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Advanced Foundation Engineering

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CHAPTER 9: MACHINE FOUNDATIONS

9.1 Introduction
9.2 Basic Definitions
9.3 Simple Harmonic Motion
9.4 Fundamentals of Vibration
9.5 Modes of Vibration
9.6 Free Vibrations and Forced Vibrations
9.6.1 Resonance
9.7 Damping
9.7.1 Negative Damping
9.7.2 Free Vibration with Damping
9.7.3 Forced Vibration without Damping
9.7.4 Free Vibrations with Damping
9.7.5 Forced Vibration with Damping
9.8 Constant Force—Amplitude Excitation
9.9 Quadratic excitation
9.10 Machine Foundations—Special Features
9.10.1 Types of Machines and Machine Foundations
9.10.2 General Criteria for Design of Machine Foundations
9.10.3 Design Approach for Machine Foundation
9.10.4 Vibration Analysis of a Machine Foundation
9.11 Elastic Half Space Theory
9.12 Mass-Spring-Dashpot Model
9.13 Foundations for Reciprocating Machines
9.13.1 Design Criteria
9.13.2 Calculation of Unbalanced Inertial Forces
9.14 Foundations for Impact Machines
9.14.1 Special Considerations
9.14.2 Design Data
9.14.3 Elastic Pad under the Anvil
9.14.4 Velocity of Anvil
9.14.5 Velocity of Tup and Anvil after Impact
9.14.6 Dynamic Analysis of Foundation for Impact Machines
9.15 Design Criteria
9.15.1 Design Approach
9.15.2 Barkan's Empirical Procedure
9.15.3 Weight of Foundation
9.15.4 Base Area of the Foundation Block
9.15.5 Minimum Thickness of Foundation
9.16 Vibration Isolation
9.16.1 Types of Isolation-Transmissibility
9.16.2 Methods of Isolation
9.16.3 Properties of Isolating Materials
9.17 Construction Aspects of Machine Foundations
9.17.1 Concrete
9.17.2 Reinforcement
9.17.3 Expansion Joints
9.17.4 Connecting Elements
9.17.5 Spring Absorbers
9.18 Provision for Tuning

Chapter 9

Machine Foundations

9.1 Introduction

Foundations may be subjected to either static loads or a combination of static and dynamic loads; the latter lead to motion in the soil and mutual dynamic interaction of the foundation and the soil.

The design of foundations subjected to dynamic forces is part of soil dynamics. 'Soil Dynamics' may be defined as that part of soil mechanics which deals with the behavior of soil under dynamic conditions. The effects of dynamic forces on soil are under this topic which is relatively a new area of Geotechnical Engineering.

The sources of dynamic forces are numerous; violent types of dynamic forces are caused by earthquakes, and by blasts engineered by man. Pile driving and landing of aircraft in the vicinity, and the action of wind and running water may be other sources. Machinery of different kinds induces different types of dynamic forces which act on the foundation soil.

Most motions encountered in Soil Dynamics are rectilinear (translational), curvilinear, rotational, two-dimensional, or three-dimensional, or a combination of these. The motion may be a periodic or periodic, and steady or transient, inducing 'vibrations' or 'oscillations'. Impact forces or seismic forces cause 'shock', implying a degree of suddenness and severity, inducing a periodic motion in the form of a 'pulse' or a transient vibration. This may lead to settlement of foundations and consequent failure of structures.

Since dynamic forces impart energy to the soil grains, several changes take place in the soil structure, internal friction, and adhesion. Shock and vibration may induce liquefaction of saturated fine sand, leading to instability.

The primary aim of Soil Dynamics is to study the engineering behavior of soil under dynamic forces and to develop criteria for the design of foundations under such conditions.

The fields of application of Soil Dynamics are varied and diverse, and include (*i*) vibration and settlement of structures, and of foundations of machinery, (*ii*) densification of soil by dynamic compaction and vibration, (*iii*) penetration of piles and sheet piles by vibration or impact, (*iv*) dynamic and geophysical methods of exploration, (*v*) effects of blasting on soil and rock materials, and (*vi*) effects of earthquakes and earthquake-resistant design of foundations. The increasing use of heavy machinery, of blasting operations in construction practice, and of various kinds of heavy transport in the context of industrial and technological progress point to the importance of 'Soil Dynamics'.

'Dynamics of Bases and Foundations' forms an important part of 'Industrial Seismology', a branch of mechanics devoted to the study of the effects of shocks and vibrations in the fields of engineering and technology; in fact, the former phrase happens to be the title of a famous book on the subject by Professor D.D. Barkan in Russian (English Translation edited by G.P. Tschebotarioff and first published by McGraw-Hill Book Company, Inc., New York, in 1962). This is a monumental reference book on the subject, based on the original research in Barkan's Soil Dynamics Laboratory. The Book "Vibration Analysis and Design of Foundations for Machines and Turbines" by Alexander Major (1962) also ranks as an excellent and authoritative reference on the subject, while a more recent Book "Vibrations of Soils and Foundations" by Richart, Hall and Woods (Prentice Hall, Inc., New York, 1970) is also an excellent treatise.

9.2 Basic Definitions

I. **Vibration (or Oscillation):** It is a time-dependent, repeated motion which may be translational or rotational.

II. **Periodic motion**: It is a motion which repeats itself periodically in equal time intervals.

III. **Period**: The time in which the motion repeats itself is called the 'Period'.

IV. **Cycle:** The motion completed in a period is called a 'Cycle'.

V. **Frequency:** The number of cycles in a unit of time is known as the 'frequency'. It is expressed in Hertz (Hz) in SI Units (cycles per second). The period and frequency are thus inversely related, one being simply the reciprocal of the other.

VI. **Degree of Freedom**: The number of independent co-ordinates required to describe the motion of a system completely is called the 'Degree of Freedom

9.3 Simple Harmonic Motion:-

The simplest form of periodic motion is the simple harmonic motion—that of a point in a straight line, such that the acceleration of the point is proportional to the distance of the point from a fixed reference point or origin. One famous example is the motion of a weight suspended by a spring and set into vertical oscillation by being pulled down beyond the static position and release (Fig. 9.1). If the spring were to be frictionless and weightless, the weight oscillates about the static position indefinitely. The maximum displacement with respect to the equilibrium position is called the 'Amplitude' of the oscillation.

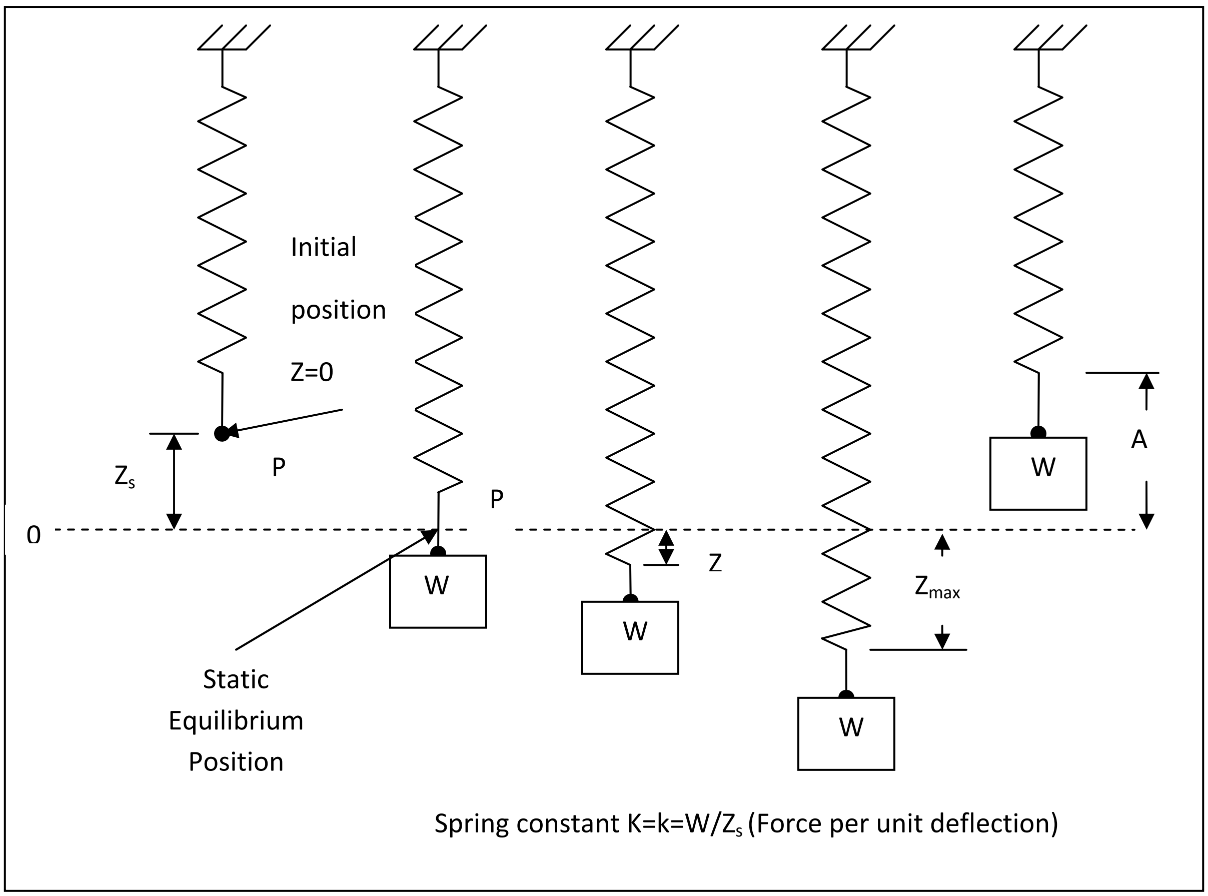


Fig.9.1: Simple Harmonic Motion of a weight suspended by a spring

A graphical representation of the simple harmonic motion of the weight is shown in Fig. 9.2. The actual line of oscillation of the point p in the vertical direction may be taken as the projection on the vertical diameter of the point 'a' rotating at uniform angular velocity about the circle with the centre at O (Fig. 9.2 (a)). The displacement versus time is shown in (Fig. 9.2 (b)).

The equation of motion is represented by a sine function

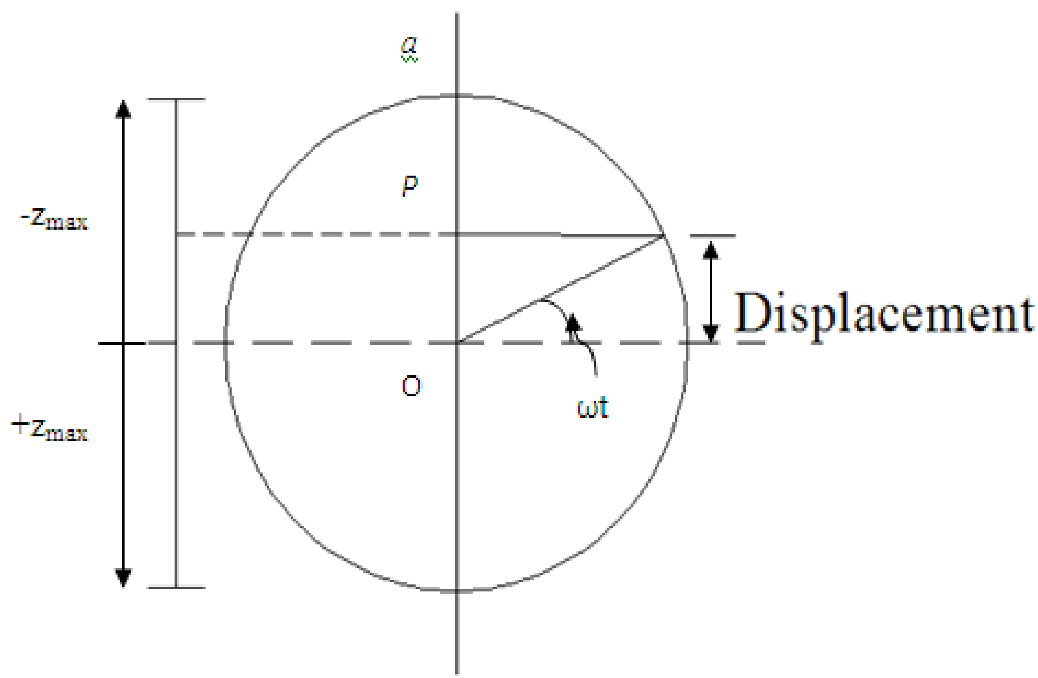
$$Z = A\sin\omega t \text{ (Eq.9.1)}$$

where, ω is the circular frequency in radians per unit time, also known as the angular velocity of point ‘a’ around O in Fig. 9.2 (a).

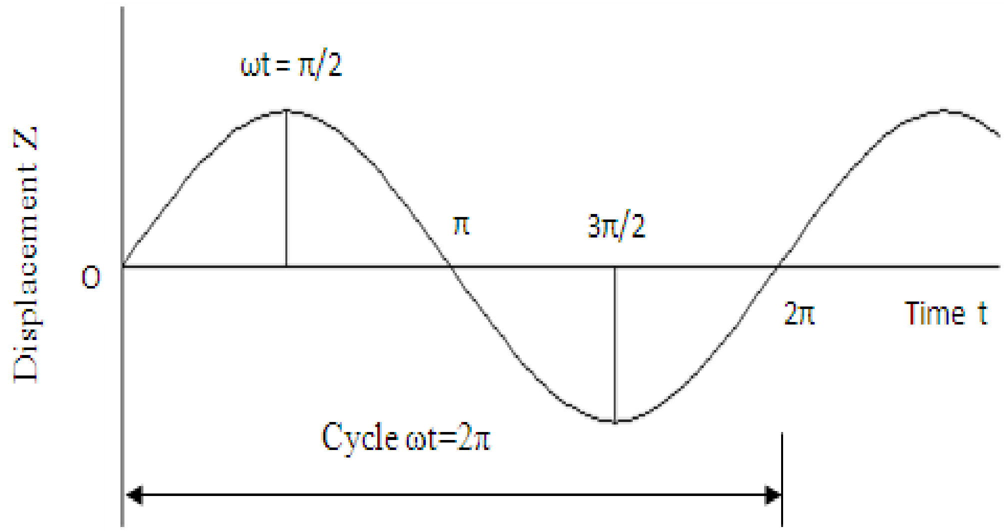
The cycle of motion is completed when $\omega t = 2\pi$

Therefore,

$$\text{The period, } T = \frac{2\pi}{\omega} \text{ (Eq.9.2)}$$



(a) Circular Motion



(b) Displacement versus Time

Fig. 9.2: Graphical representation of Simple Harmonic Motion

The number of cycles per unit of time, or the frequency, is

$$f = \frac{1}{T} = \frac{\omega}{2\pi} \qquad \text{(Eq. 9.3)}$$

The number of cycles per second is called 'Hertz' (Hz). Successive differentiation of Eq.9.1 gives

$$\frac{dZ}{dt} = \dot{Z} = \omega A \cos \omega t = \omega A \sin\left[\omega t + \frac{\pi}{2}\right] \text{(Eq. 9.4)}$$

$$\frac{d^2 Z}{dt^2} = \ddot{Z} = -\omega^2 A \sin \omega t = \omega^2 A \sin[\omega t + \pi]. \qquad \text{(Eq. 9.5)}$$

It is obvious that velocity leads the displacement by 90° and acceleration leads the displacement by 180°.

If a vector of length A is rotated counter-clockwise about the origin as shown in Fig. 9.3 its projection on to the vertical axis would be equal to $A \sin \omega t$ which is exactly the expression for displacement given by Eq.9.1. Similarly it can be easily understood that the velocity can be represented by the vertical projection of a vector of length ωA positioned 90° ahead of displacement vector, and acceleration by a vector of length $\omega^2 A$ located 180° ahead of the displacement vector.

The differential equation of motion is $\ddot{Z} + \omega^2 A \sin \omega t = 0$ or

$$\ddot{Z} + \omega^2 Z = 0 \qquad \text{(Eq.9.6)}$$

The general solution of this equation is

$$Z = C_1 \sin \omega t + C_2 \cos \omega t \qquad \text{(Eq. 9.7)}$$

where, C_1 and C_2 are constants.

