Construction Economics & Finance

Module 3

Lecture-1

Depreciation:-

It represents the reduction in market value of an asset due to age, wear and tear and obsolescence. The physical deterioration of the asset occurs due to wear and tear with passage of time. Obsolescence occurs to due to availability of new technology or new product in the market that is superior to the old one and the new one replaces the old even though the old one is still in working condition. The tangible assets for which the depreciation analysis is carried out are construction equipments, buildings, electronic products, vehicles, machinery etc. Depreciation amount for any asset is usually calculated on yearly basis. Depreciation is considered as expenditure in the cash flow of the asset, although there is no physical cash outflow. Depreciation affects the income tax to be paid by an individual or a firm as it is considered as an allowable deduction in calculating the taxable income. Generally the income tax is paid on taxable income which is equal to gross income less the allowable deductions (expenditures). Depreciation reduces the taxable income and hence results in lowering the income tax to be paid.

Before discussing about different methods of depreciation, it is necessary to know the common terms used in depreciation analysis. These terms are initial cost, salvage value, book value and useful life. **Initial cost** is the total cost of acquiring the asset. **Salvage value** represents estimated market value of the asset at the end of its useful life. It is the expected cash inflow that the owner of the asset will receive by disposing it at the end of useful life. **Book value** is the value of asset recorded on the accounting books of the firm at a given time period. It is generally calculated at the end of each year. Book value at the end of a given year equals the initial cost less the total depreciation amount till that year. **Useful life** represents the expected number of years the asset is useful in terms of generating revenue. The asset may still be in working condition after the useful life but it may not be economical. Useful life is also known as depreciable life. The asset is depreciated over its useful life.

The commonly used depreciation methods are straight-line depreciation method, declining balance method, sum-of-years-digits method and sinking fund method.

Straight-line (SL) depreciation method:-

It is the simplest method of depreciation. In this method it is assumed that the book value of an asset will decrease by same amount every year over the useful life till its salvage value is reached. In other words the book value of the asset decreases at a linear rate with the time period.

The expression for annual depreciation in a given year '*m*' is presented as follows;

$$D_m = \frac{P - SV}{n} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.1)$$

Where

D_m = depreciation amount in year 'm' (m = 1, 2, 3, 4, ………., n, i.e. $1 \le m \le n$)

P = initial cost of the asset

n = useful life or depreciable life (in years) over which the asset is depreciated.

In equation (3.1), $1/n$ is the constant annual depreciation rate which is denoted by the term 'd_m'.

Since the depreciation amount is same every year, $D_1 = D_2 = D_3 = D_4 = \ldots\ldots\ldots = D_m$

The book value at the end of 1st year is equal to initial cost less the depreciation in the 1st year and is given by;

$$BV_1 = P - D_m \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.2)$$

Book value at the end of 2nd year is equal to book value at the beginning of 2nd year (i.e. book value at the end of 1st year) less the depreciation in 2nd year and is expressed as follows;

$$BV_2 = BV_1 - D_m \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.3)$$

As already stated depreciation amount is same in every year.

Now putting the expression of 'BV_1' from equation (3.2) in equation (3.3);

$$BV_2 = (P - D_m) - D_m = P - 2D_m \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3.4)$$

Similarly the book value at the end of 3rd year is equal to book value at the beginning of 3rd year (i.e. book value at the end of 2nd year) less the depreciation in 3rd year and is given by;

$BV_3 = BV_2 - D_m$ ……………………………… (3.5)

$BV_3 = (P - 2D_m) - D_m = P - 3D_m$ …………….. (3.6)

In the same manner the generalized expression for book value at the end of any given year '*m*' can be written as follows;

$BV_m = P - mD_m$ ………………………………. (3.7)

Declining balance (DB) depreciation method:-

It is an accelerated depreciation method. In this method the annual depreciation is expressed as a fixed percentage of the book value at the beginning of the year and is calculated by multiplying the book value at the beginning of each year with a fixed percentage. Thus this method is also sometimes known as fixed percentage method of depreciation. The ratio of depreciation amount in a given year to the book value at the beginning of that year is constant for all the years of useful life of the asset. When this ratio is twice the straight-line depreciation rate i.e. $2/n$, the method is known as double-declining balance (DDB) method. In other words the depreciation rate is 200% of the straight-line depreciation rate. Double-declining balance (DDB) method is the most commonly used declining balance method. Another declining balance method that uses depreciation rate equal to 150% of the straight-line depreciation rate i.e. $1.5/n$ is also used for calculation of depreciation. In declining balance methods the depreciation during the early years is more as compared to that in later years of the asset's useful life.

