Module 2
Lecture 8

Permeability and Seepage -4

Topics

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1.2.1 Equation of Continuity

In many practical cases, the nature of the flow of water through soil is such that the velocity and gradient vary throughout the medium. For these problems, calculation of flow is generally made by use of graphs referred to as flow nets. The concept of the flow net is based on Laplace's equation of continuity, which describes the steady flow condition for a given point in the soil mass.

To derive the equation of continuity of flow, consider an elementary soil prism at point A (**Figure2.27b)** for the hydraulic structure shown in **Figure2.27a**. The flows entering the soil prism in the $x, y, and\ z$ directions can be given from Darcy's law as

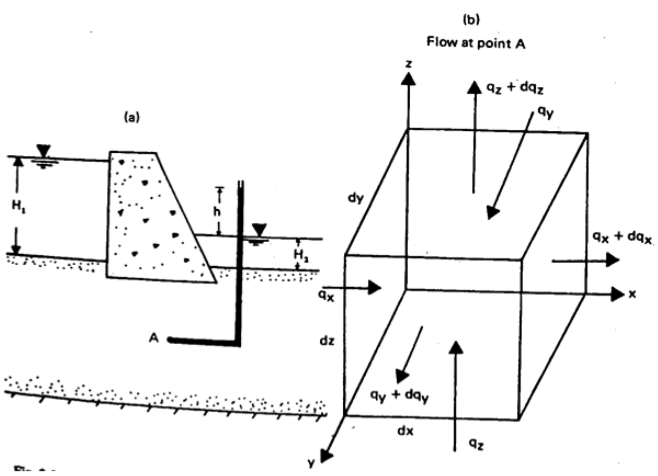

Figure2.27 Derivation of continuity equation

$$q_x = k_x i_x A_x = k_x \frac{\partial h}{\partial x} dy\, dz \quad (1.78)$$

$$q_y = k_y i_y A_y = k_y \frac{\partial h}{\partial y} dx\, dz \quad (1.79)$$

$$q_z = k_z i_z A_z = k_z \frac{\partial h}{\partial z} dx\, dy \quad (1.80)$$

Where $q_x, q_y, q_z =$Flow entering in directions $x, y, z,$ respectively

$k_x, k_y, k_z =$ Coefficient of permeability in directions $x, y, z,$ respectively

$h =$ Hydraulic head at point A

The flows leaving the prism in the $x, y, and\ z$ directions are, respectively,

$$q_x + dq_x = k_x(i_x + di_x)A_x$$

$$= k_x \left(\frac{\partial h}{\partial x} + \frac{\partial^2 h}{\partial x^2} dx\right) dy\, dz \quad (1.81)$$

$$q_y + dq_y = k_y \left(\frac{\partial h}{\partial x} + \frac{\partial^2 h}{\partial x^2} dy\right) dx\, dz \quad (1.82)$$

$$q_z + dq_z = k_z \left(\frac{\partial h}{\partial z} + \frac{\partial^2 h}{\partial z^2} dz\right) dx\, dy \quad (1.83)$$

For steady flow through an incompressible medium, the flow entering the elementary prism is equal to the flow leaving the elementary prism. So,

$$q_x + q_y + q_z = (q_z + dq_x) + (q_y + dq_y) + (q_z + dq_z) \quad (1.84)$$

Combining equations (1.78) to (1.84), we obtain

$$k_x \frac{\partial^2 h}{\partial x^2} + k_y \frac{\partial^2 h}{\partial y^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0 \quad (1.85)$$

For two-dimensional flow in the xz plane, equation (1.85) becomes

$$k_x \frac{\partial^2 h}{\partial x^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0 \quad (1.86)$$

If the soil is isotropic with respect to permeability, $k_x = k_z = k$, and the continuity equation simplifies to

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0 \quad (1.87)$$

This is generally referred to as Laplace's equation.

Potential and stream functions. Consider a function $\emptyset(x, z)$ such that

$$\frac{\partial \emptyset}{\partial x} = v_x = -k \frac{\partial h}{\partial x} \quad (1.88)$$

And

$$\frac{\partial\emptyset}{\partial z} = v_z = -k\frac{\partial h}{\partial z} \quad (1.89)$$

If we differentiate equation (1.88) with respect to x and equation (1.89) with respect to z and substitute in equation (1.87), we get

$$\frac{\partial^2\emptyset}{\partial x^2} + \frac{\partial^2\emptyset}{\partial z^2} = 0 \quad (1.90)$$

Therefore, $\emptyset(x, z)$ satisfies the Laplace equation. From equation (1.88) and (1.89),

$$\emptyset(x, z) = -kh(x, z) + f(z) \quad (1.91)$$

, And $\emptyset(x, z) = -kh(x, z) + g(x)$ (1.92)

Since x and z can be varied independently, $f(z) = g(x) = C,$ a constant. So

$$\emptyset(x, z) = -kh(x, z) + c$$

Or $h(x, z) = \frac{1}{k}[C - \emptyset(x, z)]$ (1.93)

If $h(x, z)$ is a constant equal to h_1, equation (1.93) represents a curve in the xz plane. For this curve, $\emptyset$ will have a constant value $\emptyset_1$. This is an equipotnetial line. So, by assigning to $\emptyset$ a number of values such as $\emptyset_1, \emptyset_2, \emptyset_3, \ldots.$, we can get a number of equipotential lines along which $h = h_1, h_2, h_3, \ldots \ldots \ldots$, respectively. The slope along an equipotential line $\emptyset$ can now be derived:

$$d\emptyset = \frac{\partial_\emptyset}{\partial_x}dx + \frac{\partial_\emptyset}{\partial_z}dz \quad (1.94)$$

If $\emptyset$ is a constant along a curve, $d\emptyset = 0.$ hence,

$$\left(\frac{dz}{dx}\right)_\emptyset = -\frac{\partial\emptyset/\partial x}{\partial\emptyset/\partial z} = -\frac{v_x}{v_z} \quad (1.95)$$

Again, let $\psi(x, z)$ be a function such that

$$\frac{\partial\psi}{\partial z} = v_x = -k\frac{\partial h}{\partial x} \quad (1.96)$$