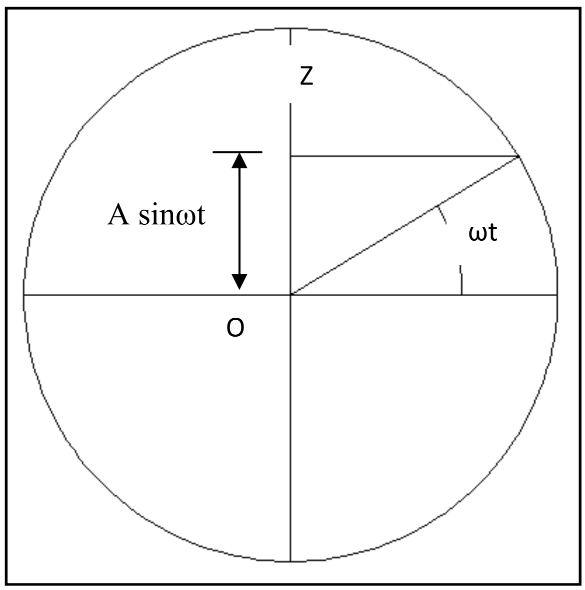


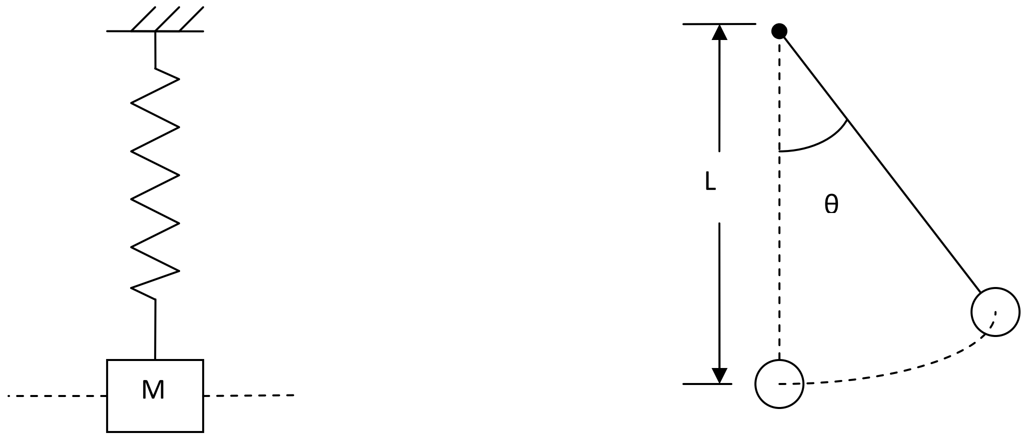
Fig. 9.3: Vector representing simple harmonic motion

9.4 Fundamentals of Vibration

Certain fundamental aspects of Vibration essential to the study of Soil Dynamics are considered in the following subsections.

i) **Degree of Freedom:**

The 'Degree of Freedom' for a system is defined as the minimum number of independent co-ordinates required to describe the motion of the system mathematically. A mass supported by a spring and constrained to move in only one direction is a system with a single degree of freedom. Similarly, a simple pendulum oscillating in one plane is also an example of a system with a single degree of freedom (Fig. 9.4).



(a) Mass supported by a spring

(b) Simple pendulum oscillating in one plane

Fig. 9.4: Systems with single degree of freedom

However, if the spring-supported mass of Fig. 9.4 (*a*) can also rotate in one plane its degree of freedom is two. A two-mass two-spring system, constrained to move in one direction without rotation, is also an example of a system with a degree of freedom of two (Fig. 9.5).A body in space has a degree of freedom of six—three translational and three rotational (Fig. 9.6). A flexible beam between two supports has an infinite number of degrees of freedom (Fig. 9.7).

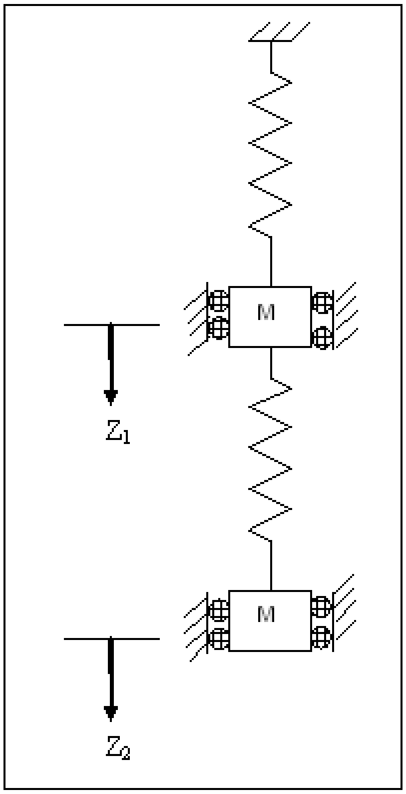


Fig. 9.5: A two-mass two-spring mass system

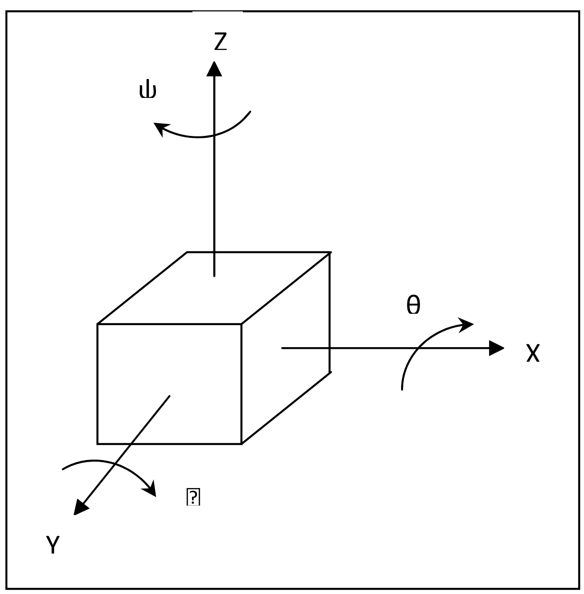


Fig. 9.6: Body in space with six degrees of freedom

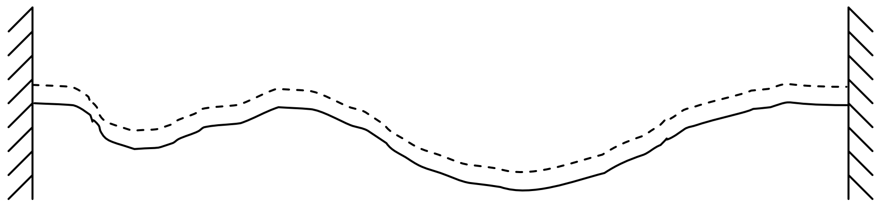


Fig. 9.7: A beam with infinite degree of freedom

9.5 Modes of Vibration

A system with more than one degree of freedom vibrates in complex modes. However, if each point in the system follows a definite pattern of vibration, the mode is systematic and orderly, and is known as a 'principal mode of vibration'. The vibration of a block can be reduced to six modes for the purpose of analysis. These are

(*i*) Translation along *X*-axis (lateral)

(*ii*) Translation along *Y*-axis (longitudinal)

(*iii*) Translation along *Z*-axis (vertical)

(*iv*) Rotation about *X*-axis (pitching)

(*v*) Rotation about *Y*-axis (rocking)

(*vi*) Rotation about *Z*-axis (yawing or torsional)

These are known as the Principal modes of vibration of the block and are shown schematically in Fig. 9.8.

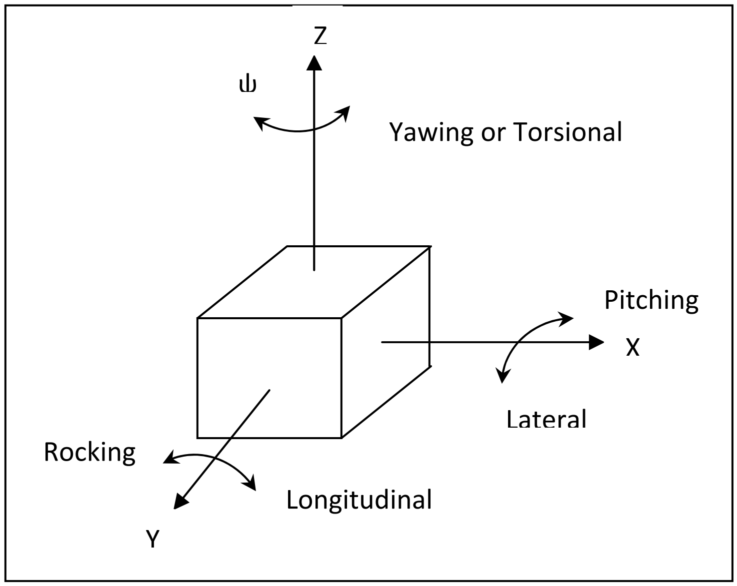


Fig. 9.8: Modes of vibration of a block

Vertical and torsional vibrations can occur independently but not others. This is because rotation about *X*-axis or *Y*-axis is always accompanied by translation along *Y*- or *X*-axis and vice-versa, producing what is known as ‘coupled motion’. If a combination of more than one mode of vibration occurs in a particular case, it is referred to as ‘coupled mode’ of vibration. The analysis of such modes requires the use of complex mathematical treatment.

9.6 Free Vibrations and Forced Vibrations

Bodies which have both mass and elasticity are capable of undergoing vibrations. The vibrations of a body or a system may be classified as ‘Free Vibrations’ and ‘Forced Vibrations’.

‘Free Vibration’ is a vibration that occurs under the influence of forces inherent in the system itself, without any external force. Of course, an external force or natural disturbance is required to initiate the free vibration which continues without an external force acting continuously.

If the vibration is un-damped by friction or any other forces, the body undergoes free vibration with a frequency known as the ‘Natural frequency’ of the body or system. It is considered as the property of the body or system. Depending upon the particular mode of vibration, the body will have a particular value of natural frequency. Thus a body or system can have as many natural frequencies as the possible modes of vibration.

‘Forced Vibration’ is a vibration that occurs under the continuous influence of an external force. This obviously depends upon the nature of the external force, also known as the ‘exciting force', which may be caused by an impulsive force or a continuous periodic force. Hammer foundation produces an impulsive force causing forced vibration of the system. A foundation for a machine with rotating masses will be subjected to a vibration caused by a continuous periodic force.

In practice it is extremely rare that a body has free vibration at its natural frequency un-damped, since it is always subjected to some form of damping.

9.6.1 Resonance

When the frequency of the exciting force in a forced vibration of a body or a system equals one of the natural frequencies of the body or system, the amplitude of motion tends to become excessively large. This condition or phenomenon is called 'Resonance'. The particular value of the frequency of the exciting force producing resonant conditions is called the 'resonant frequency' under that specific mode of vibration. Since resonance produces excessively large amplitudes, it has dangerous implications for any engineering structure, machine, or system in causing failure. Hence one of the important endeavours of an Engineer dealing with Soil Dynamics and Design of a Machine Foundation is to avoid resonant conditions.

9.7 Damping

'Damping' in a physical system is resistance to motion, and may be one of the several types mentioned in the following paragraphs.

Viscous Damping: - This type of damping occurs in lubricated sliding surfaces, dashpots with small clearances etc. Eddy current damping is also of viscous nature. The magnitude of damping depends upon the relative velocity and upon the parameters of the damping system. For a particular system, the damping resistance is proportional to the velocity:

$$F = c\frac{dz}{dt} \quad \text{(Eq. 9.8)}$$

where, F =damping force, $\frac{dz}{dt}$ = velocity, and c= damping coefficient.

This affords relatively easy analysis of the system, since the differential equation of the system becomes linear with this type of damping. This is why a system is often represented to include an equivalent viscous damper even if the damping is not truly viscous.

a) Friction or Coulomb Damping: - This kind of damping occurs when two machine parts rub against each other, dry or un-lubricated. The damping force in this case is practically constant and is independent of the velocity with which the parts rub each other.

b) Solid, Internal or Structural Damping: -This type of damping is due to the internal friction of the molecules. The stress-strain diagram for a vibrating body is not a straight line but forms a hysteresis loop, the area of which represents the energy dissipated due to molecular friction per cycle per unit volume. The area of the loop depends upon the material of the vibrating body, frequency, and the magnitude of the stress. Since this involves internal loss of energy by absorption, it is also called ‘internal damping’.

c) Slip or Interfacial Damping. Energy of vibration is dissipated by microscopic slip on the interfaces of machine parts in contact under fluctuating loads. Microscopic slip also occurs on the interfaces of the machine elements forming various types of joints. The magnitude of damping depends, amongst other things, upon the surface roughness of the parts, the contact pressure, and the amplitude of vibration. This type of damping is essentially of a non-linear type.

d) Radiation’, ‘dispersion’ or ‘geometric’ damping:-In the case of machine foundation resting on soil, damping occurs due to the loss of energy on two counts. First, some energy loss occurs by the absorption of energy into the system, reflected by the hysteresis in the stress~strain relationship; damping caused by this internal loss of energy is called ‘internal damping’ already given in (b). Next, the dissipation of energy by wave propagation, radiating away into the soil mass, causes damping effect. This is known as ‘radiation’, ‘dispersion’, or ‘geometric’ damping.

9.7.1 Negative Damping

Generally speaking, damping is positive, so that energy is always absorbed from the system by damping devices. If the system draws energy from some source or is supplied energy, the amplitude continues to increase, leading to instability. Such a system is said to be negatively damped. The build-up of amplitudes of transmission line wires, or tall poles or suspension bridges under the action of uniform wind flow at critical speeds are examples of negatively damped systems. In structural systems subjected to dynamic forces due to an earthquake or a blast, the damping is always positive.

9.7.2 Free Vibration with Damping

The mathematical model consists of a mass supported by a weightless spring (Fig. 9.9) with single degree freedom.

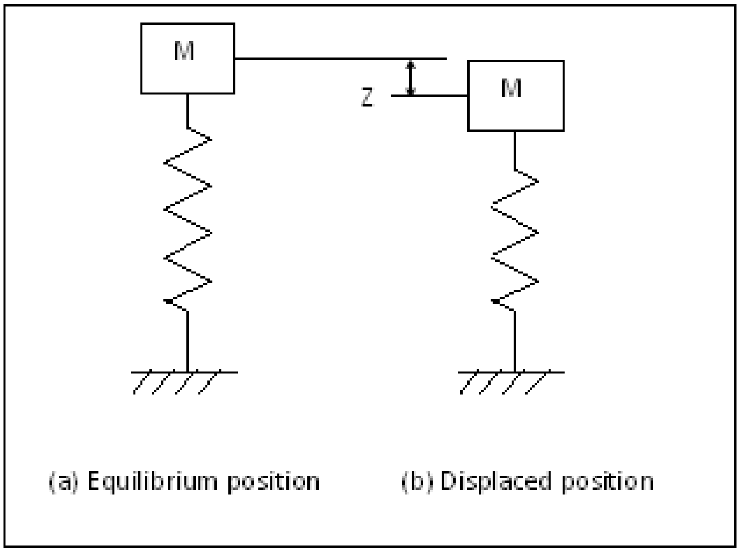


Fig. 9.9: Free vibrations- undamped-mass spring system

If z is the vertical displacement of the system from its equilibrium position, and k is the spring constant, applying Newton's law of motion, the equation of motion is

$$M\ddot{Z} + kZ = 0 \quad \text{(Eq. 9.9)}$$

Or, $\ddot{Z} + \left(\frac{k}{M}\right)Z = 0$

Or $\ddot{Z} + \omega_n^2 Z = 0$ (Eq. 9.10)

where $\omega_n^2 = \frac{k}{M}$ (Eq. 9.11)

Eq. 9.10 is a homogeneous linear differential equation and the solution is given by;

$$Z = C_1 \sin\omega t + C_2 \cos\omega t \quad \text{(Eq. 9.12)}$$

where, C_1 and C_2 are constants and can be evaluated from the initial conditions of the system.

The equation also represents simple harmonic motion expressed by Eq. 9.7, ω_n being the circular frequency. Therefore, the free vibration of a mass resting on a spring and subjected to inertial forces only can be represented by a simple harmonic motion.

