rpm (40 Hz) and is viewed under a fluorescent light in India. The AC line frequency in India is 50 Hz at which the fluorescent lamp flickers. This is equivalent to sampling a 40 Hz signal at a sampling frequency of 50 Hz. Hence, the resulting sampled signal contains components at 40-n50 Hz,

where n is any integer. Therefore, the frequencies present are 40 Hz, -10 Hz, -60 Hz etc. Since the eye is usually sensitive to the lowest frequency, the wagon wheel will be observed to rotate at -10 Hz i.e. 10 Hz anti-clockwise. This is termed as the wagon wheel effect.

Ans d

7. The pulse amplitude modulated signal $m(t)$, with pulse $p(t)$ and sampling duration T_s can be expressed as $\sum_{n=-\infty}^{\infty} m(nT_s)p(t - nT_s)$.

   Ans c

8. Given triangular pulse $p(t) = \left(1 - \frac{|t|}{\tau}\right)$ for $|t| \leq \tau$ and 0 otherwise, with $\tau << T_s$. This corresponds to a triangle of height 1 from $-\tau$ to τ. This can be obtained as the convolution to two rectangular pulses of height $\frac{1}{\sqrt{\tau}}$ from $-\frac{\tau}{2}$ to $\frac{\tau}{2}$ i.e. $\frac{1}{\sqrt{\tau}}p_\tau(t) * \frac{1}{\sqrt{\tau}}p_\tau(t)$. The spectrum of $\frac{1}{\sqrt{\tau}}p_\tau(t)$ is $\sqrt{\tau}sinc\,(\tau f)$. Hence, spectrum of given triangular pulse is $(\sqrt{\tau}sinc\,(\tau f))^2 = \tau sinc^2(\tau f)$. Since there is no aliasing, original signal can be reconstructed by filtering with system with overall response $\frac{1}{\tau sinc^2(\tau f)}$ for $|f| \leq \frac{1}{2T_s}$ and 0 otherwise.

   Ans d

9. Uniform quantizers with odd and even number of quantization levels respectively are known as mid-tread and mid-rise quantizers respectively.

   Ans a

10. As described in class lectures, the resulting quantization noise power is well approximated by the formula $-10\log_{10} 3 + 20\log_{10}\nu - 6R$.

    Ans b