And $-\frac{\partial\psi}{\partial x} = v_z = -k\frac{\partial h}{\partial z}$ (1.97)

Combining equation (1.88) and (1.96), we obtain

$$\frac{\partial\emptyset}{\partial x} = \frac{\partial\psi}{\partial z}$$

$$\frac{\partial^2\psi}{\partial z^2} = \frac{\partial^2\emptyset}{\partial x\,\partial z} \quad (1.98)$$

Again, combining equation (1.89 and 1.97),

$$-\frac{\partial \emptyset}{\partial z}=\frac{\partial \psi}{\partial x}$$

$$-\frac{\partial^2 \psi}{\partial x\,\partial z}=\frac{\partial^2 \psi}{\partial z^2} \qquad (1.99)$$

From equations (1.98 and (1.99),

$$\frac{\partial^2 \psi}{\partial x^2}+\frac{\partial^2 \psi}{\partial z^2}=-\frac{\partial^2 \emptyset}{\partial x\,\partial z}=\frac{\partial^2 \emptyset}{\partial x\,\partial z}=0$$

So $\psi(x,z)$ also satisfies Laplace's equation. If we assign to $\psi(x,z)$ various values $\psi_1,\psi_2,\psi_3,\ldots..$, we get a family of curves in the xz plane. Now

$$d\psi=\frac{\partial \psi}{\partial x}dx+\frac{\partial \psi}{\partial z}dz \qquad (2.100)$$

For a given curve, if ψ is constant, then $d\psi=0$. Thus, from equation (2.100),

$$\left(\frac{dz}{dx}\right)_{\psi}=-\frac{\partial\psi/\partial x}{\partial\psi/\partial z}=\frac{v_z}{v_x} \qquad (2.101)$$

Note that the slope, $(dz/dx)\psi$, is in the same direction as the resultant velocity. Hence, the curves $\psi=\psi_1,\psi_2,\psi_3,\ldots\ldots$, are the flow lines.

From equations (1.95) and (2.101), we can see that at a given point (x,z) the equipotential line and the flow line are orthogonal.

The functions $\emptyset(x,z) and\ \psi(x,z)$ are called the potential function and the stream function, respectively.

Use Continuity Equation for Solution of Simple Flow Problem

To understand the role of the continuity equation [equation (1.87)], consider a simple case of flow of water through two layers of soil as shown in **Figure2.28**. The flow is in one direction only i.e., in the direction of the x axis. The lengths of the two soil layers $(L_A\ and\ L_B)$ and their coefficients of permeability in the direction of the x axis $(k_A\ and\ k_B)$ are known. The total heads at sections 1 and 3 are known. It is required to plot the total head at any other section for $0<x<L_A+L_B$.

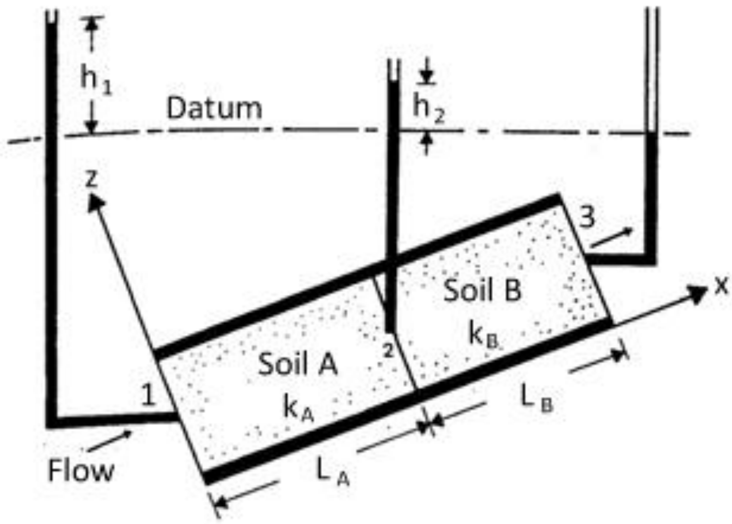

Figure2.28 One-directional flow through two layers of soil

For one-dimensional flow, equation (1.87) becomes

$$\frac{\partial^2 h}{\partial x^2} = 0 \qquad (2.102)$$

Integration of equation (2.102) twice gives

$$h = C_2 x + C_1 \qquad (2.103)$$

Where C_1 *and* C_2 are constants.

For flow through soil A, the boundary conditions are:

$$at\ x = 0, h = h_1$$

$$at\ x = L_A, h = h_2$$

However, h_2 is unknown$(h_1 > h_2)$. From the first boundary condition and equation (2.103), $C_1 = h_1$. So,

$$h = C_2 x + h_1 \qquad (2.104)$$

From the second boundary condition and equation (2.103),

$h_2 = C_2 L_A + h_1$ *or* $C_2 = (h_2 - h_1)/L_A$ so,

$$h = -\frac{h_1 - h_2}{L_A} x + h_1 \qquad (for\ 0 \le x \le L_A) \qquad (2.105)$$

For flow through soils B, the boundary condition for solution of C_1 *and* C_2 in equation (2.102) are

$$at\ x = L_{A,}\ h = h_2$$

$$at\ x = L_A + L_{B,}\ h = 0$$

From the first boundary condition and equation (2.103), $h_2 = C_2 L_A + C_1$, or

$$C_1 = h_2 - C_2 L_A \tag{2.106}$$

Again, from the secondary boundary condition and equation (2.103), $0 = C_2(L_A + L_B) + C_1$, or

$$C_1 = -C_2(L_A + L_B) \tag{2.107}$$

Equating the right-hand sides of equation (2.106) and (2.107),

$$h_2 - C_2 L_A = -C_2(L_A + L_B)$$

$$C_2 = -\frac{h_2}{L_B} \tag{2.108}$$

And then substituting equation (2.108) into equation (2.106), gives

$$C_1 = h_2 + \frac{h_2}{L_B} L_A = h_2\left(1 + \frac{L_A}{L_B}\right) \tag{2.109}$$

Thus, for flow through soil B,

$$h = \frac{h_2}{L_B} x + h_2\left(1 + \frac{L_A}{L_B}\right) \quad (for L_A \leq x \leq L_A + L_B) \tag{2.110}$$

With equation (2.105) and (2.110), we can solve for h for any value of x from 0 to $L_A + L_B$, provided that h_2 is known, however,

$$q = rate\ of\ flow\ through\ soil\ A = rate\ of\ flow\ through\ soil\ B$$

So, $q = k_A\left(\frac{h_1 - h_2}{L_A}\right) A = k_B\left(\frac{h_2}{L_B}\right) A$ (2.111)

Where $k_A\ and\ k_B$ are the coefficients of permeability of soils A and B, respectively, and A is the area of cross section of soil perpendicular to the direction of flow.