ω_n in this case is called 'Natural Circular Frequency' of the system.

$$\omega_n = \sqrt{\frac{k}{M}} rad/\sec \quad \text{(Eq. 9.13)}$$

$$\text{And, } f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{M}} \quad \text{(Eq. 9.14)}$$

$$\text{The Period, } T_n = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{M}{k}} \quad \text{(Eq. 9.15)}$$

9.7.3 Forced Vibration without Damping

If a mass supported by a spring is subjected to an exciting force, the system undergoes forced vibrations. Such an exciting force may be caused by unbalanced rotating machinery or by other means. In the analysis that follows, it is assumed that the exciting force is periodic and that it may be expressed as

$$P = P_0 \sin \omega t \quad \text{(Eq. 9.16)}$$

where, P_0 is the maximum value of the exciting force and ω is the circular frequency of the exciting force in rad/sec. The system is shown in Fig. 9.10.

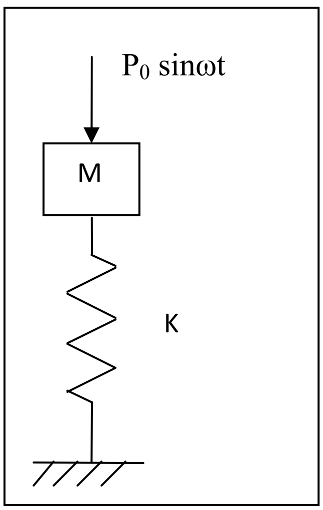


Fig. 9.10: Forced Vibration- undamped mass-spring system

The equation of motion for the system may be written as;

$$M\ddot{Z} + kZ = P_0 \sin \omega t \quad \text{(Eq.9.17)}$$

$$\text{Or, } \ddot{Z} + \omega_n^2 Z = \frac{P_0}{M}\sin \omega t$$

$$\text{Since, } \omega_n^2 = \frac{k}{M}$$

The solution of Eq. 9.17 includes the solution for free vibrations (Eq. 9.12), along with the solution which satisfies the right hand side of Eq. 9.17. The solution may be obtained by parts as the sum of the complementary function and the particular integral. The complementary function which represents the free vibration does not exist in this situation and the particular integral alone is of interest.

Since the applied force is harmonic, the motion of the system may be taken as being harmonic. Thus the particular integral may be taken as

$$Z = A \sin \omega t \quad \text{(Eq. 9.18)}$$

By substituting this in Eq. 9.17, we may show that

$$A = \frac{P_0}{M\left(\omega_n^2 - \omega^2\right)} \quad \text{(Eq.9.19)}$$

It follows that the frequency of a forced vibration is equal to that of the exciting force. (This is the same as the speed of machine, in case it is a machine that is being dealt with). Equation 9.19 may be rewritten as;

$$A = \frac{P_0}{M\omega_n^2\left(1 - \frac{\omega^2}{\omega_n^2}\right)} \quad \text{(Eq. 9.20)}$$

$$\text{But} \quad \frac{P_0}{M\omega_n^2} = A_{st} \quad \text{(Eq.9.21)}$$

where, A_{st} =deflection of the system under P_0, applied statically. The ratio $\left(\frac{\omega}{\omega_n}\right)$ is called the frequency ratio, ξ.

$$A = \frac{A_{st}}{1 - \xi} \quad \text{(Eq. 9.22)}$$

The factor $\left(\frac{1}{1-\xi^2}\right)$ is called the 'magnification factor', η_0. It is the ratio of the dynamic amplitude to the static displacement. A plot between ξ and η_0 is shown in fig. 9.11.

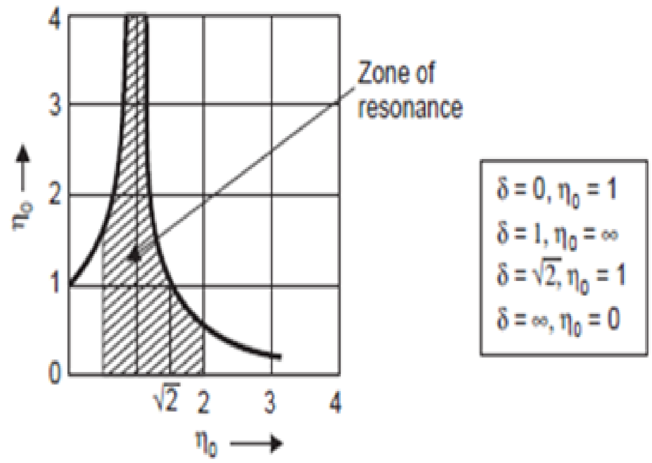


Fig. 9.11: Frequency ratio vs. magnification factor

When the exciting frequency approaches the natural frequency of the system the magnification factor ($\xi = 1$) and hence the amplitude of vibration tend to become infinite, leading to resonance. If the frequency ratio is more than 1, there will be steep decrease of the magnification factor.

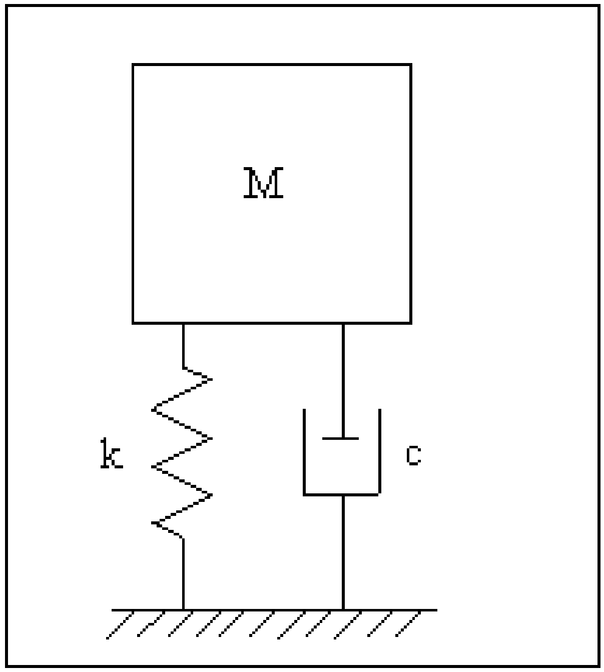
It is obvious that resonant conditions should be avoided.

9.7.4 Free Vibrations with Damping

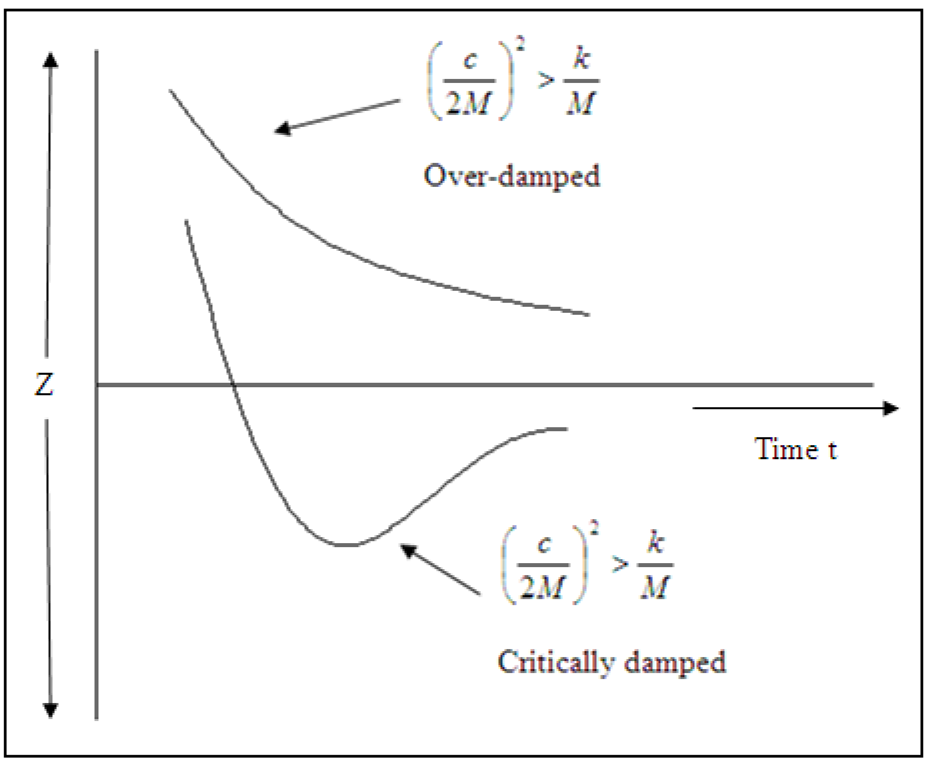
Assuming that in a system undergoing free vibrations viscous damping is present, a "mass-spring dashpot" system can serve as the relevant mathematical model for analysis (Fig. 9.12). The 'dashpot' is the simplest mathematical element to simulate a viscous damper. The force in the dashpot under dynamic loading is directly proportional to the velocity of the oscillating mass.

The equation of motion is

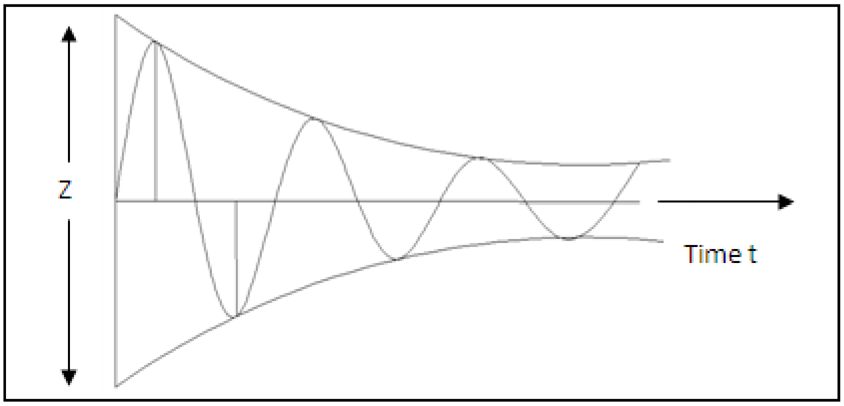
$$M\ddot{Z} + c\dot{Z} + kZ = 0 \text{(Eq. 9.23)}$$



(a) Mass-spring dash pot system



(b) Different damping conditions- over-damped and critically damped systems



(c) Undamped system

Fig 9.12 A mathematical model for free vibrations with damping

This can be rewritten as

$$\ddot{Z}+\frac{c}{M}\dot{Z}+\frac{k}{M}Z=0 \qquad \text{(Eq.9.24)}$$

Putting $\alpha=\frac{c}{M}$

$$\ddot{Z}+\alpha\dot{Z}+\omega_n^2 Z=0 \qquad \text{(Eq.9.25)}$$

Let the solution to equation 9.25 be in the form

$$Z=e^{\lambda t} \qquad \text{(Eq. 9.26)}$$

λ being a constant to be determined.

Substituting this in Eq. 9.25 we get

$$\left(\lambda^2+\alpha\lambda+\omega_n^2\right)e^{\lambda t}=0 \qquad \text{(Eq.9.27)}$$

Or, $$\lambda^2+\alpha\lambda+\omega_n^2=0 \qquad \text{(Eq.9.28)}$$

The roots of this equation are

$$\lambda_1=-\frac{\alpha}{2}+\sqrt{\left(\frac{\alpha}{2}\right)^2-\omega_n^2} \qquad \text{(Eq.9.29 (a))}$$

$$\lambda_2=-\frac{\alpha}{2}-\sqrt{\left(\frac{\alpha}{2}\right)^2-\omega_n^2} \qquad \text{(Eq.9.29 (b))}$$

Three possible case of damping arise from these roots, these are:

Case-1: Roots are real and negative if

$$\left(\frac{\alpha}{2}\right)^2 > \omega_n^2 \text{ or } \left(\frac{c}{2M}\right)^2 > \frac{k}{M}$$

The general solution is $\qquad Z = C_1 e^{(\lambda_1 t)} + C_2 e^{(\lambda_2 t)}$ (Eq. 9.30)

Since both λ_1 and λ_2 are negative, z will decrease exponentially with time without any change in sign as shown in Fig. 9.12(b). The motion is not periodic and the system is said to be over-damped.

Case-2: Roots are equal if

$$\left(\frac{\alpha}{2}\right)^2 = \omega_n^2 \text{ or } \left(\frac{c}{2M}\right)^2 = \frac{k}{M}$$

The general solution is $\qquad Z = e^{\left[-\left(\frac{c}{2M}\right)t\right]}(C_1 + C_2 t)$ (Eq. 9.31)

This is similar to the over-damped case except that it is possible for the sign to change once as shown in Fig. 9.12 (b). This is also not a periodic motion and with increase in time, approaches zero. The value of 'c' for this condition is called the 'critical damping coefficient 'c_c'.

Since $\qquad \left(\frac{c_c}{2M}\right)^2 = \frac{k}{M}$

$c_c = 2\sqrt{kM}$ (Eq. 9.32)

Using Eq. 9.13 we may write

$c_c = 2M\omega_n$ (Eq. 9.33)

c_c is the limiting value for the motion to be periodic.

Case-3: Roots are complex conjugates if

$$\left(\frac{\alpha}{2}\right)^2 < \omega_n^2 \text{ or } \left(\frac{c}{2M}\right)^2 < \frac{k}{M}$$

By using Eq. 9.32 the roots λ_1 & λ_2 become

$\lambda_1 = \omega_n\left(-D + i\sqrt{1-D^2}\right)$ (Eq.9.34 (a))

$\lambda_2 = \omega_n\left(-D - i\sqrt{1-D^2}\right)$ (Eq.9.34 (b)) where, $D = \frac{c}{c_c}$ and is called “Damping Ratio” or “Damping factor”.Substituting these into Eq. 9.30 and simplifying, the general solution becomes

$$Z = e^{-\omega_n D t}(C_3 \sin \omega_n t\sqrt{1-D^2} + C_4 \cos \omega_n t\sqrt{1-D^2}) \qquad \text{(Eq. 9.35)}$$

where C_3 and C_4 are arbitary constants

Eq. 9.35 indicates that the motion is periodic and the decay in amplitude will be proportional to $e^{-\omega_n D t}$ as shown by the dashed curve in Fig. 9.12 (*c*). Further Eq. 9.35 indicates that the frequency of free vibrations with damping is less than the natural frequency for undamped free vibrations, and that as $D \rightarrow 1$, the frequency approaches zero. The relation between these two frequencies is given by

$$\omega_{dn} = \omega_n \sqrt{1-D^2} \text{ (Eq.9.36)}$$

where, ω = frequency of free vibration with damping. Fig. 9.12(c) shows that there is a decrement in the successive peak amplitudes. Using Eq. 9.35, ratios of successive peak amplitudes may be found.

Let Z_1 and Z_2 be the amplitudes of successive peaks at times $t_1 \& t_2$ respectively as shown in Fig. 9.12 c.

$$\frac{Z_1}{Z_2} = \exp\left(\frac{2\pi D}{\sqrt{1-D^2}}\right) \qquad \text{(Eq.9.37)}$$

‘Logarithmic Decrement’ is defined as

$$\delta = \ln\frac{Z_1}{Z_2} = \frac{2\pi D}{\sqrt{1-D^2}} \qquad \text{(Eq.9.38)}$$

In words, logarithmic decrement is defined as the natural logarithm of the ratio of any two successive amplitudes of same sign in the decay curve obtained in free vibration with damping.

δ is approximately 2πD, when D is small. Eq. 9.38 also indicates that, in viscous damping, the ratio of amplitudes of any successive peaks is a constant. It follows that the logarithmic decrement may be obtained from any two peak amplitudes Z_1 and Z_{1+n} from the equation,

$$\delta = \frac{1}{2} \ln \frac{Z_1}{Z_{1+n}} \qquad \text{(Eq. 9.39)}$$

9.7.5 Forced Vibration with Damping

A system which undergoes forced vibrations, and in which viscous damping is present, may be analysed by the mass-spring-dashpot model shown in Fig. 9.13.