In case of declining balance method, for calculating annual depreciation amount, the salvage value is not subtracted from the initial cost. It is important to ensure that, the asset is not depreciated below the estimated salvage value. In declining-balance method the calculated book value of the asset at the end of useful life does not match with the salvage value. If the book value of the asset reaches its estimated salvage value before the end of useful life, then the asset is not depreciated further.

Representing constant annual depreciation rate in declining balance method as 'd_m', the expressions for annual depreciation amount and book value are presented as follows;

The depreciation in 1st year is calculated by multiplying the initial cost (i.e. book value at beginning) with the depreciation rate and is given by;

$D_1 = P \times d_m = P(1 - d_m)^0 \times d_m$ ……………………….…. (3.8)

Where

D_1 = depreciation amount in 1st year

P = initial cost of the asset as already mentioned

d_m = constant annual depreciation rate

The book value at the end of 1st year is equal to initial cost less the depreciation in the 1st year and is calculated as follows;

$$BV_1 = P - D_1$$

Now putting the expression of 'D_1'from equation (3.8) in the above expression;

$$BV_1 = P - D_1 = P - P \times d_m = P(1 - d_m) \quad \text{…………………} \quad (3.9)$$

The depreciation in 2nd year i.e. 'D_2' is calculated by multiplying the book value at the beginning of 2nd year (i.e. book value at the end of 1st year) with the depreciation rate 'd_m' and is given as follows;

$$D_2 = BV_1 \times d_m \quad \text{……………..…………………………} \quad (3.10)$$

Now putting the expression of 'BV_1'from equation (3.9) in equation (3.10) results in the following;

$$D_2 = BV_1 \times d_m = P(1 - d_m) \times d_m \quad \text{…………………….…} \quad (3.11)$$

Book value at the end of 2nd year is equal to book value at the beginning of 2nd year (i.e. book value at the end of 1st year) less the depreciation in 2nd year and is expressed as follows;

$$BV_2 = BV_1 - D_2$$

Now putting the expressions of 'BV_1' and 'D_2'from equation (3.9) and equation (3.11) respectively in above expression results in the following;

$$BV_2 = BV_1 - D_2 = [P(1 - d_m)] - [P(1 - d_m) \times d_m] = P(1 - d_m)(1 - d_m) = P(1 - d_m)^2 \quad \text{...} \quad (3.12)$$

Similarly the depreciation in 3rd year i.e. 'D_3' is calculated as follows;

$$D_3 = BV_2 \times d_m \quad \text{……………..…………………………} \quad (3.13)$$

Now putting the expression of 'BV_2'from equation (3.12) in equation (3.13);

$$D_3 = BV_2 \times d_m = P(1 - d_m)^2 \times d_m \quad \text{…………………….} \quad (3.14)$$

Book value at the end of 3rd year is calculated as follows;

$$BV_3 = BV_2 - D_3$$

$$BV_3 = BV_2 - D_3 = \left[P(1-d_m)^2\right] - \left[P(1-d_m)^2 \times d_m\right] = P(1-d_m)^2(1-d_m) = P(1-d_m)^3 \quad \text{………..} \quad (3.15)$$

In the same manner the generalized expression for depreciation in any given year 'm' can be written as follows (referring to equations (3.8), (3.11) and (3.14));

$$D_m = P(1-d_m)^{m-1} \times d_m \quad \text{………………………………...} \quad (3.16)$$

Similarly the generalized expression for book value at the end of any year 'm' is given as follows (referring to equations (3.9), (3.12) and (3.15));

$$BV_m = P(1-d_m)^m \quad \text{…………………………………..} \quad (3.17)$$

The book value at the end of useful life i.e. at the end of 'n' years is given by;

$$BV_n = P(1-d_m)^n \quad \text{…………………………………...} \quad (3.18)$$

The book value at the end of useful life is theoretically equal to the salvage value of the asset. Thus equating the salvage value (SV) of the asset to its book value (BV_n) at the end of useful life results in the following;