From equation (2.111),

$$h_2 = \frac{k_A h_1}{L_A(k_A/L_A + k_B/L_B)} \tag{2.112}$$

Substitution of equation (2.112) into equation (2.105) and (2.110) yields, after simplification,

$$h = h_1\left(1 - \frac{k_B x}{k_A L_B + k_B L_A}\right) \quad (for\ x = 0\ to\ L_A) \tag{2.113}$$

$$h = h_1\left[\frac{k_A}{k_A L_B + k_B L_A}(L_A + L_B - x)\right] \quad (for\ x = L_A\ to\ L_A + L_B) \tag{2.114}$$

1.2.2 Flow Nets

A set of flow lines and equipotential lines is called a flow net. A flow line is a line along which a water particle will travel. An equipotential line is a line joining the points that show the same piezometric elevation (i.e., hydraulic head $= h(x,z) = constant$). **Figure2.29** shows an example of a flow net for a single row of sheet piles. The permeable layer is isotropic with respect to the coefficient of permeability i.e., $k_x = k_z = k$. Note that the solid lines in **Figure2.29** are the flow lines, and the broken lines are the equipotential lines. In drawing a flow net, the boundary conditions must be kept in mind. For example, in **Figure2.29**,

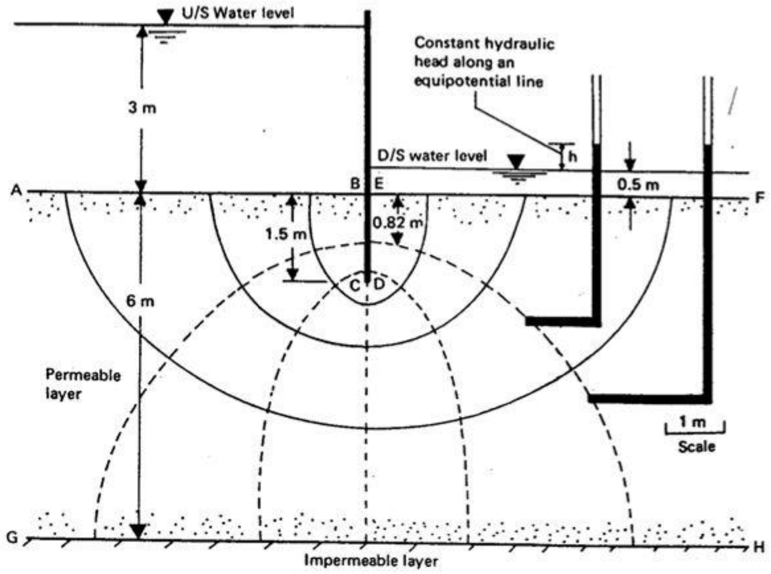

Figure2.29 Flow net around a single row of sheet piles

AB is an equipotential line

EF is an equipotential line

BCDE (i.e., the sides of the sheet pile) is a flow line

GH is a flow line

The flow lines and the equipotential lines are drawn by trial and error. It must be remembered that the flow lines intersect the equipotential lines at right angles. The flow and equipotential lines are usually drawn in such a way that the flow elements are approximately squares. Drawing a flow net is time consuming the tedious because of the trial-and-error process involved. Once a satisfactory flow net has been drawn, it can be traced out.

Some other examples of flow nets are shown in **Figure2.30 and 2.31** for flow under dams.

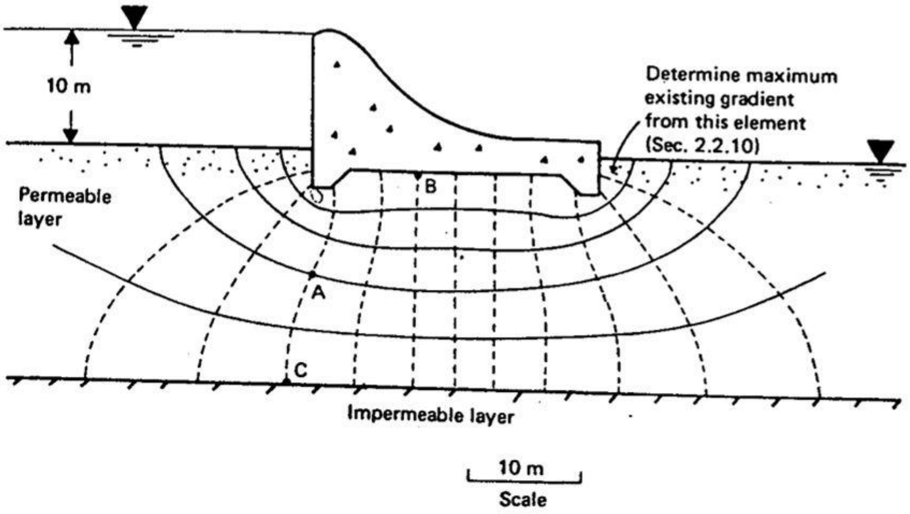

Figure2.30 Flow net under a dam

Calculation of seepage from a flow net under a hydraulic structure. A flow channel is the strip located between two adjacent flow lines. To calculate the seepage under a hydraulic structure, consider a flow channel as shown in **Figure2.32**. The equipotential lines crossing the flow channel are also shown, along with their corresponding hydraulic heads. Let Δq be the flow through the flow channel per unit length of the hydraulic structure (i.e., perpendicular to the section shown). According to Darcy's law