The equation of motion for this system may be written as follows:-

$$M \ddot{Z} + c \dot{Z} + kZ = P_0 \sin \omega t \qquad \text{(Eq. 9.40)}$$

This may be written as

$$\ddot{Z} + \frac{c}{M} \dot{Z} + \frac{k}{M} Z = \frac{P_0}{M} \sin \omega t \qquad \text{(Eq.9.41)}$$

Or, $$\ddot{Z} + \alpha \dot{Z} + \omega_n^2 Z = \frac{P_0}{M} \sin \omega t \qquad \text{(Eq.9.42)}$$

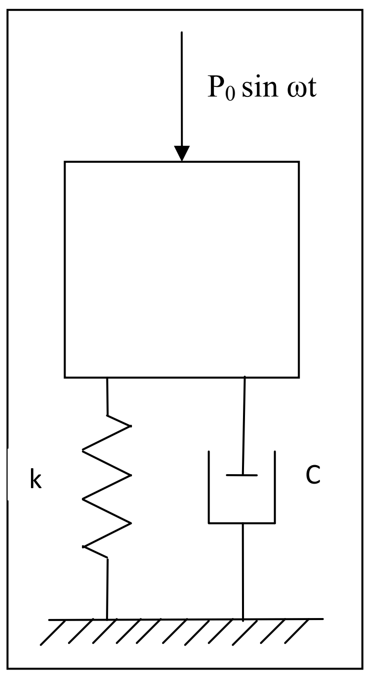


Fig. 9.13: Forced vibration with damping

where, $\alpha = \frac{c}{M}$ & $\omega_n^2 = \frac{k}{M}$

The particular solution is a steady state harmonic oscillation having a frequency equal to that of the excitation, and the displacement vector lags the force vector by some angle.

Let us therefore assume that the particular solution is

$$Z = A\sin(\omega t - \phi) \qquad \text{(Eq.9.43)}$$

where $$A = \frac{P_0}{M\omega_n^2\sqrt{\alpha^2\omega^2 + \left(\omega_n^2 - \omega^2\right)}} \qquad \text{(Eq. 9.44)}$$

and, $$\phi = \tan^{-1}\left\{\frac{\alpha\omega}{\left(\omega_n^2 - \omega^2\right)}\right\} \qquad \text{(Eq. 9.45)}$$

'A' may also be expressed as

$$A = \frac{P_0}{M\omega_n^2\sqrt{\left(1 - \xi^2\right)^2 + 4D^2\xi^2}} \qquad \text{(Eq. 9.46)}$$

where, $\xi = \dfrac{\omega}{\omega_n}$ the frequency ratio

There are two kinds of excitation:

i) Constant force-amplitude excitation and

ii) Quadratic excitation.

9.8 Constant Force—Amplitude Excitation

This type is caused by an electro-magnetic vibrator, the exciting force being generated mainly from the magnetic attraction or repulsion due to the change in intensity or direction of magnetic flux linking several flux-carrying elements (Fig. 9.14). The magnetic flux is produced by passing an electric current through a winding on one part of the magnetic circuit. The resultant magneto-motive force is proportional to the current passing through the coil. The other winding is placed in order to generate a force having fundamental frequency of the magnetic circuit and to eliminate the rectification process. An electromagnetic vibrator is driven by a frequency oscillator and power amplifier.

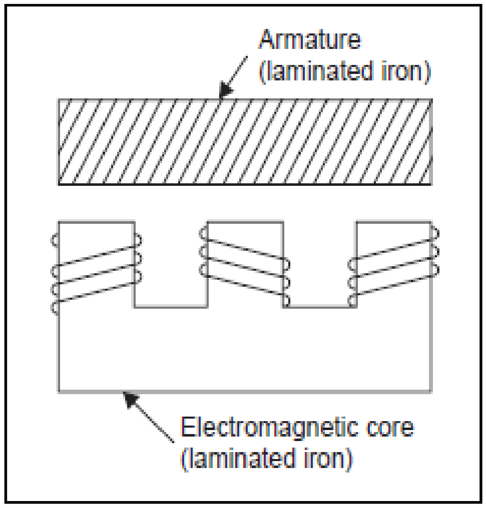


Fig. 9.14: Electromagnetic Vibrator

Eq. 9.46 may be rewritten as follows:

$$A = \eta_1 A_{st} \quad \text{(Eq. 9.47)}$$

where, $$\eta_1 = \frac{1}{\sqrt{(1-\xi^2)^2 + 4D^2\xi^2}} \quad \text{(Eq. 9.48)}$$

Since A is a constant for given spring and excitation, amplitude of motion A is directly proportional to η_1 .

To determine the conditions corresponding to maximum amplitude, Eq. 9.48 may be differentiated with respect to ξ , equated to zero, and solved for ξ. One obtains

$$\xi = \sqrt{(1-2D^2)} \quad \text{(Eq. 9.49)}$$

It is clear from this equation that if D decreases, ξ increases and vice versa. Resonance condition is said to occur when the peak amplitude occurs. Hence, the magnification factor at resonance $\eta_{1\max}$ is got by substituting the value of ξ from Eq. 9.49 in Eq. 9.48.

$$\eta_{1\max} = \frac{1}{2D\sqrt{1-D^2}} \quad \text{(Eq. 9.50)}$$

From this equation, it can be seen that the larger the damping ratio, the smaller the magnification factor at resonance, and vice versa. The relationship between ξ and η_1 (or ***A)*** [for varying ***D***] is shown in Fig. 9.15.

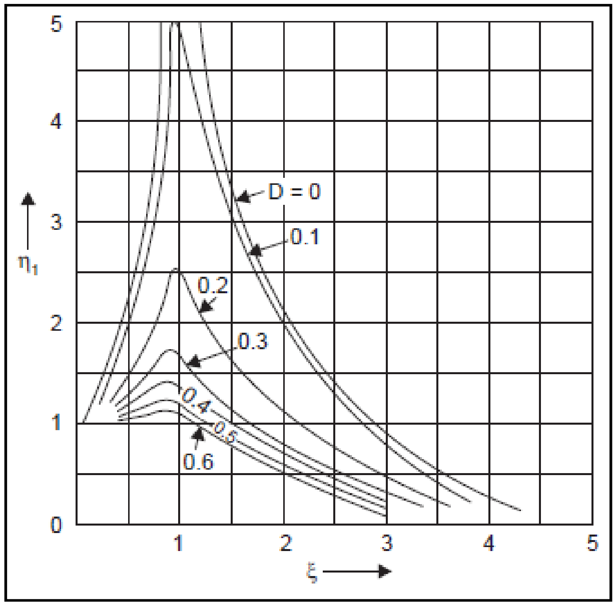


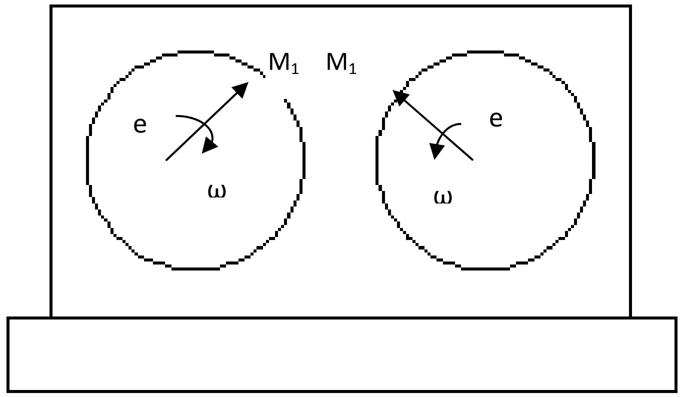
Fig. 9.15: Magnification factor vs. frequency ratio

It can be observed that the maximum value of η_1, and hence the peak amplitude occurs at a value of ξ less than unity when damping is present. As the Damping ratio, *D* increases the value of ξ for peak amplitude deviates more from unity. The corresponding frequency at which peak amplitude occurs at a certain value of damping is known as the resonant frequency for the damped case.

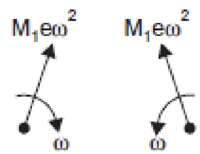
It may be recalled that, without damping, the peak amplitude which occurs when $\xi = 1$, is infinite. The effect of damping is to make the peak amplitude finite and make the frequency ratio for peak amplitude deviate from unity. In other words, what is called the resonant frequency is different in the undamped and damped cases.

9.9 Quadratic excitation

In this type of excitation, the exciting force is proportional to the square of the frequency. This is caused by the rotation of unbalanced masses (Fig. 9.16) in an oscillator.



(a) Rotation of unbalanced masses



(b) Counteracting forces

Fig. 9.16: Quadratic excitation due to rotation of unbalanced mass

The exciting moment, M_e, may be varied by varying either the total unbalanced mass M_e or the eccentricity e. The periodic force is not constant unlike the previous case. The rotating force of each mass is $M_1e\omega^2$. The total force in the vertical position is $2M_1e\omega^2$ or $M_ee\omega^2$ where $M_e$is the total unbalanced mass (equal to 2 M_1). The vibrating force at any position may be represented by

$$P = M_e e\omega^2 \sin \omega t = \overline{P_0} \sin \omega t \qquad \text{(Eq.9.51)}$$

$$\overline{P_0} = M_e e\omega^2 \qquad \text{(Eq.9.52)}$$

The periodic force is expressed by Eq. 9.16 replacing P_0 by $\overline{P_0}$ for frequency-dependent exciting force;

$$P = \overline{P_0} \sin \omega t \qquad \text{(Eq. 9.53)}$$

We may write

$$\frac{\overline{P_0}}{k} = \frac{M_e e\omega^2}{k} = \left(\frac{M_e e}{M}\right)\left(\frac{M}{k}\right)\omega^2 = \frac{M_e e}{M}\left(\frac{\omega}{\omega_n}\right)^2 = \frac{M_e e}{M}\xi^2 \qquad \text{(Eq. 9.54)}$$

The differential equation of motion and its solution are the same as those in the previous case in as much as these are independent of the method of applying the exciting force. The amplitude may be got as follows, using Eq. 9.46, substituting P_0 for $\overline{P_0}$, and using Eq. 9.54, and simplifying further:

$$A = \frac{M_e e}{M} = \frac{\xi^2}{\sqrt{\left(1-\xi^2\right)^2 + 4D^2\xi^2}} \qquad \text{(Eq. 9.55)}$$

Analyzing in the same manner as in the previous case, the maximum amplitude occurs when

$$\xi = \frac{1}{\sqrt{1-2D^2}} \qquad \text{(Eq. 9.56)}$$

From this, it can be seen that as D decreases, ξ decreases and vice-versa. Defining magnification factor η_2 as;

$$\eta_2 = A.\frac{M}{M_e e} = \frac{\xi^2}{\sqrt{\left(1-\xi^2\right)^2 + 4D^2\xi^2}} \qquad \text{(Eq. 9.57)}$$

That means $\eta_2 = \eta_1 \xi^2$ (Eq. 9.58)

It can also be shown that, $\eta_{2_{max}} = \frac{1}{2D\sqrt{1-D^2}}$ (Eq. 9.59)

The relationship between ξ & η_2 (or A) for different values of D is shown in fig. 9.17.

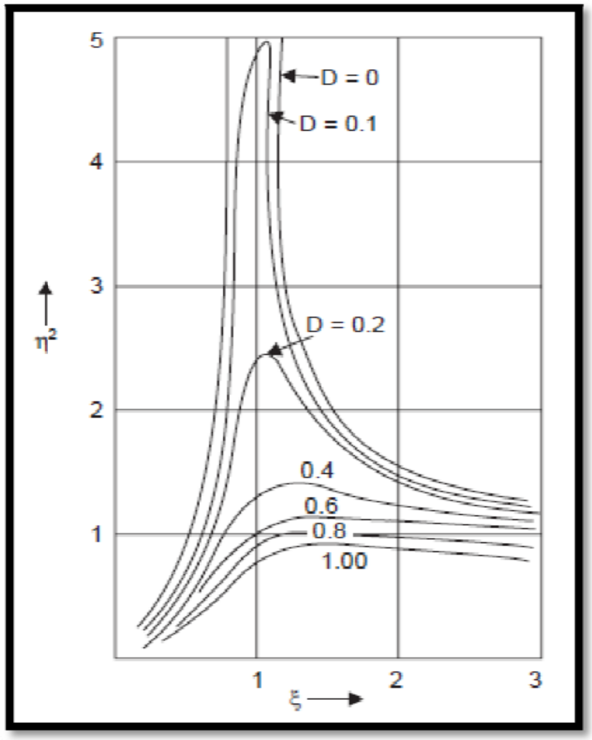


Fig. 9.17: η_2 versus ξ

It can be seen from this figure that for quadratic excitation the maximum value of η_2 (or A) occurs at a value of ξ greater than unity when damping is present. As the value of D increases, the peak η_2 (or A) deviates more from $\xi = 1$. Thus resonant conditions tend to occur at a frequency ratio more than unity.

In this case also, the effect of damping is to make the peak amplitude finite and to make the ξ-value corresponding to the peak amplitude deviate from unity. It can also be seen that the influence of damping is large when resonance condition occurs and it decreases when the amplitude of motion is different from the peak amplitude; the greater the difference the smaller the influence of damping ratio.

9.10 Machine Foundations—Special Features

Machine foundations, being of a special kind, fall into a separate class of their own. For example, the general criteria for ensuring stability of a machine foundation are rather different from those for other foundations. Also the design approach and methods of analysis are totally different in view of the dynamic nature of the forces. The types of machine foundations are also different.

Responsibility for satisfactory performance of a machine is divided between the machine designer, who is usually a mechanical engineer, and the foundation designer, who is usually a civil engineer, more specifically a geotechnical engineer. The latter's task is to design a suitable foundation consistent with the requirements and tolerance limits imposed by the machine designer. It is therefore imperative that the machine designer and the geotechnical engineer work in close co-ordination right from the stage of planning until the machinery is installed and commissioned for its intended use.

Until recently, design of machine foundations has been mostly based on empirical rules, before the evolution of Soil Dynamics as a discipline. With the developments in the fields of structural and soil dynamics, sound principles of design were gradually established. The relevant aspects with regard to the design of machine foundations will be dealt with in the subsections that follow.

9.10.1 Types of Machines and Machine Foundations

Machines may be classified as follows, based on their dynamic effects and the design criteria:

(*i*) Those producing periodical forces—reciprocating machines or engines, such as compressors.

(*ii*) Those producing impact forces—forge hammers and presses.

(*iii*) High speed machines such as turbines and rotary compressors.

(*iv*) Other miscellaneous kinds of machines.

Based on their operating frequency, machines may be divided into three categories:

(*a*) Low to medium frequency machines up to 500 rpm: Large reciprocating engines, compressors, and blowers fall in this category. Usual operating frequencies range from 50 to 250 rpm.

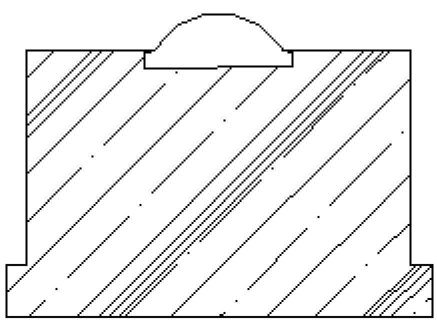
(*b*) Medium to high frequency machines- 300 to 1000 rpm. Medium-sized reciprocating engines such as diesel and gas engines come under this category.

(*c*) Very high frequency machines-greater than 1000 rpm: High-speed internal combustion engines, electric motors, and turbo-generators fall in the category.

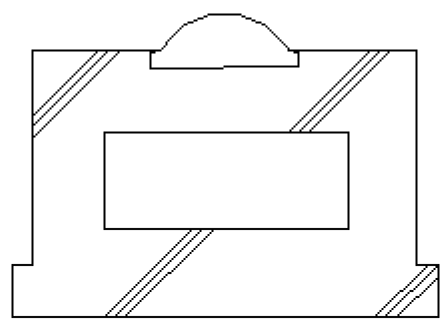
Machine foundations are generally classified as follows, based on their structural form:

- I–Block-type foundations, consisting of a pedestal of concrete on which the machine rests (Fig. 9.18 (*a*)). Reciprocating machinery falling into category (*a*) above are supported on block-type foundations with large contact area with the soil.
- Reciprocating machinery of category (*b*) above may also be supported on block-type foundations, but these are made to rest on springs or suitable elastic pads to reduce their natural frequencies.
- High-speed machinery of category (*c*) above may also be supported on massive block foundations; small contact surfaces with suitable isolation pads are desirable to reduce the natural frequencies.
- II–Box or caisson type foundations, consisting of a hollow concrete block (Fig 9.18 (*b*)). III–Wall-type foundations, consisting of a pair of walls which support the machinery on their top (Fig. 9.18 (*c*))
- IV–Framed-type foundations, consisting of vertical columns supporting on their top a horizontal frame work which forms the seat of essential machinery (Fig. 20.26 (*d*)).
- Turbo-machinery requires this type of foundations, which accommodate the necessary auxiliary equipment between the columns.