$$SV = BV_n = P(1-d_m)^n \quad \text{…………………………….....} \quad (3.19)$$

Thus for calculating the depreciation of an asset using declining balance method, equation (3.19) can be used to find out the constant annual depreciation rate, if it is not stated for the asset. From equation (3.19), the expression for constant annual depreciation rate 'd_m' from known values of initial cost 'P' and salvage value (SV > 0) is obtained as follows;

$$SV = P(1-d_m)^n$$

$$\frac{SV}{P} = (1-d_m)^n$$

$$d_m = 1 - \left(\frac{SV}{P}\right)^{1/n} \quad \text{……………………………….…...} \quad (3.20)$$

In double-declining balance (DDB) method, for calculating annual depreciation amount and book value at the end of different years, the value of constant annual depreciation rate 'd_m' is replaced by '$2/n$' in the above mentioned equations.

Example -1

The initial cost of a piece of construction equipment is Rs.3500000. It has useful life of 10 years. The estimated salvage value of the equipment at the end of useful life is Rs.500000. Calculate the annual depreciation and book value of the construction equipment using straight-line method and double-declining balance method.

Solution:

Initial cost of the construction equipment = P = Rs.3500000

Estimated salvage value = SV = Rs.500000

Useful life = n = 10 years

For straight-line method, the depreciation amount for a given year is calculated using equation (3.1).

$$D_m = D_1 = D_2 = \cdots\cdots = D_9 = D_{10} = \frac{3500000 - 500000}{10} = \text{Rs.}300000$$

The book value at the end of a given year is calculated by subtracting the annual depreciation amount from previous year's book value.

Book value at the end of 1st year = $BV_1 = \text{Rs.}3500000 - \text{Rs.}300000 = \text{Rs.}3200000$

Book value at the end of 2nd year = $BV_2 = \text{Rs.}3200000 - \text{Rs.}300000 = \text{Rs.}2900000$

Similarly the book values at the end of other years have been calculated in the same manner. The annual depreciation amount and book values at end of years using straight-line depreciation method are presented in Table 3.1. For straight-line method, the book value at the end of different years can also be calculated by using equation (3.7). For example, the book value at the end of 2nd year is given by;

$$BV_2 = 3500000 - 2 \times 300000 = \text{Rs.}2900000$$

For double-declining balance method, the constant annual depreciation rate 'd_m' is given by;

$$d_m = \frac{2}{n} = \frac{2}{10} = 0.2$$

The depreciation amount for a given year is calculated using equation (3.16).

Depreciation for 1st year = $\text{D}_1 = 3500000 \times (1-0.2)^{1-1} \times 0.2 = \text{Rs.}700000$

Depreciation for 2nd year = $\text{D}_2 = 3500000 \times (1-0.2)^{2-1} \times 0.2 = \text{Rs.}560000$

The book value at the end of a given year is calculated by subtracting the annual depreciation amount from previous year's book value.

Book value at the end of 1st year = $BV_1 = \text{Rs.}3500000 - \text{Rs.}700000 = \text{Rs.}2800000$

Book value at the end of 2nd year = $BV_2 = \text{Rs.}2800000 - \text{Rs.}560000 = \text{Rs.}2240000$

Similarly the annual depreciation and book value at the end of other years have been calculated in the same manner and are presented in Table 3.1.

The book value at the end of different years can also be calculated by using equation (3.17). Using this equation, the book value at the end of 2nd year is given by;

$$BV_2 = 3500000(1-0.2)^2 = Rs.2240000$$

Table 3.1 Depreciation and book value of the construction equipment using straight-line method and double-declining balance method

Year	Straight-line method		Double-declining balance method	
	Depreciation (Rs.) D_m	Book value (Rs.) BV_m	Depreciation (Rs.) D_m	Book value (Rs.) BV_m
0	-	3500000	-	3500000
1	300000	3200000	700000	2800000
2	300000	2900000	560000	2240000
3	300000	2600000	448000	1792000
4	300000	2300000	358400	1433600
5	300000	2000000	286720	1146880
6	300000	1700000	229376	917504
7	300000	1400000	183500.80	734003.20
8	300000	1100000	146800.64	587202.56
9	300000	800000	117440.51	469762.05
10	300000	500000	93952.41	*375809.64*