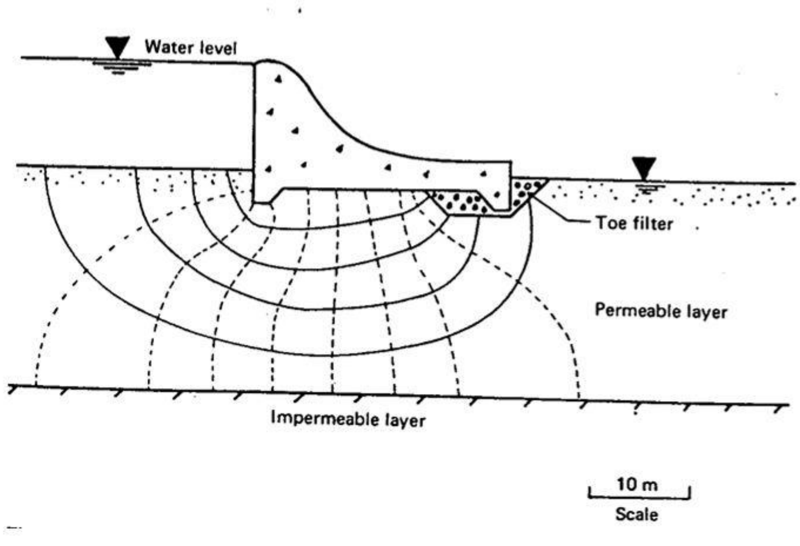

Figure 2.31 Flow net under a dam with a toe filter

$$\Delta q = kiA = k\left(\frac{h_1-h_2}{l_1}\right)(b_1 \times 1) = k\left(\frac{h_2-h_3}{l_2}\right)(b_2 \times 1)$$

$$= k\left(\frac{h_3-h_4}{l_3}\right)(b_3 \times 1) = \cdots \quad (2.115)$$

If the flow elements are drawn as squares, then

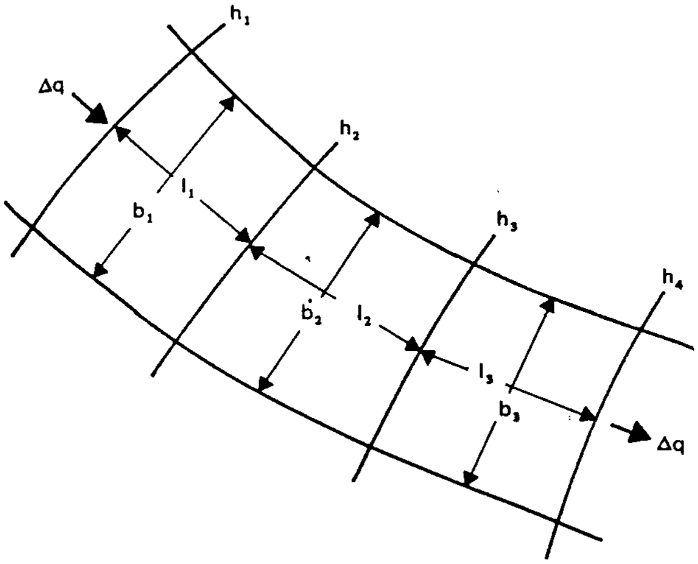

Figure 2.32

$l_1 = b_1$

$l_2 = b_2$

$l_3 = b_3$

So, from equation (2.115), we get

$$h_1 - h_2 = h_2 - h_3 = h_3 - h_4 = \cdots = \Delta h = \frac{h}{N_d} \quad (2.116)$$

Where

$\Delta h =$ Potential drop = drop in piezometric elevation between two consecutive equipotential lines

$h =$ Total hydraulic head = difference in elevation of water between the upstream and downstream side

$N_d =$ Number of potential drops

Equation (2.116) demonstrates that the loss of head between any two consecutive equipotential lines is the same. Combining equation (2.115) and (2.116),

$$\Delta q = k\frac{h}{N_d} \quad (2.117)$$

If there are N_f flow channels in a flow net, the rate of seepage per unit length of the hydraulic structure is

$$q = N_f \Delta q = kh\frac{N_f}{N_d} \quad (2.118)$$

Although flow nets are usually constructed in such a way that all flow elements are approximately squares, that need not always be the case. We could construct flow nets with all the flow elements drawn as rectangles. In that case, the length-to-width ratio of the flow nets has to be a constant, i.e.,

$$\frac{b_1}{l_1} = \frac{b_2}{l_2} = \frac{b_3}{l_3} = \cdots = n \quad (2.119)$$

For such flow nets, the rate of seepage per unit length of hydraulic structure can be given by

$$q = kh\frac{N_f}{N_d}n \quad (2.120)$$

Example 1.4 For the flow rate net shown in **Figure2.30**:

How high would water rise if a piezometer is placed at (i) A, (ii) B, (iii) C?

If $k = 0.01\ mm/s$, determine the seepage loss of the dam in $m^3/(day \cdot m)$.

Solution. The maximum hydraulic head h is 10m. In **Figure2.30**, $N_d = 12, \Delta h = h/N_d = 10/12 = 0.833$.

Part (a), (i): To reach$A$, water has to go through three potential drops. So head lost is equal to $3 \times 0.833 = 2.5m$.. Hence the elevation of the water level in the piezometer at A will be $10 - 2.5 = 7.5\ m$ above the ground surface.

Part (a), (ii): The water level in the peizometer above the ground level is $10 - 5(0.833) = 5.84m$.

Part (a), (iii): Points A and C are located on the same equipotential line. So water in a peizometer at C will rise to the same elevation as at $A, i.e., 7.5\ m$ above the ground surface.