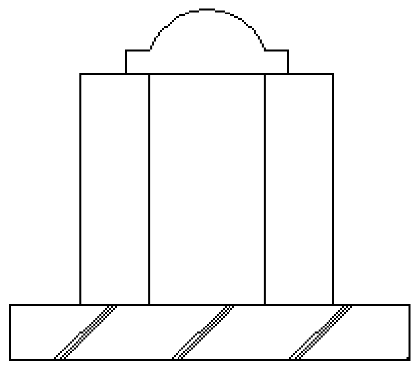
- Some machines such as lathes, which induce very little dynamic forces, do not need any foundations; such machines may be directly bolted to the floor.



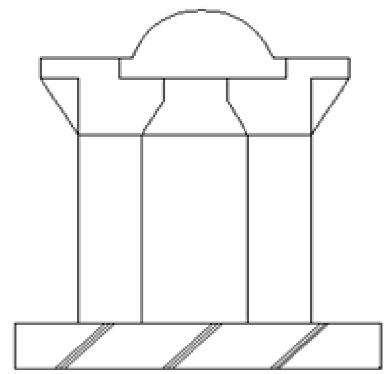
(a) Block-type



(b) Box or caisson-type



(c) Wall Type



(d) Framed-type

Fig. 9.18: Types of machine foundations

9.10.2 General Criteria for Design of Machine Foundations

The following criteria should be satisfied by a machine foundation:

i) The foundation should be able to carry the superimposed loads without causing shear failure. The bearing capacity under dynamic loading conditions is generally considered to be less than that for static loading, the reduction factor ranging from0.25 to 1.0.

ii) The settlement should be within permissible limits.

iii) The combined centre of gravity of machine and foundation should be, to the extent possible, in the same vertical line as the centre of gravity of the base line.

iv) Resonance should be avoided; hence the natural frequency of the foundation-soil system should be far different from the operating frequency of the machine. For low-speed machines, the natural frequency should be high and vice-versa. The operating frequency must be either less than 0.5 times or greater than 1.5 times the resonant frequency so as to ensure adequate margin of safety.

v) The amplitude under service conditions should be within the permissible limits, generally prescribed by the manufacturers.

vi) All rotating and reciprocating parts of the machine should be so balanced that the unbalanced forces and moments are minimised. (This, of course, is the responsibility of the mechanical engineers).

vii) The foundation should be so planned as to permit subsequent alteration of natural frequency by changing the base area or mass of the foundation, if found necessary subsequently.

From the practical point of view, the following additional requirements should also be fulfilled:

viii) The ground water table should be below the base plane by at least one-fourth of the width of the foundation. Since ground-water is a good conductor of waves, this limits the propagation of vibration.

ix) Machine foundations should be separated from adjacent building components by means of expansion joints.

x) Any pipes carrying hot fluids, if embedded in the foundation, must be properly isolated.

xi) The foundation should be protected from machine oil by means of suitable chemical treatment, which is acid-resistant.

xii) Machine foundations should be taken to a level lower than the level of foundations of adjoining structures. In this connection, it is perhaps pertinent to remember Richart's chart.

9.10.3 Design Approach for Machine Foundation

The dimensions of machine foundations are fixed according to the operational requirements of the machine. The overall dimensions of the foundation are generally specified by the manufacturers of the machine. If there is choice to the foundation designer, the minimum possible dimensions satisfying the design criteria should be chosen.

Once the dimensions of the foundation are decided upon, and site conditions are known, the natural frequency of the foundation-soil system and the amplitudes of motion under operating conditions have to be determined.

The requirements specified in the previous subsection should be satisfied to the possible extent for a good design. Thus, the design procedure is one of 'trial and error'. The specific data required for design vary for different types of machines. However, certain general requirements of data may be given as follows:

i) Loading diagram, showing the magnitudes and positions of static and dynamic loads exerted by the machine.

ii) Power and operating speed of the machine.

iii) Line diagram showing openings, grooves for foundation bolts, details of embedded parts, and so on.

iv) Nature of soil and its static and dynamic properties, and the soil parameters required for the design.

9.10.4 Vibration Analysis of a Machine Foundation

Although the machine foundation has six-degree freedom, it is assumed to have single degree of freedom for convenience of simplifying the analysis Fig. 9.19 shows a machine foundation supported on a soil mass.

M_f is the lumped mass of the machine and of the foundation, acting at the centre of gravity of the system. Along with M_f, a certain mass, M_s, of soil beneath the foundation will participate in the vibration. The combined mass M (the sum of M_f and M_s) is supposed to be lumped at the centre of gravity of the entire system.

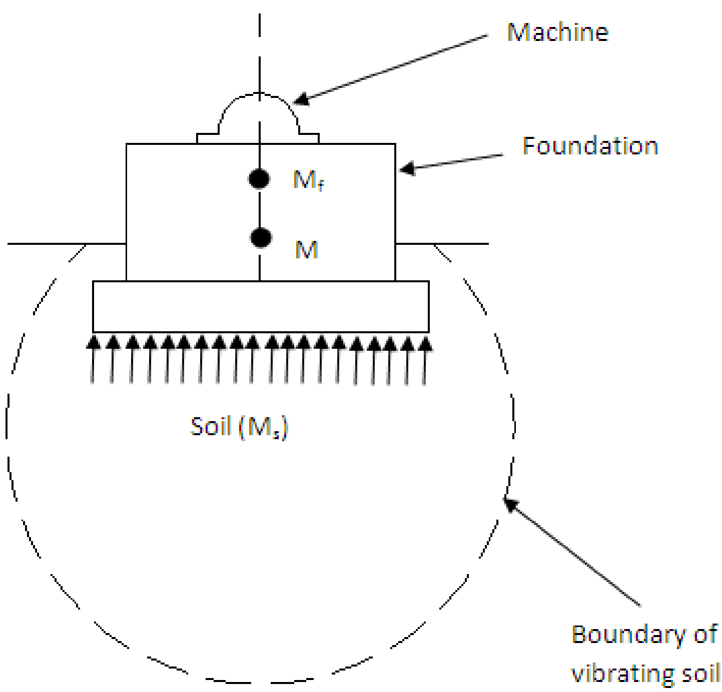


Fig. 9.19: Machine foundation-soil system

The system is taken to be undergoing purely vertical vibrations and thus considered to be a system with single degree of freedom. The vibration analysis of a machine foundation may be performed based on either one of the broad approaches, namely, the Elastic Half-space theory and the Mass-spring-dash pot model. Depending upon the approach selected the values of the appropriate soil parameters have to be determined by a suitable method.

However, it may be noted that, unfortunately, there is no rational method to determine the magnitude of the mass of soil participating in the vibration. A general guideline is to choose this value to be ranging between zero and the magnitude of the mass of machine and of the foundation. In other words, the total mass, M, is taken to be varying between M_f and 2 M_f in most cases.

Empirical approaches, based on different criteria such as type, speed or power of the machine have been advanced by some research workers; however, all such approaches may now be considered to be obsolete.

9.11 Elastic Half Space Theory

In this theory, a rigid body of known mass is taken to rest upon the surface of an ideal soil, i.e., elastic, homogeneous, and isotropic material. It is termed 'half-space' because the soil is assumed to extend infinitely in all directions including the depth, with the top surface as a boundary. For mathematical convenience, the foundation/footing is taken to be of circular shape. The basic soil parameters used in the development of the theory are the shear modulus, G, the mass density, ρ, and the Poisson's ratio, ν.

The elastic half-space theory may be used to predict the resonant frequency and the peak amplitude of motion of the system from a single field vibration test. Although the theory does not explicitly take into account the damping effect of the system, the amplitudes obtained are finite, indicating that the effect is considered, indirectly (In fact, the nature of damping in this case may be 'radiation' and/or 'internal'). Further, contact pressure distribution is required in the analysis.

Reissner (1936) presented an analytical solution for vertical vibrations of a circular disc resting on an elastic half-space; he considered the contact pressure distribution to be uniform. Reissner was the first to use elastic half-space theory for soil dynamics problems.

Quinlan (1953) and Sung (1953) gave independently mathematical solutions for three types of contact pressure distributions, *viz.,* uniform, parabolic, and rigid base distributions. (Note:-Parabolic distribution means zero pressure at the edge with maximum at the centre, the distribution across a diameter being parabolic; rigid base distribution means infinite pressure at the edge with minimum finite value at the centre, the distribution being parabolic again).

Sung's approach involves the use of the data from a single field vibration test to determine dimensionless parameters for peak amplitude and for resonant frequency, along with another dimensionless parameter, called 'mass ratio'. Sung presented design charts for machine foundations based on his work. His dimensionless parameters are used to predict the response of a proposed machine foundation at the particular site where the single field vibration test has been conducted. Subrahmanyam (1971) has extended Sung's work. Richart and Whitman (1967) have concluded that the elastic half-space theory is qualitatively satisfactory, by analysing vast test data from the U.S. Army Engineer Waterways Experiment Station, and also their own test data. A number of other investigators have also come up with their own solutions, but these are beyond the scope of this book. Readers who are interested may refer Richart et al (1970).

9.12 Mass-Spring-Dashpot Model

The mass-spring-dashpot model, or the 'lumped parameter system', has been widely used to predict machine foundation response for vertical vibrations as well as other modes of vibration, including coupled modes. In this approach also, the soil is assumed to be an ideal material, on the surface of which a machine foundation rests. The soil has been characterised as a linear weightless spring, in which damping is present. Although it is well known that the damping effect of soil is due to radiation and internal loss of energy, it is considered to be viscous damping for mathematical convenience. Thus, the theory of free, and particularly forced, vibrations with damping is used to analyse the behaviour of machine foundation.

Since the spring is considered weightless for mathematical convenience, but the soil has weight, the results from this analysis do not exactly match the experimental values obtained for a machine foundation. But this model may be considered as a first approximation to the machine foundation-soil system (Sankaran and Subrahmanyam, 1971). Non-linear models have also been proposed by some investigators to simulate the nonlinear constitutive relationship of soil, but no effective solution has been given to evaluate the nonlinear stiffness of the spring.

Pauw (1953) considered the soil as a truncated pyramid extending to infinite depth; he tried to evaluate the effect of the spring constant on the size and shape of control area and the effect of variation of soil modulus with depth. He assumed the soil modulus to increase linearly with depth for cohesionless soils, while it is taken to be a constant with depth for cohesive soils.

A modified mass-spring dashpot model, involving the use of the mass of soil participating in the vibration in the evaluation of the spring constant, k_z, and the damping ratio, D, has been proposed by Subrahmanyam (1971).

9.13 Foundations for Reciprocating Machines

Reciprocating engines having crank-type mechanism include steam engines, diesel engines, displacement compressors, and displacement pumps. Vibrations are caused due to conversion of rotary motion to linear motion. Reciprocating machines may operate either vertically or horizontally. These may have three modes of vibration-vertical, sliding, and rocking. Generally block-type foundations (with openings where necessary for functional reasons) are provided for reciprocating machinery. The main problem in the design is to successfully evaluate the unbalanced inertial forces from the mechanical details of the engine.

9.13.1 Design Criteria

The principal design criteria for foundations for reciprocating machinery are as follows:

i) The natural frequency should be at least 30 percent away from the operating speed of the machine.
ii) The amplitude of motion of the foundation should not exceed 0.2 mm.
iii) The pressure on soil (or other elastic layers such as cork, springs, etc., where used) should be within the respective permissible values. For preliminary design, the maximum pressure on soil due to static load alone may be taken as 0.4 times the corresponding safe bearing capacity.

The design data to be supplied by the manufacturer of the machine include the following:

i) Normal speed and power of engine.
ii) Magnitude and position of static loads of the machine and the foundation.
iii) Magnitude and position of dynamic loads which occur during the operation of the machine; alternatively, the designer should be supplied with all the data necessary for computing such forces.

iv) Position and size of openings provided in the foundation for anchor bolts, pipe line, flywheel, etc.

v) Any other specific information considering the special nature of the machine. These may include permissible differential settlements, permissible amplitudes of motion, etc. The relevant IS Code–IS: 2974-Pt I-1982 (Revised)-contains further details.

9.13.2 Calculation of Unbalanced Inertial Forces

A simple crank mechanism for a single-cylinder engine is shown in Fig. 9.20: It consists of a piston which moves inside a cylinder, a crank which rotates about a point *O* and a connecting rod which is attached to the piston at point *P* (known as "wrist pin")and to the crank shaft at point *C* (known as "crank pin"). The crank pin follows a circular path while the wrist pin oscillates along a linear path. Points on the connecting rod between *P* and *C* follow an elliptical path. Designating the total reciprocating mass which moves with the piston as M_{rec} and the rotating mass moving with the crank as M_{rot}, the unbalanced inertial forces P_z (along the direction of the piston) and P_x (along a perpendicular direction) may be written as;

$$P_z = \left(M_{rec} + M_{rot}\right)R\omega^2 \cos \omega t + M_{rec} \frac{R^2\omega^2}{L} \cos 2\omega t \qquad \text{(Eq. 9.60)}$$

$$P_x = M_{rot} R\omega^2 \sin \omega t \qquad \text{(Eq. 9.61)}$$

Here ω is the angular velocity and *R* is the radius of the crank.

For the simple crank-mechanism shown, the reciprocating and rotating masses are given by the following equations:

$$M_{rec} = M_2 + M_3\left(\frac{L_1}{L}\right) \qquad \text{(Eq. 9.62)}$$

$$M_{rot} = M_1 \frac{R_1}{R} + M_3\left(\frac{L_2}{L}\right) \qquad \text{(Eq. 9.63)}$$

where M_1 = mass of crank,

M_2 = mass of reciprocating parts, *i.e.,* piston, piston rod, and crank-head,

M_3 = mass of connecting rod,

L = length of connecting rod,

L_1 = distance between the *CG* of the connecting rod and the crank pin *C*,

L_2 = distance between the *CG* of the connecting rod and the wrist pin *P*,

and R_1 = distance between the *CG* of the crank shaft and the centre of rotation.

(These equations are based on a simplifying assumption regarding the distribution of the mass of crank shaft).

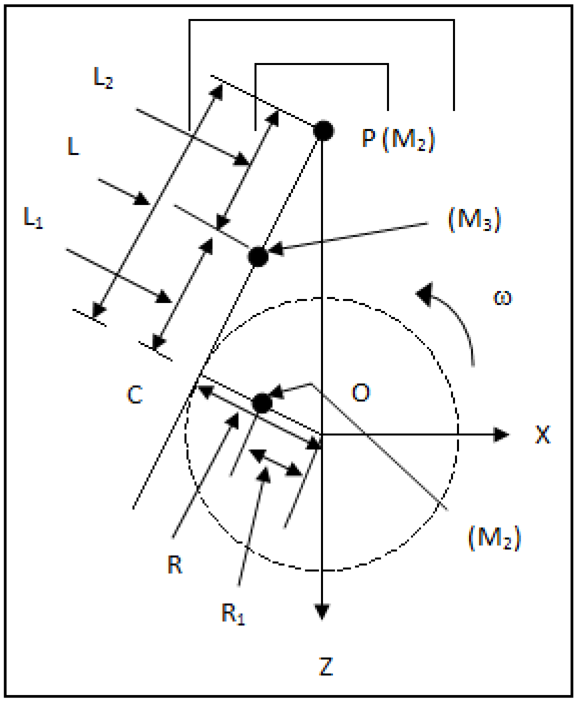


Fig. 9.20: Simple crank-mechanism

The first term for P_z in Eq. 9.60 involving cos ω*t* (first harmonic) is called the "primary" inertial force and the second involving cos 2ω*t* (second harmonic) is called the "secondary" inertial force. Generally speaking, the contribution to the total inertial force from the secondary components (due to second and higher harmonics) is considered negligible compared with that from the primary component.

The inertial force due to rotating masses may be eliminated by what is known as "counter-balancing"; however, that due to reciprocating mass cannot be avoided. The above analysis is applicable only for reciprocating machines with a single cylinder. But most machines have more than one cylinder, that is to say, most machines are multi-cylinder engines, all cylinders being usually housed in single plane. The analysis can be extended to a multi-cylinder engine and the unbalanced inertial forces may be derived as follows:

$$P_z = \left(M_{rec} + M_{rot} R\omega^2 \sum_{i=1}^{n} \cos(\omega t + \alpha_i) \right) \text{ [Neglecting secondary inertial forces]} \qquad \text{(Eq.9.64)}$$

$$P_x = M_{rot} R\omega^2 \sum_{i=1}^{n} (\sin \omega t + \alpha_i) \qquad \text{(Eq.9.65)}$$

where α_i = the angle between the crank of the *i*-th cylinder and that of the first cylinder (this is known as the "crank angle" of the *i*-th cylinder) and *n* = number of cylinders.