Part (b): The seepage loss is given by $q = kh(N_f/N_d)$. From **Figure2.30**, $N_f = 5\ and\ N_d = 12$. Since

$$k = 0.01\ mm/s = \left(\frac{0.01}{1000}\right)(60 \times 60 \times 24) = 0.864\ m/day$$

$$q = 0.864(10)(5/12) = 3.6m^3/(day \cdot m)$$

1.2.3 Hydraulic Uplift Force under a Structure

Flow nets can be used to determine the hydraulic uplifting force under a structure. The procedure can best be explained through a numerical example. Consider the dam section shown in **Figure2.30**, the cross section of which has been replotted in net shown in **Figure2.33**. To find the pressure head at point D (**Figure2.33**), we refer to the flow net shown in **Figure2.30**; the pressure head is equal to (10 + 3.34 m) minus the hydraulic head loss. Point D coincides with the third equipotential line beginning with the upstream side, which means that the hydraulic head loss at that point is $2(h/N_d) = 2(10/12) = 1.67m$. So,

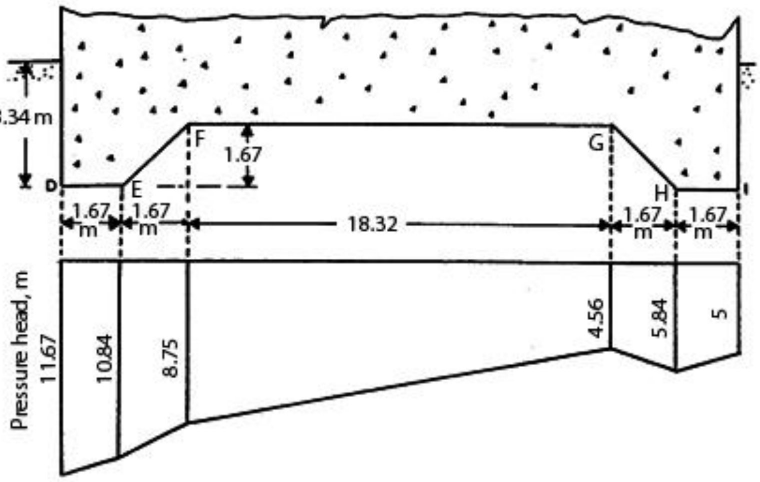

Figure2.33 Pressure head under the dam section shown in Figure1.30

Pressure head at $D = 13.34 - 1.67 = 11.67\ m$

Similarly,

Pressure head at $E = (10 + 3.34) - 3(10/12) = 10.84m$

Pressure head at $F = (10 + 1.67) - 3.5(10/12) = 8.75m$

(Note that point F is approximately midway between the fourth and fifth equipotential lines starting from the upstream side).

Pressure head at $G = (10 + 1.67) - 8.5(10/12) = 4.56\ m$

Pressure head at $H = (10 + 3.34) - 9(10/12) = 5.84m$

Pressure head at $I = (10 + 3.34) - 10(10/12) = 5m$

The pressure heads calculated above are plotted in **Figure2.33**. Between points F and G, the variation of pressure heads will be approximately linear. The hydraulic uplift force per unit length of the dam, U, can now be calculated as

$U = \gamma_w$ (Area of pressure head diagram)(1)

$$= 9.81\left[\left(\frac{11.67 + 10.84}{2}\right)(1.67) + \left(\frac{10.84 + 8.75}{2}\right)(1.67) + \left(\frac{8.75 + 4.56}{2}\right)(18.32) + \left(\frac{4.56 + 5.84}{2}\right)(1.67) + \left(\frac{5.84 + 5}{2}\right)(1.67)\right]$$

$$= 9.81(18.8 + 16.36 + 121.92 + 8.68 + 9.05)$$

$$= 1714.9\ kN/m$$

1.2.4 Flow Nets in Anisotropic Material

In developing the procedure described in section. 2.2.3 For plotting flow nets, we assumed that the permeable layer is isotropic, i.e., $k_{horizontal} = k_{vertical=k}$. let us now consider the case of constructing flow nets for seepage through soils that show anisotropy with respect to permeability. For two-dimension a flow problems, we refer to equation (1.86):

$$k_x \frac{\partial^2 h}{\partial x^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0$$

Where $k_x = k_{horizontal}$ and $k_z = k_{vertical}$. This equation can be rewritten as

$$\frac{\partial^2 h}{(k_z/k_x)\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0 \qquad (2.121)$$

Let $x' = \sqrt{k_z/k_x}\ x$; then

$$\frac{\partial^2 h}{(k_z/k_x)\partial x^2} + k_z \frac{\partial^2 h}{\partial x'^2} = 0 \qquad (2.122)$$

Substituting equation (2.122) into equation (2.121), we obtain

$$\frac{\partial^2 h}{\partial x'^2} + \frac{\partial^2 h}{\partial z^2} = 0 \qquad (2.123)$$

Equation (2.123) is of the same form as equation (1.87), which governs the flow, is isotropic soils and should represent two sets of orthogonal lines in the $x'z$ plane. The steps for construction of a flow net in an anisotropic medium are as follows:

To plot the section of the hydraulic structure, adopt a vertical scale.

Determine $\sqrt{\frac{k_z}{k_x}} = \sqrt{\frac{k_{vertical}}{k_{horizontal}}}$

Adopt a horizontal scale such that $Scale_{horizontal} = \sqrt{\frac{k_z}{k_x}}\ (scale_{vertical})$

With the scales adopted in steps 1 and 3, plot the cross section of the structure.

Draw the flow net for the transformed section plotted in step 4 in the same manner as is done for seepage through isotropic soils.

Calculate the rate of seepage as

$$q = \sqrt{k_x k_z}\, h \frac{N_f}{N_d} \qquad (2.124)$$

Compare equation (2.117) and (2.124). Both equations are similar, except for the fact that k in equation (2.117) is replaced by $\sqrt{k_x k_z}$ in equation (2.124).

Example 1.5 A dam section is shown **in Figure2.34a**. The coefficient of permeability of the permeable layer in the vertical and horizontal directions are $2 \times 10^{-2}\ mm/s\ and\ 4 \times 10^{-2} mm/s$, respectively. Draw a flow net and calculate the seepage loss of the dam in $ft^3(day \cdot ft)$.

Solution From the given data,

$k_z = 2 \times 10^{-2} mm/s = 5.67 ft/day$

$k_x = 4 \times 10^{-2} mm/s = 11.34 ft/day$ And

$h = 20\ ft$

for drawing the flow net,

$horizontal\ scale = \sqrt{\frac{2\times10^{-2}}{4\times10^{-2}}}\ (vertical\ scale) = \frac{1}{\sqrt{2}}(vertical\ scale)$

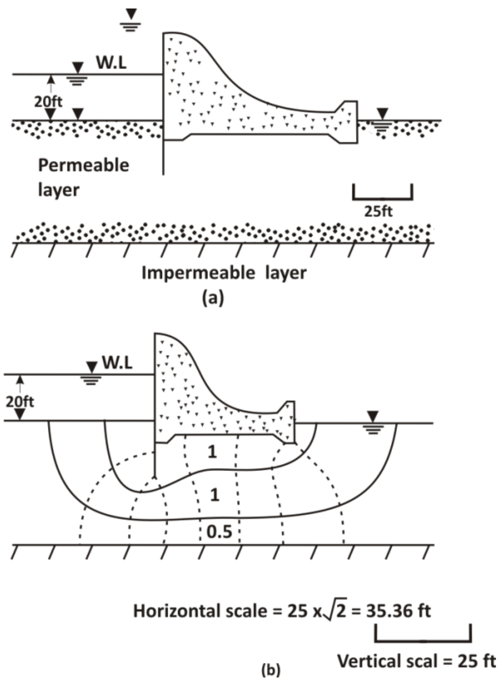

Figure2.34

On the basis of this, the dam section is replotted and the flow net drawn as in **Figure2.34b**. The rate of seepage is given by $q = \sqrt{k_x k_z}\ h(N_f/N_d)$. From Figure2.34b, $N_d = 8\ and\ N_f = 2.5$ (the lowermost flow channels a width-to-length ratio of 0.5). So,

$q = \sqrt{(5.67)(11.34)}\ (20)(2.5/8) = 50.12 ft^3/(day \cdot ft)$

Example 1.6. A single row of sheep pile structure is shown in Figure2.35a. Draw a flow net for the transformed section. Replot this flow net in the natural scale also. The relationship between the permeability's is given as $k_x = 6k_z$.

Solution For the transformed section,

$$Horizontal\ scale = \sqrt{\frac{k_z}{k_x}}\ (vertical\ scale)$$

$$= \frac{1}{\sqrt{6}}(vertical\ scale)$$

The transformed section and the corresponding flow net are shown in **Figure 2.35b. Figure 2.35c** shows the flow net constructed to the natural scale. One important fact to be noticed from this is that when the soil is anisotropic with respect to permeability, the flow and equipotential lines are not necessarily orthogonal.

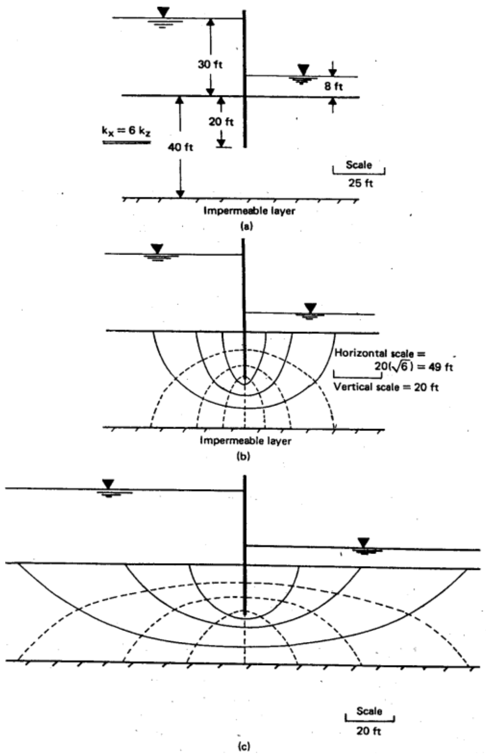

Figure 2.35

1.2.5 Construction of Flow Nets for Hydraulic Structures on Nonhomogeneous Subsoils

The flow-net construction technique is for the condition where the subsoil is homogeneous. Rarely in nature do such ideal conditions occur; in most cases, we encounter stratified soil deposits such as those shown in Figure1.38. When a flow net is constructed across the boundary of two soils with different permeability's, the flow net deflects at the boundary. This is called a transfer condition. **Figure2.36** shows a general condition where a flow channel crosses the boundary of two soils. Soil layers 1 and 2 have permeability is of k_1 and k_2, respectively. The broken lines drawn across the flow channel are the equipotential lines.

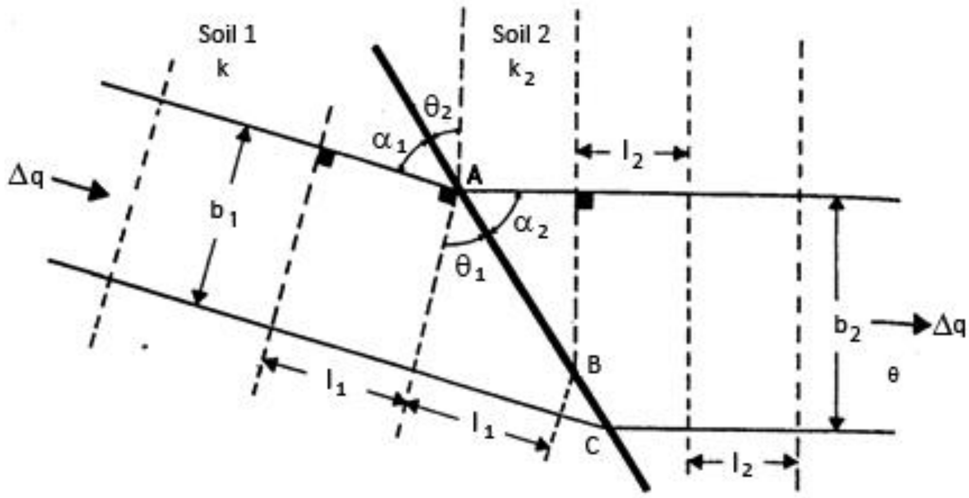

Figure 2.36

Let Δh be the loss of hydraulic head between two consecutive equipotential lines. Considering a unit length perpendicular to the section shown, the rate of seepage through the flow channel is

$$\Delta q = k_1 \frac{\Delta h}{l_1}(b_1 \times 1) = k_2 \frac{\Delta h}{l_2}(b_2 \times 1)$$

Or $\frac{k_1}{k_2} = \frac{b_2/l_2}{b_1/l_1}$ (2.125)

Where l_1 and b_1 are the length and width of the flow elements is soil layer 1, and l_2 and b_2 are the length and width of the flow elements in soil layer 2.