(Note:- Moments of these inertial forces about the relevant centroidal axis of the machine may be determined if the exact relative locations of the engines, and hence the lever arms, are known).For a vertical two-cylinder engine, for example, the resultant unbalanced inertial forces for different crank angles may be obtained as follows:

Crank Angle $\frac{\pi}{2}$ (or phase difference is $\frac{\pi}{2}$)

This is the most common case.

$$P_{z_1} = (M_{rec} + M_{rot}) R\omega^2 \cos\omega t \text{ (approx.)}$$

$$P_{z_2} = (M_{rec} + M_{rot}) R\omega^2 \cos\left(\omega t + \frac{\pi}{2} \right)$$

Therefore, $$P_z = P_{z_1} + P_{z_2} = (M_{rec} + M_{rot}) R\omega^2 \left[\cos\omega t + \cos\left(\omega t + \frac{\pi}{2} \right) \right]$$

Or, $$P_z = (M_{rec} + M_{rot}) R\omega^2 \cos\left(\omega t + \frac{\pi}{4} \right) \qquad \text{(Eq.9.66)}$$

Similarly, $$P_{x_1} = M_{rot} R\omega^2 \sin\omega t$$

$$P_{x_2} = M_{rot} R\omega^2 \sin\left(\omega t + \frac{\pi}{2} \right)$$

Therefore $$P_x = P_{x_1} + P_{x_2} = M_{rot} R\omega^2 \left[\sin\omega t + \sin\left(\omega t + \frac{\pi}{2} \right) \right]$$

Or, $$P_x = \sqrt{2} M_{rot} R\omega^2 \sin\left(\omega t + \frac{\pi}{4} \right) \qquad \text{(Eq. 9.67)}$$

It may be noted that the peak values of the inertial forces in this case are $\sqrt{2}$ times those for a single cylinder engine. The moments about the principal axes may be easily obtained if the exact positions of the cylinders and lever arms are known in a given case.

Crank Angle π

The resultant inertial forces P_z and P_x are obviously zero in this case. However, the moments may not be zero and have to be computed.

Crank Angle 3π/2

The resultant inertial forces are

$$P_z = \sqrt{2}(M_{rec} + M_{rot})R\omega^2 \sin\left(\omega t + \frac{\pi}{4}\right) \qquad \text{(Eq. 9.68)}$$

$$P_x = \sqrt{2}M_{rot}R\omega^2 \sin(\omega t - \frac{\pi}{4}) \qquad \text{(Eq. 9.69)}$$

Crank Angle 2π (Cranks in parallel directions)

The resultant inertial forces in this case are

$$P_z = 2(M_{rec} + M_{rot})R\omega^2 \cos \omega t \qquad \text{(Eq. 9.70)}$$

$$P_x = 2M_{rot}R\omega^2 \sin \omega t \qquad \text{(Eq. 9.71)}$$

These are double the respective forces for a single-cylinder engine.

(Note:-Corresponding expressions for the unbalanced inertial forces may be derived for different sets of crank angles for vertical reciprocating machines with three, four, and six cylinders, using the same principles).

Similar treatment is applicable even for horizontal reciprocating machines, except that *x* and *z* have to be interchanged; horizontal machines are generally two-cylinder engines with 90°-crank angle.

If the engines have auxiliary cylinders such as a compressor and an exhaust, the loads imposed by the auxiliaries should also be considered; however, these are usually small, relatively speaking, and hence may be ignored. If vibration absorbers such as springs are interposed between the machine and the foundation in order to limit the amplitudes, the system has to be considered at least as one with two-degree freedom depicted in Fig. 9.5, 9.6, 9.7 although strictly speaking, its degree of freedom is twelve.

9.14 Foundations for Impact Machines

Hammers are typical examples of impact type machines. The design principles of foundations for hammers are entirely different from those for reciprocating machinery. In a hammer, a ram falls from a height on the anvil executing either forging or stamping a material placed on the anvil.

Hammer foundations are generally reinforced concrete block type of construction (Fig. 9.21). The anvil on which the tup falls repeatedly is usually placed on an elastic layer which may be of timber grillage or cork. The foundation may be placed directly on soil or on a suitable elastic layer, the purpose of which is to isolate the vibration and minimise the harmful effects of the impact.

The frame of the hammer may either rest directly on the foundation or it may be supported from outside depending on the convenience. The frame is essentially to guide the ram and house the arrangement for the movement of the ram. The practice of design of the foundation for hammers has been to provide a massive block foundation.

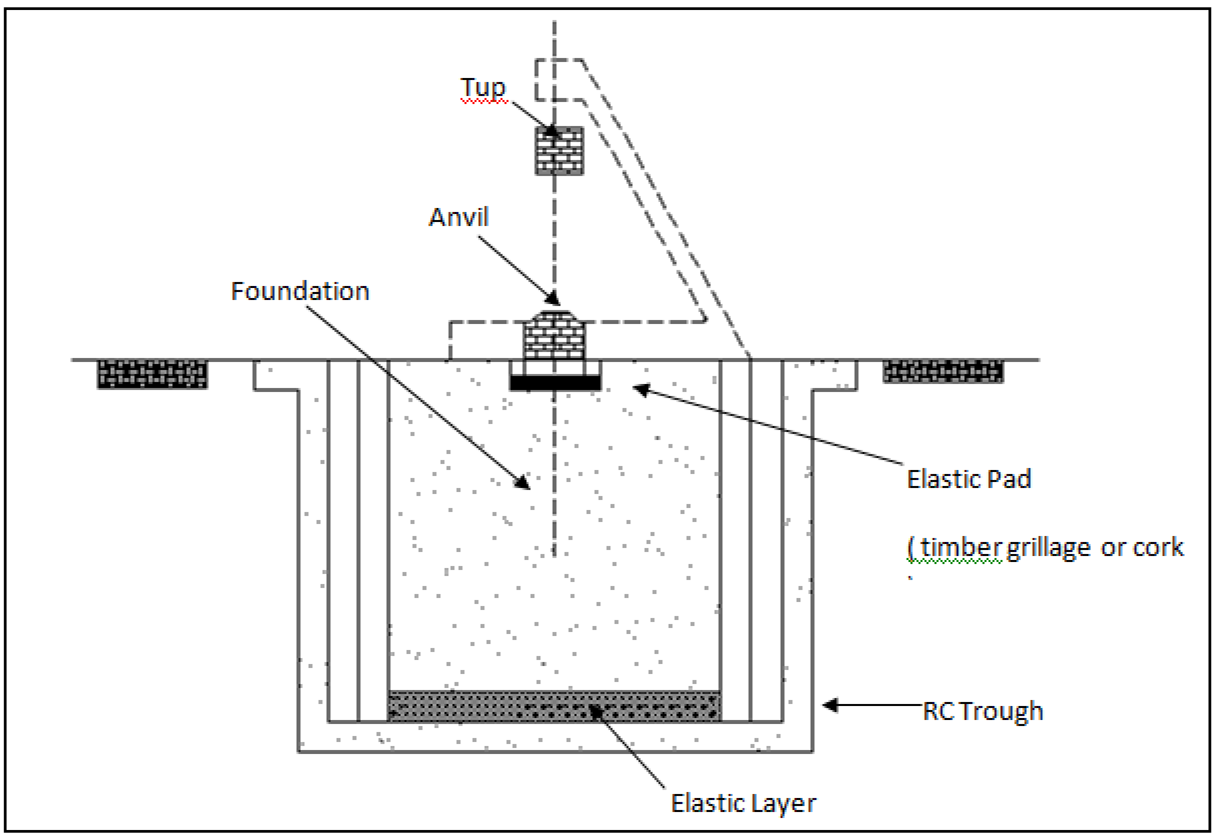


Fig. 9.21: Schematic of a typical hammer foundation (IS 2974-1980)

9.14.1 Special Considerations

The following are the special considerations in planning the foundation for impact machines:

i) The centre line of the anvil and the centroid of the base area should lie on the vertical line passing through the common centre of gravity of the machine and its foundation.

ii) Where elastic pad is used under the anvil and the base of the foundation, care should be taken to ensure uniform distribution of loading and protection of the pad against water, oil, etc. It is recommended that the foundation be laid in a reinforced concrete trough formed by retaining walls on all sides. The foundation may be separated from the side walls by means of an air gap.

iii) If timber is used for elastic pad, the timber joists should be laid horizontally in the form of a grillage. The joists must be impregnated with preservative for protection against moisture.

iv) The thickness of the elastic pad is governed by the permissible stresses in the respective materials. Guidelines in this regard are given in Table 9.1 (Major, 1962):

v) When two adjacent foundations are laid at different depths, the straight line connecting edges should form an angle not exceeding 25^0 to the horizontal (fig. 9.22). However, if foundations are too close, they may be laid to the same depth and a common raft provided as base.

Table 9.1 Thickness of timber pads under anvil (after Major, 1962)

Type of Hammer	Thickness of pad (m) for a falling weight of		
	Upto 10 kN	10-30 kN	30 kN
Double acting drop hammers	0.2	0.2 to 0.6	0.6 to 1.2
Single acting drop hammers	0.1	0.1 to 0.4	0.4 to 0.9
Forge hammers	0.2	0.2 to 0.6	0.6 to 1.0

9.14.2 Design Data

The following data are required to be supplied to the designer:

(i) Type of hammer

(ii) Weight of falling tup (W_t)

(iii) Weight of anvil (W_a)

(iv) Weight of the hammer stand supported on the foundation (W_f), to be added to W_a only if the stand is directly resting on the foundation.

(v) Base area of anvil ($L_a \times B_a$)

(vi) Stroke or fall of hammer (h)

(vii) Effective working pressure (p) on the piston and area of piston (a)

(viii) Outline of the foundation showing the position of anchor bolts, floor level, position of adjacent foundations, etc.

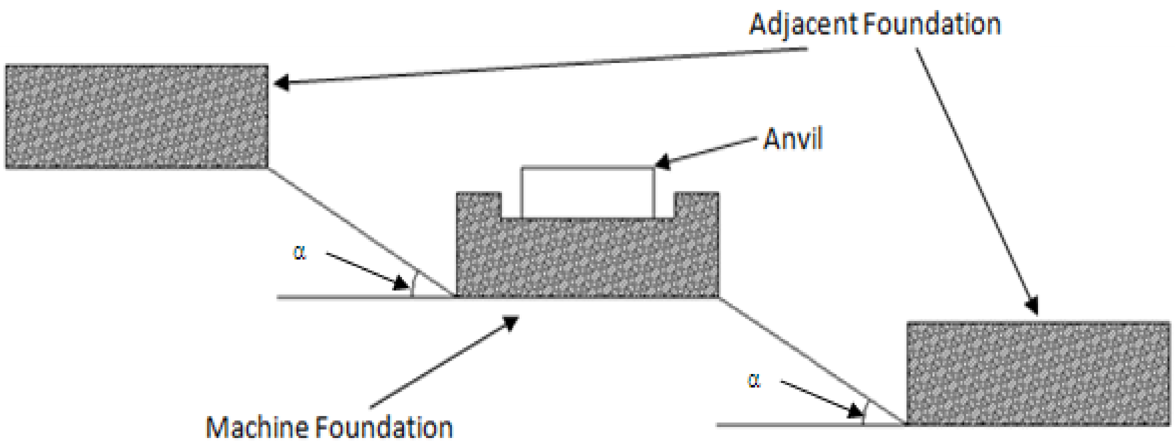


Fig. 9.22 Criteria for locating adjacent foundations (IS: 2974- Pt II-1980)

9.14.3 Elastic Pad under the Anvil

The thickness of elastic pad varies with the weight of the dropping parts and the type of hammer from about 200 mm for 10 kN hammer to a maximum of about 200 mm for hammers of over 30 kN (Table 9.1).

The thickness of the pad should be so selected that the dynamic stresses induced in the pad by impact do not exceed the permissible values, which are as follows (Barkan, 1962):

Oak: 30,000 to 35,000 kN/m^2

Pine: 20,000 to 25,000 kN/m^2

Larch: 15,000 to 20,000 kN/m^2

9.14.4 Velocity of Anvil

The velocity of anvil after impact is required to be determined for the dynamic analysis of a hammer foundation. This may be obtained as follows:

Velocity of Tup before Impact

For free fall hammer, the velocity v before impact is given by

$$v = \alpha\sqrt{2gh} \qquad \text{(Eq. 9.72)}$$

where, h = height of fall, and α = correction factor which characterises the resistance of exhaust steam (α = 1 for well adjusted hammer according to Barkan, 1962).

For double-acting hammer, v is given by

$$v = \alpha\sqrt{\frac{2g(W_t + pa)h}{W_t}} \qquad \text{(Eq. 9.73)}$$

where, W_t= weight of tup, p = pressure on the piston, a = area of piston, h = stroke, and α = correction factor which varies from 0.5 to 0.8. Barkan (1962) recommends an average value of 0.65.

9.14.5 Velocity of Tup and Anvil after Impact

Let v be the velocity of tup before impact, v_1 be the velocity of tup after impact, and v_a be the velocity of anvil after impact. (It may be remembered the velocity of the anvil and foundation is zero before impact).

Applying the principle of conservation of momentum

$$M_t v = M_t v_1 + M_a v_a \qquad \text{(Eq. 9.74)}$$

where M_t= mass of tup, and

M_a= mass of anvil (including the weight of frame, if mounted on it).

Another equation is obtained by using Newton's hypothesis concerning the restitution of impactwhich states that "the relative velocity after impact is proportional to that before impact". The ratio between these two, which is known as the coefficient of elastic restitution (e) depends only on the materials of the bodies involved in the impact. Therefore we may write

$$e = \frac{(v_a - v_1)}{v}$$

or, $$v_1 = v_a - ev \qquad \text{(Eq.9.75)}$$

Substituting for v_1 in Eq. 9.74 and simplifying

$$v_1 = \frac{(1+e)}{(1+\lambda_a)}v \qquad \text{(Eq.9.76)}$$

$$\text{where } \lambda_a = \frac{M_a}{M_t} \qquad \text{(Eq.9.77)}$$

This analysis is applicable for a 'central blow' or 'centred impact' as it is called. For an 'eccentric blow' or 'eccentric impact' the moment of momentum equation also has to be used in addition to the two equations used for central blow. Proceeding on similar lines, one obtains the following equations for the initial velocity of anvil after impact (v_0) and the initial angular velocity after impact (ω_0):

$$v_0 \frac{(1+e)}{\left(1+\frac{M}{M_t}+\frac{e_1^{\,2}}{i^2}\right)} v \qquad \text{(Eq.9.78)}$$

$$\text{and,} \quad \omega_0 = \frac{(1+e)e_1}{i^2\left(1+\frac{M_a}{M_t}\right)+e_1^{\,2}} \qquad \text{(Eq.9.79)}$$

where e_1 = eccentricity of blow or impact

and, $i^2 = \frac{l_m}{M_a} I_m$ is the mass moment of inertia of the moving system about the axis of rotation.

The coefficient of restitution, e, is unity for perfectly elastic bodies and zero for plastic bodies. For real bodies, e lies between zero and one. Barkan (1962) observed from his experiments that the value of e does not exceed 0.5. Since higher values of e lead to greater amplitudes of motion, Barkan recommends that a value of 0.5 be chosen for hammers stamping steel parts. Values of e for large hammers proper are much smaller than those for stamping hammers, and corresponding design value may be taken as 0.25.

For hammers forging nonferrous metals, e is considerably smaller, and may be considered to equal zero (Barkan, 1962).

9.14.6 Dynamic Analysis of Foundation for Impact Machines

The hammer-anvil-pad-foundation-soil system is assumed to have two degrees of freedom. The elastic pad is taken to be an elastic body with spring constant k_2 and the soil below the foundation another elastic body with a spring constant k_1.