Referring again to **Figure 2.36**,

$l_1 = AB\, sin\, \theta_1 = AB\, cos\, \alpha_1$ (2.126a)

$l_2 = AB\, sin\, \theta_2 = AB\, cos\, \alpha_2$ (2.126b)

$l_1 = AC \cos\theta_1 = AC \sin\alpha_1$ (2.126c)

$b_2 = AC \cos\theta_2 = AC \sin\alpha_2$ (2.126d)

From equation (2.126a) and (2.126c),

$\frac{b_1}{l_1} = \frac{\cos\theta_1}{\sin\theta_1} = \frac{\sin\alpha_1}{\cos\alpha_1}\ or\ \frac{b_1}{l_1} = \frac{1}{\tan\theta_1} = \tan\alpha_1$ (2.127)

Also, from equations (2.126b) and (2.126d).

$\frac{b_2}{l_2} = \frac{\cos\theta_2}{\sin\theta_2} = \frac{\sin\alpha_2}{\cos\alpha_2}\ or\ \frac{b_2}{l_2} = \frac{1}{\tan\theta_2} = \tan\alpha_2$ (2.128)

Combining equations (2.125), (2.127), and (2.128),

$\frac{k_1}{k_2} = \frac{\tan\theta_1}{\tan\theta_2} = \frac{\tan\alpha_2}{\tan\alpha_1}$ (2.129)

Flow nets in nonhomogeneous subsoil's can be constructed using the relations given by equation (2.129) and other general principles outlined in section 2.2.3. It is useful to keep the following points in mind while constructing the flow nets:

If $k_1 > k_2$, we may plot square flow elements in layer 1. This means that $l_1 = b_1$ in equation (2.125). So$k_1/k_2 = b_2/l_2$. Thus the flow elements in layer 2 will be rectangles and their width-to-length ratios will be equal to k_1/k_2. This is shown in **Figure 2.37a**.

If $k > k_2$, we may plot square flow elements in layer 1(i.e., $l_1 = b_1$). From equation (2.125), $k_1/k_2 = b_2/l_2$. So the flow elements in layer 2 will be rectangles. This is shown in **Figure 2.37b**.

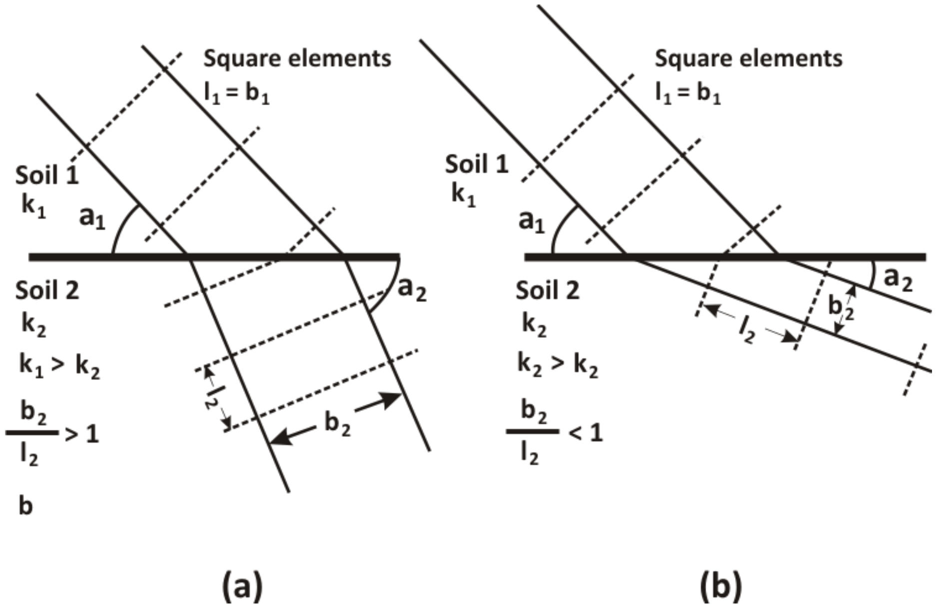

Figure 2.37 Flow channel at the boundary between two soils with different coefficient of permeability

An example of the construction of a flow net for a dam section resting on a two-layered soil deposit is given in **Figure 2.38**. Note that $k_1 = 5 \times 10^{-2}\ mm/s\ and\ k_2 = 2.5 \times 10^{-2} mm/s$. So,

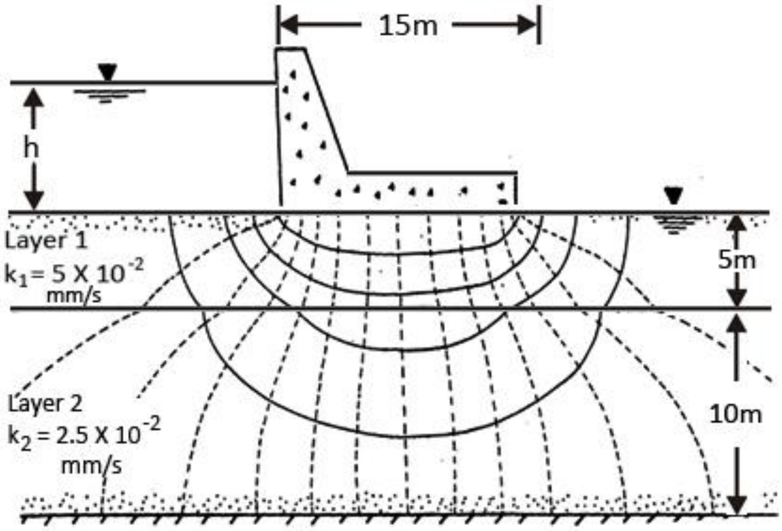

Figure 2.38 Flow net under a dam

$$\frac{k_1}{k_2} = \frac{5.0\times10^{-2}}{2.5\times10^{-2}} = 2 = \frac{\tan\alpha_2}{\tan\alpha_1} = \frac{\tan\theta_1}{\tan\theta_2}$$

In soil layer 1, the flow elements are plotted as squares; and since $k_1/k_2 = 2$, the length-to-width ratio of the flow elements in soil layer 2 is $1/2$.