This model for dynamic analysis is shown in Fig. 9.23. The impact caused by the ram (tup) of the hammer causes free vibrations in the system. Since the soil has also damping effect, the system undergoes free vibrations with damping. The equations of motion may be written as follows using Newton's laws or D' Alembert's Principle:

$$M_f \ddot{Z}_1 + k_1 Z_1 - k_2(Z_2 - Z_1) = 0 \quad \text{(Eq.9.80 (a))}$$

and

$$M_a \ddot{Z}_2 + k_2(Z_2 - Z_1) = 0 \quad \text{(Eq.9.80 (b))}$$

where

Z_1, Z_2 = displacements of foundation and anvil from their equilibrium positions,

M_f = mass of the foundation,

k_1 = soil spring constant.

k_2 = spring constant of elastic pad

also $k_1 = C_u' A_1$ (Eq. 9.81)

and $k_2 = \frac{E_p A_p}{t_p}$ (Eq. 9.82)

where C_u' = coefficient of elastic uniform compression of soil under impact,

A_1 = contact area of foundation,

A_p = base area of pad,

E_p = Young's modulus of the material of the pad,

and t_p = thickness of pad.

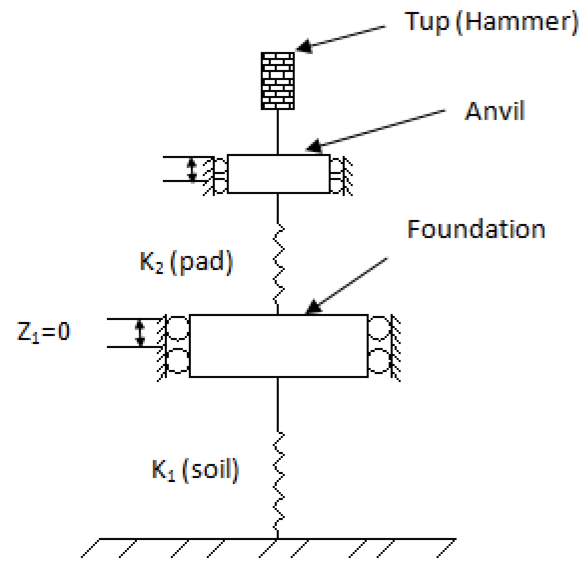


Fig. 9.23: Model for analysis of a hammer foundation

Starting with possible solutions for Z_1 and Z_2

such as $$Z_1 = C_1 \sin \omega_n t$$

and $$Z_2 = C_2 \sin \omega_n t$$

C_1 and C_2 being constants, and substituting in the differential equations of motion (Eq. 9.80), and simplifying, it is possible to develop a frequency equation of the fourth degree as follows:

$$\omega_n^{\ 4} - (1+\lambda_1)(\omega_a^{\ 2} + \omega_l^{\ 2})\omega_n^{\ 2} + (1+\lambda_1)\omega_a^{\ 2}\omega_l^{\ 2} = 0 \qquad \text{(Eq. 9.83)}$$

where $$\lambda_1 = \frac{M_a}{M_f} \qquad \text{(Eq. 9.84)}$$

$$\omega_n^{\ 2} = \frac{k_2}{M_a} = \frac{E_p A_p}{t_p M_a} \qquad \text{(Eq. 9.85)}$$

$$\omega_l^{\ 2} = \frac{k_1}{(M_a + M_f)} = \frac{C_u' A_1}{(M_a + M_f)} \qquad \text{(Eq. 9.86)}$$

Here, ω_a is the limiting natural frequency of the anvil, assuming the soil to be infinitely rigid ($k_1 = \infty$) and, ω_l is the limiting natural frequency of the entire system (anvil and foundation), assuming the anvil to be infinitely rigid ($k_2 = \infty$).The positive roots of Eq. 9.83 are designated as ω_{n1} and ω_{n2}. These may be expressed as

$$\omega_{n1,2}^2 = \frac{1}{2}\left((1+\lambda_1)(\omega_a^{\ 2} + \omega_l^{\ 2})\right) \pm \sqrt{\left((1+\lambda_1)(\omega_a^{\ 2} + \omega_l^{\ 2})\right)^2 - 4(1+\lambda_1)\omega_a^{\ 2}\omega_l^{\ 2}} \qquad \text{(Eq. 9.87)}$$

The differential equations of motion (Eq.9.80) may be solved for the known initial conditions:

when $t = 0, Z_1 = Z_2 = 0, \dot{Z}_1 = 0$ and $\dot{Z}_2 = v_a$

The solution is

$$Z_1 = \frac{\left(\omega_a^2 - \omega_{n2}^2\right)\left(\omega_a^2 - \omega_{n1}^2\right)}{\omega_a^2\left(\omega_{n1}^2 - \omega_{n2}^2\right)} v_a \left(\frac{\sin\omega_{n1}t}{\omega_{n1}} - \frac{\sin\omega_{n2}t}{\omega_{n2}}\right) \quad \text{(Eq.9.88 (a))}$$

$$Z_2 = \frac{v_a}{\left(\omega_{n1}^2 - \omega_{n2}^2\right)}\left(\frac{\left(\omega_a^2 - \omega_n^2\right)}{\omega_{n1}}\sin\omega_{n1}t - \frac{\left(\omega_a^2 - \omega_{n1}^2\right)}{\omega_{n2}}\sin\omega_{n2}t\right) \quad \text{(Eq.9.88 (b))}$$

Barkan (1962) observed from his experiments, that vibrations occurred at the lower principal frequency only, and as such, it may be assumed that the amplitude of vibrations for sin $\omega_{n1}t$ (where $\omega_{n1} > \omega_{n2}$) equals zero.

Then the approximate expressions for Z_1 and Z_2 are as follows:

$$Z_1 = -\frac{\left(\omega_a^2 - \omega_{n1}^2\right)\left(\omega_a^2 - \omega_{n2}^2\right)}{\omega_a^2\left(\omega_{n1}^2 - \omega_{n2}^2\right)\omega_{n2}} v_a \sin\omega_{n2}t \quad \text{(Eq.9.89 (a))}$$

$$Z_2 = -\frac{\left(\omega_a^2 - \omega_{n1}^2\right)}{\left(\omega_{n1}^2 - \omega_{n2}^2\right)\omega_{n2}} v_a \sin\omega_{n2}t \quad \text{(Eq.9.89 (b))}$$

The maximum amplitude of motion occurs when $\sin \omega_{n2} t = 1$, and these are;

For foundation soil system

$$A_1\left(= Z_{1\max}\right) = -\frac{\left(\omega_a^2 - \omega_{n2}^2\right)\left(\omega_a^2 - \omega_{n1}^2\right)}{\omega_a^2\left(\omega_{n1}^2 - \omega_{n2}^2\right)\omega_{n2}} v_a \quad \text{(Eq. 9.90 (a))}$$

For Anvil

$$A_a\left(= Z_{2\max}\right) = -\frac{\left(\omega_a^2 - \omega_{n1}^2\right)}{\omega_{n2}\left(\omega_{n1}^2 - \omega_{n2}^2\right)} v_a \quad \text{(Eq.9.90 (b))}$$

The stress in the elastic pad σ_p, is given by

$$\sigma_p = \frac{k_2}{A_p}\left(Z_2 - Z_1\right) \text{ Or, } \frac{k_2}{A_p}\left(A_a - A_1\right) \quad \text{(Eq.9.91)}$$

The basic model is applicable if there is a uniform contact between the elastic pad and the anvil as well as between the pad and the top surface of the foundation block. However, it has been generally observed that the contacts are not uniform by virtue of the fact that the bottom surface of anvil and the top surface of the foundation are relatively rough. Also, the hammer foundation-soil system is the case of free vibration with damping. Satisfactory solutions are not available to analyse the system as mass-spring-dash-pot-model with two-degree of freedom. Further, hammer foundations are generally embedded in the soil either partially or completely. Embedment makes the analysis rather complex. Therefore, one has to resort to the use of empirical correlations of Barkan (1962), based on his experimental investigations, to take care of the influence of damping of the system, non-uniform contact of the elastic pad, and depth of embedment.

In this connection Barkan gives the following equation:

$$C_u' = k_c C_u \qquad \text{(Eq.9.92)}$$

where, C_u' = coefficient of elastic uniform compression to be used in the design of hammer foundation, C_u= coefficient of elastic uniform compression of the soil obtained from tests, and k_c= a correction coefficient. Barkan (1962) recommends a value of 3 for $k_c$for use in the design of hammer foundations based on the observations in the extensive experimental program carried out by him.

9.15 Design Criteria

The following are the primary criteria for the design of a hammer foundation:

(i) The amplitudes of the foundation block and anvil should not exceed the permissible values given hereunder:

For the Foundation Block (A_1)

The maximum amplitude of the foundation should not exceed 1.2 mm. In the case of foundations resting on sand below ground water table, this should be limited to 0.8 mm.

For the Anvil (A_a)

The permissible amplitudes which depend upon the weight of the falling tup are given in Table9.2:

Table 9.2 Permissible amplitudes for anvil (After Barkan, 1962)

Weight of Tup (W_t)	Up to 10 kN	20 kN	30 kN
Maximum permissible amplitude	1 mm	2 mm	3 to 4 mm

The maximum stresses in the soil and other elastic layers shall be less than the permissible values for the respective materials.

9.15.1 Design Approach

The design is a trial and error process. Certain dimensions are assumed for the foundation block and elastic pad. The stress in the elastic pad and the amplitudes of motion are calculated. These values are compared with the respective permissible values, and if necessary, the dimensions are changed, and the analysis revised.

9.15.2 Barkan's Empirical Procedure

Based on his experimental investigations, Barkan (1962) recommended the following empirical equations for the determination of the tentative weight of the foundation and the base area of the foundation block in terms of the coefficient of restitution and the velocity of the dropping parts:

9.15.3 Weight of Foundation

$$n_f = [8.0(1+e)v - n_a] \qquad \text{(Eq.9.93)}$$

where, n_f = ratio of the weight of the foundation (W_f) to that of the dropping weight or tup (W_t) or $\left(W_f / W_t \right)$

e = coefficient of restitution

v = velocity of the dropping weight just before impact, (meters/sec)

and, n_a = ratio of the weight of anvil (W_a) to that of the dropping weight (W_t) or $\frac{W_a}{W_t}$

Numerical values of some hammer coefficients are given in table 9.3.

Weight of the foundation can now be got by multiplying the value of η_f obtained from Eq. 9.93 by the weight of the falling tup, as this and the weight of the anvil would have been decided earlier.

Table 9.3 Values of some hammer coefficients (After Barkan, 1962)

Type of Hammer	*v (m/s)*	*e*	n_a	n_f
Stamping Hammers				
Double acting (Stamping of steel)	6.5	0.5	20	48
Single-acting (Stamping of steel)	4.5	0.5	20	34
Single-acting (Stamping of non-ferrous metal)	4.5	0.0	...	16
Forge Hammers				
Double-acting	6.5	0.25	30	35
Single-acting	4.5	0.25	20	25

9.15.4 Base Area of the Foundation Block

$$a_f = \frac{20(1+e)v}{q_a} \qquad \text{(Eq.9.94)}$$

where, a_f = ratio of the base area of the foundation block (A) to the weight of the dropping weight (W_t) or $\frac{A}{W_t}$

and q_a= allowable bearing pressure of soil.

Values of a_f have been found to vary from 2 to 13 for different types of hammers resting on a variety of soils of different strengths. The required base area of the foundation block may be got by multiplying the value of a_f obtained from Eq. 9.94 by the weight of the falling tup. It is important to note that Equations 9.93 and 9.94 are not dimensionally correct; therefore, these equations shall be used with the weight, length, and time expressed in tonnes, metres, and seconds, respectively, the units in which Barkan derived them.

9.15.5 Minimum Thickness of Foundation

The minimum thickness of the foundation below the anvil for different weights of hammer, as recommended by Major (1962) are given in Table 9.4:

Table 9.4 Minimum thickness of foundation (After Major, 1962)

Weight of Hammer (kN)	Minimum thickness of foundation (m)
Up to 10	1.00
20	1.25
40	1.75
60	2.25
>60	>2.25

9.16 Vibration Isolation

If a machine is rigidly bolted to the floor, the vibration of the machine itself may be reduced, but that transmitted to the floor and soil will be large, producing harmful effects even at large distances. On the other hand, if an elastic support of sufficient flexibility is provided under the machine or its foundation, the vibration transmitted to the floor and soil will be reduced, but this may cause significant vibration to the machine itself. A judicious compromise is, therefore, to be struck; this is achieved usually through an appropriate frequency ratio, by adjusting the natural frequency of the machine foundation to a suitable value.

To avoid excessive vibration due to the working of a machine, the following points should be considered in the planning stage:

(*i*) *Selection of Site:* The machinery should be located far away from the area meant for precision work.

(*ii*) *Balancing of dynamic loads:* The machine should be dynamically balanced to limit the unbalanced forces produced during its operation.

(*iii*) *Adopting suitable foundations:* The foundation for the machine should be designed using accepted criteria, after evaluating the necessary design parameters at the site.

(*iv*)*Providing isolation:* Machine foundations should be completely separated from adjoining floors and other components by providing suitable isolating layers in between.

9.16.1 Types of Isolation-Transmissibility

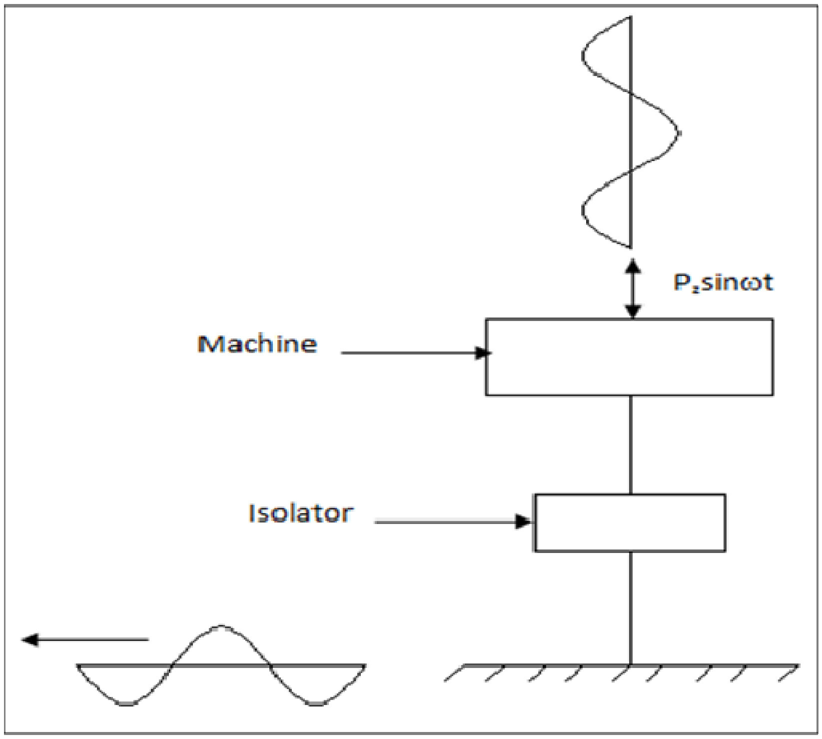
Two types of vibration problems are encountered in practice from the point of view of isolation. The first is the one in which isolation is required against vibration caused by the machine itself, and is called “Active Isolation”. The second is the one in which the foundation for a delicate machinery is designed in such a way that the amplitude of its motion due to floor vibration, caused by a disturbing force in the vicinity, is reduced to an acceptable limit; this is called “Passive Isolation”.

The schematic for active isolation and the mathematical model for it are shown in Fig. 9.24 (*a*) and (*b*): Active isolation is also called ‘force isolation’, since the attempt here is to reduce the force transmitted by the machine to the foundation in order to prevent vibration of adjacent machines and structures.