1.2.6 Directional Variation of Permeability in Anisotropic Medium

In anisotropic soils, the directions of the maximum and minimum permeability are generally at right angles to each other. However, the equipotential lines and the flow lines are not necessarily orthogonal, as was shown in Figure2.35c.

Figure 2.39 shows a flow line and an equipotential line. m is the direction of the tangent drawn to the flow line at O, and thus that is the direction of the resultant discharge velocity. Direction n is perpendicular to the equipotential line at O, and so it is the direction of the resultant hydraulic gradient. Using Darcy's law,

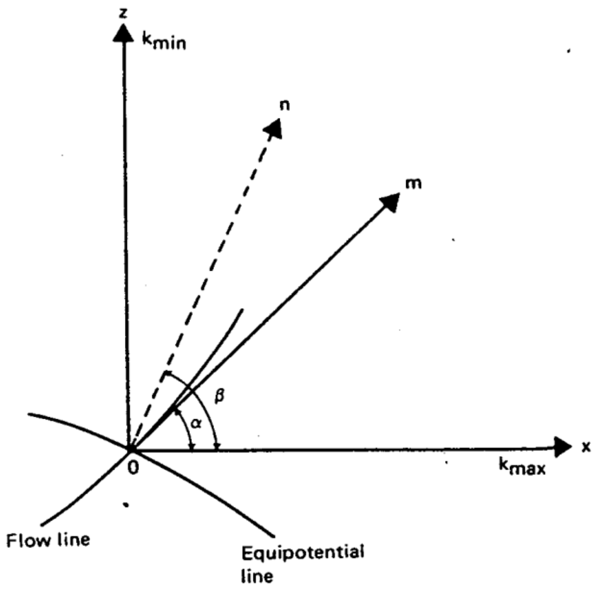

Figure 2.39 Directional variation of the coefficient of permeability

$$v_x = -k_{max}\frac{\partial h}{\partial x} \quad (2.130)$$

$$v_z = -k_{min}\frac{\partial h}{\partial z} \quad (2.131)$$

$$v_m = -k_{\alpha}\frac{\partial h}{\partial m} \quad (2.132)$$

$$v_n = -k_{\beta}\frac{\partial h}{\partial n} \quad (2.133)$$

Where

k_{max} = Maximum coefficient of permeability (in the horizontal x direction)

k_{min} = Maximum coefficient of permeability (in the vertical z direction)

k_{α}, k_{β} = Coefficients of permeability in m, n directions, respectively

Now we can write

$$\frac{\partial h}{\partial m} = \frac{\partial h}{\partial x}\cos\alpha + \frac{\partial h}{\partial z}\sin\alpha \quad (2.134)$$

From equation (2.130), (2.131), and (2.132), we have

$$\frac{\partial h}{\partial x} = -\frac{v_x}{k_{max}} \quad \frac{\partial h}{\partial x} = -\frac{v_z}{k_{min}} \quad \frac{\partial h}{\partial m} = -\frac{v_m}{k_{\alpha}}$$

Also, $v_x = v_m\, cos\alpha\; and\; v_z = v_m \sin\alpha$.

Substitution of these into equation (2.134) gives

$$-\frac{v_m}{k_\alpha} = -\frac{v_x}{k_{max}}\cos\alpha - \frac{v_z}{k_{min}}\sin\alpha \;\; or \;\; \frac{v_m}{k_\alpha} = \frac{v_m}{k_{max}}\cos^2\alpha + \frac{v_m}{k_{min}}\sin^2\alpha$$

So $\frac{1}{k_\alpha} = \frac{cos^2\alpha}{k_{max}} + \frac{sin^2\alpha}{k_{min}}$ (2.135)

The nature of the variation of k_α with α as determined by equation (2.135) is shown in **Figure 2.40**.

Again, we can say that

$$v_n = v_x \cos\beta + v_z \sin\beta \qquad (2.136)$$

Combining equation (2.130), (2.131), and (2.133),

$$k_\beta \frac{\partial h}{\partial n} = k_{max}\frac{\partial h}{\partial x}\frac{\partial h}{\partial z}\cos\beta + k_{min}\sin\beta \qquad (2.137)$$

But $\frac{\partial h}{\partial x} = \frac{\partial h}{\partial n}\cos\beta$ (2.138)

But $\frac{\partial h}{\partial z} = \frac{\partial h}{\partial n}\sin\beta$ ` (2.139)

Substitution of equation (2.138) and (2.139) into equation yields

$$k_\beta = k_{max}cos^2\beta + k_{min}sin^2\beta \qquad (2.140)$$

The variation of $k_\beta\; with\; \beta$ is also show in **Figure 2.40**. It can be seen that, for gives values of $k_{max}\; and\; k_{min}$, equation (2.135) and (2.140) yields slightly different values of the directional permeability. However, the maximum difference will not be more than 25%.

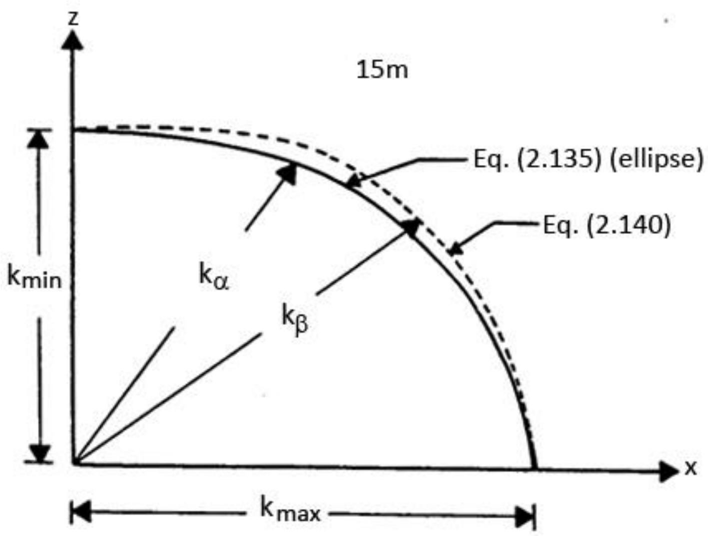

Figure 2.40 Directional variation of permeability