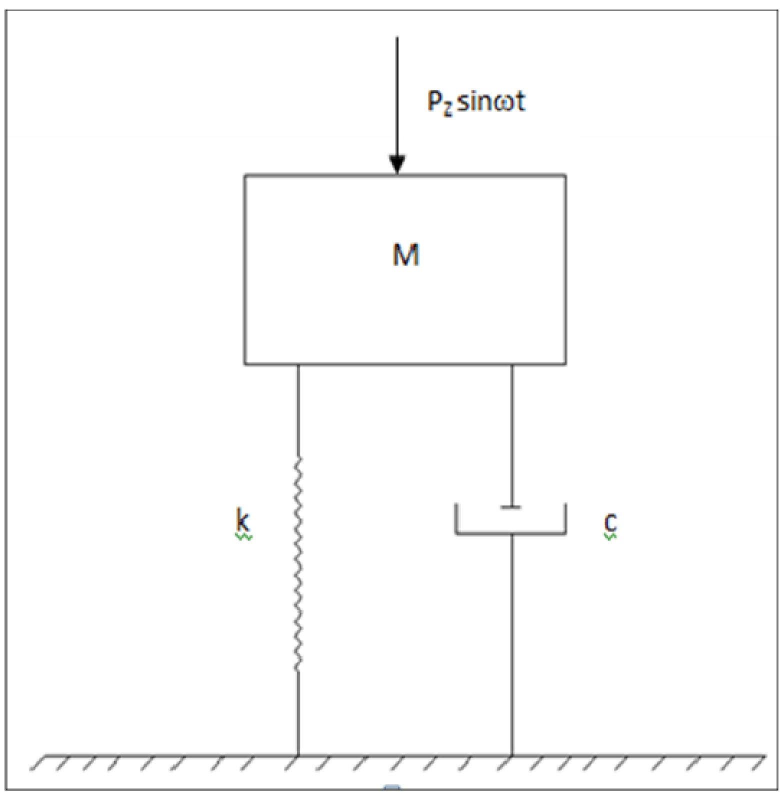
The schematic for passive isolation and the corresponding mathematical model are shown in Fig. 9.25 (*a*) and (*b*).

Passive isolation is also called ‘motion or amplitude isolation’, since the attempt here is to reduce the motion or amplitude of the machine (which may affect its performance) induced by ground vibration caused by disturbing sources in the vicinity.

The term “Transmissibility” is defined in the case of active isolation as the ratio of force transmitted to the foundation to the vibratory force developed by the machine itself. In the case of passive type of isolation, the term is defined as the ratio of the amplitude of the sensitive machine to the amplitude of the base.

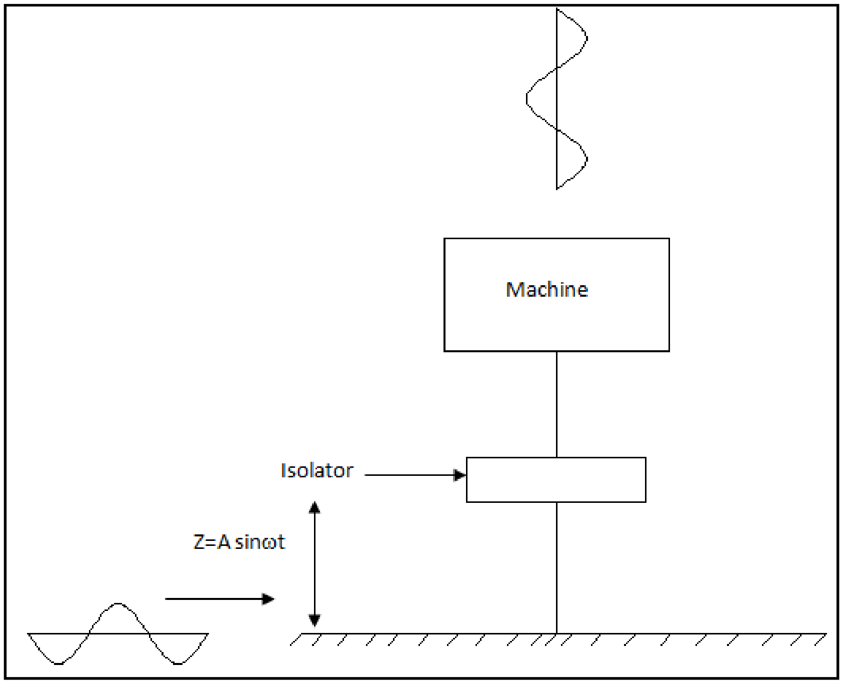


(a) Schematic for active isolation

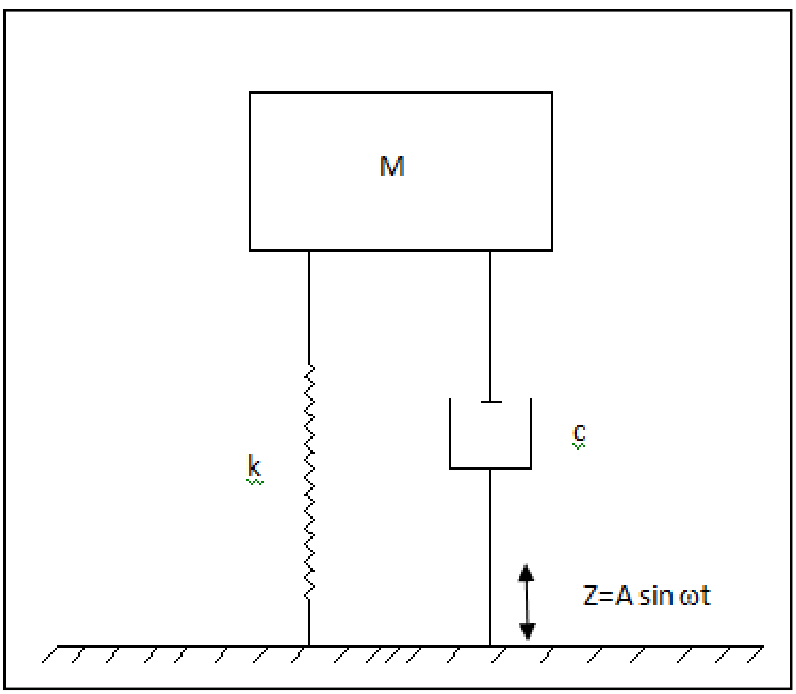


(b) Mathematical model for active isolation

Fig. 9.24: Active type of vibration isolation



(a) Schematic for passive isolation



(b) Mathematical model for passive isolation

Fig. 9.25: Passive type of vibration isolation

A common expression for transmissibility, *T*, can be derived for both these cases from the theory of vibration:

$$T = \sqrt{\frac{\left(1+4D^2\xi^2\right)}{\left(1-\xi^2\right)^2+4D^2\xi^2}} \qquad \text{(Eq.9.95)}$$

where ξ is frequency ratio and *D* is the damping factor. If the damping is very small, a simpler expression for transmissibility can be used:

$$T = \left|\frac{1}{\left(1-\xi^2\right)}\right| = \left|\frac{f_n{}^2}{\left(f^2-f_n{}^2\right)}\right| \qquad \text{(Eq.9.96)}$$

It is obvious that with greater values of ξ ($\xi > 2$), the transmissibility will be less. This means that the natural frequency of the isolated system should be made as low as possible relative to the forcing frequency. It is recommended that the frequency ratio be at least two in all cases of vibration isolation.

Also, the design should ensure adequate isolation in all possible modes of vibration. Eq.9.95 for transmissibility applies to translatory as well as rotatory modes of vibration.

9.16.2 Methods of Isolation

Different methods are available both for active and passive types of isolation. The following are the various types practised:

- *Counterbalancing the exciting forces:* One of the best ways of reducing the vibration is to treat the source itself. In the case of rotating type machinery, it is possible to counter-balance completely the exciting forces perpendicular to the direction of motion of piston and partly in the direction of motion of piston. The efficiency of a certain method of counter-balancing depends on the type of engine and nature of vibration. Counter-balancing does not require long interruption in the operation of the machine; the time required for attaching the counter weights is adequate.
- *Stabilisation of Soil:* Stabilization of soil increases the rigidity of the base and, hence, increases the natural frequencies of the foundation resting directly on soil. This possible only for sandy soils for which chemical or cement stabilization is generally adopted. The nature of vibration determines the limits of stabilized zones of soil. This method also does not involve prolonged interruption of the working of the machine.
- *Use of structural measures:* Suitable structural measures may be adopted to change the natural frequency of a foundation and ensure the required margin of safety against

resonant conditions. The choice of structural measures depends on the nature of vibration and the frequency ratio.

The following are some of the structural measures that may be adopted:

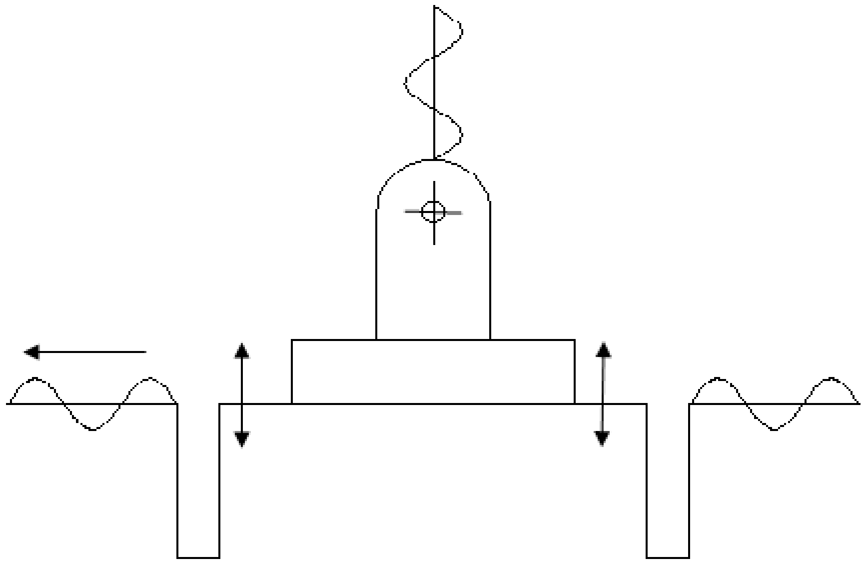
- *Increasing base area or mass of foundation:* Depending upon the frequency ratio, either increasing the base area or increasing the mass of the foundation, whichever is considered appropriate, may be adopted.
- *Use of slabs attached to foundation:* The dimension of the attached slab is so chosen that the amplitude of motion of the system is reduced to the required limit.
- *Use of auxiliary spring-mass systems:* Auxiliary spring-mass systems may be added to the primary system to reduce the vibrations. These systems without damping are called "Vibration neutralizers" and those with damping are known as "Vibration dampers".
- *Isolation by Trench barriers:* It has been found that the presence of a trench in the path of a wave reduces the onward transmission of vibration. According to Barkan, for effective isolation the depth of the trench should be at least one-third of the wavelength of vibration. This may not always be practicable. Trenches filled with bentonite slurry are known to show better isolation characteristics.
- *Isolation in Buildings:* Vertical separation between parts of a building would help to prevent vibrations from machines located in one part from causing damage in other parts.
- *Interposing isolating media:* Isolating media such as rubber carpets, steel helical springs, and air-bellow mounting systems are introduced between the machine and the foundation to effectively reduce the vibrations.

  Trenches for isolation of the active and the passive types are shown in fig. 9.26

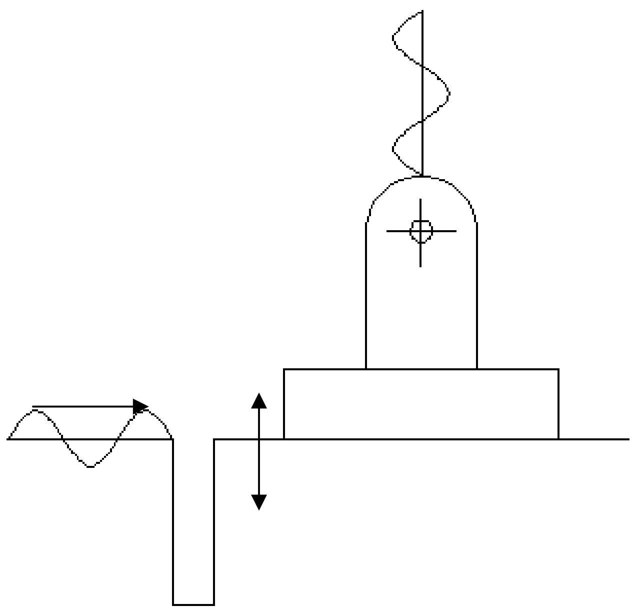
9.16.3 Properties of Isolating Materials

Important properties of a few isolating materials used in machine foundations are given below:

I. *Cork:* Cork is an effective isolating medium against vibration. It has low unit weight, high compressibility and high impermeability. Cork slabs are placed either directly under base of machine or under the concrete foundation. The stiffness of cork is relatively large. It is available only in slab form and is capable of resisting compression only. Cork sheets need to be enclosed in a steel frame to prevent lateral expansion. The resilient properties of cork deteriorate when it comes into contact with water or soil. Preservatives may be used to enhance its life.



(a) Active Case



(b) Passive Case

Fig. 9.26 Vibration isolation by trench barriers

II. *Felt:* Felt consists of a fabric with interlocking fibres of wool or other synthetic fibres. It is used in the form of small pads. The compressive strength is around 8 N/mm^2 and its elastic modulus about 80 N/mm^2. Under conditions of alternate wetting and drying, it tends to lose its elastic properties.

III. *Rubber:* Rubber springs have the advantage of resisting compression as well as shear. The allowable stress may be taken as 0.8 to 1.6 N/mm^2 in compression and 0.3 to 0.5 N/mm^2 in shear. A property known as “shore hardness” decides the quality of rubber.

IV. *Steel spring:* Steel springs have the advantage that their properties are known more precisely than other materials. Hence a more accurate design of spring isolators is possible and hence they are generally preferred. Springs are often used in groups.

9.17 Construction Aspects of Machine Foundations

Apart from the normal requirements of reinforced concrete construction as given in relevant codes of practice, a few additional points especially applicable to the construction of machine foundations are pertinent here.

9.17.1 Concrete

M 150 concrete should be used for block foundations and M 200 concrete for framed foundations. The concreting should preferably be done in a single operation. The location of construction joints should be judiciously chosen. Proper treatment of the joints with a suitable number of dowels and shear keys is required. Cement grout with non-shrinkable additive should be used under the machine bed-plate and for pockets of anchor-bolts.

9.17.2 Reinforcement

Reinforcement should be used on all surfaces, openings, cavities, etc., required to be provided in the machine foundation. In block-type foundation, reinforcements should be used in the three directions. The minimum reinforcement should be 250 N/cum of concrete. The reinforcement usually consists of 16 to 25 mm bars kept at 200 to 300 mm spacing in both directions, and also on the lateral faces. The concrete cover should be a minimum of 75 mm at bottom and50 mm on sides and at top. Around all openings, steel reinforcement equal to 0.50 to 0.75% of cross-sectional area of the opening shall be provided, in the form of a cage.

9.17.3 Expansion Joints

Machine foundations should be separated from adjoining structural elements by expansion joints to prevent transmission of vibration.

9.17.4 Connecting Elements

Base plates and anchor bolts are used to fix machines to the foundation. For this purpose, concreting should be stopped at the level of the base plate. This gap will be filled later by cement mortar. A 150 mm × 150 mm hole is generally sufficient for bolt holes. A minimum clearance of 80 mm should be provided from the edge of the bolt hole to the nearest edge of the foundation. The length of a bolt to be concreted is generally 30 to 40 times the diameter. Boltholes should be invariably filled with concrete. Concreting the spaces under the machines should be done with extreme care using 1:2 mortar mix. Machines should not be operated for at least 15 days after under-filling, since vibrations are harmful to fresh mortar. The edges of the foundation should be protected by providing a border of steel angles.

9.17.5 Spring Absorbers

Spring absorbers are commonly used for providing isolation in machine foundations. These can be installed by using either 'supported system' or 'suspended system'. In the former, the springs are placed directly under the machine or the foundation; in the latter, the foundation is suspended from springs located at or close to the floor level. In the suspended system, access to the springs becomes easy for future maintenance or replacement. For well-balanced machines, relatively smaller springs are adequate; in such cases, the supported system may be used. For machines with large exciting forces, heavy springs will be required; in this case, the suspended system is preferred.

9.18 Provision for Tuning

When the necessary margin of safety cannot be realised in design to avoid resonance, it is desirable to give due provision in the construction for tuning the foundation at a later stage. By "tuning" is meant changing the natural frequency of the foundation system if found necessary at a later stage. To facilitate subsequent enlargement of the foundation, dowels should be let projecting. It has been suggested that hollows be left in the foundation block which may be subsequently concreted, if required, to increase the mass of the foundation with the same base area.