From Table 3.1 it is observed that the book value at the end of useful life i.e. 10 years in double-declining balance method is less than salvage value. Thus the equipment will be depreciated fully before the useful life. In other words the book value reaches the estimated salvage value before the end of useful life. As the book value of the equipment is not depreciated further after it has reached the estimated salvage value, the depreciation amount is adjusted accordingly. The book value of the construction equipment at the end of 8th, 9th, and 10th years are Rs.587202.56, Rs.469762.05 and Rs.375809.64 respectively (from Table 3.1, Double-declining balance method). Similarly the depreciation amounts for 9th and 10th years are Rs.117440.51 and Rs.93952.41 respectively. Thus to match the book value with the estimated salvage value, the depreciation in 9th year will be Rs.87202.56 (Rs.587202.56 – Rs.500000) and that in 10th year will be zero. This will lead to a book value of Rs.500000 (equal to salvage value) at the end of 9th year and 10th year. For this case, the switching from double-declining balance method to straight-line method can be done to ensure that the book value does not fall below the estimated salvage value and this procedure of switching is presented in Lecture 3 of this module.

Lecture-2

Depreciation:-

Sum-of-years-digits (SOYD) depreciation method:-

It is also an accelerated depreciation method. In this method the annual depreciation rate for any year is calculated by dividing the number of years left (from the beginning of that year for which the depreciation is calculated) in the useful life of the asset by the sum of years over the useful life.

The depreciation rate 'd_m' for any year 'm' is given by;

$$d_m = \frac{(n-m+1)}{SOY} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.21)$$

Where n = useful life of the asset as stated earlier

SOY = sum of years' digits over the useful life = $\frac{n(n+1)}{2}$

Rewriting equation (3.21);

$$d_m = \frac{(n-m+1)}{\frac{n(n+1)}{2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.22)$$

The depreciation amount in any year is calculated by multiplying the depreciation rate for that year with the total depreciation amount (i.e. difference between initial cost 'P' and salvage value 'SV') over the useful life.

Thus the expression for depreciation amount in any year 'm' is represented by;

$$D_m = d_m \times (P - SV) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.23)$$

Putting the value of 'd_m' from equation (3.21) in equation (3.23) results in the following;

$$D_m = \frac{(n-m+1)}{SOY} \times (P - SV) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.24)$$

The depreciation in 1st year i.e. 'D_1' is obtained by putting 'm' equal to '1' in equation (3.24) and is given by;

$$D_1 = \frac{(n-1+1)}{SOY} \times (P - SV) = \frac{(P - SV)}{SOY} \times n \quad \ldots\ldots\ldots\ldots \quad (3.25)$$

The book value at the end of 1st year is equal to initial cost less the depreciation in the 1st year and is given by;

$$BV_1 = P - D_1$$

Now putting the expression of 'D_1'from equation (3.25) in the above expression results in following;

$$BV_1 = P - \frac{(P - SV)}{SOY} \times n \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (3.26)$$

The depreciation in 2nd year i.e. 'D_2' is given by;

$$D_2 = \frac{(n-2+1)}{SOY} \times (P - SV) = \frac{(P - SV)}{SOY} \times (n-1) \quad \ldots\ldots \ (3.27)$$

Book value at the end of 2nd year is equal to book value at the beginning of 2nd year (i.e. book value at the end of 1st year) less the depreciation in 2nd year and is given by;

$$BV_2 = BV_1 - D_2$$

Now putting the expressions of 'BV_1' and 'D_2'from equation (3.26) and equation (3.27) respectively in above expression results in the following;

$$BV_2 = BV_1 - D_2 = \left[P - \frac{(P - SV)}{SOY} \times n\right] - \left[\frac{(P - SV)}{SOY} \times (n-1)\right]$$

$$BV_2 = P - \left[\frac{(P - SV)}{SOY} \times (n + n - 1)\right] = P - \frac{(P - SV)}{SOY} \times (2n-1) \quad \ldots\ldots\ldots\ldots \ (3.28)$$

Similarly the expressions for depreciation and book value for 3rd year are presented below.

$$D_3 = \frac{(n-3+1)}{SOY} \times (P - SV) = \frac{(P - SV)}{SOY} \times (n-2) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (3.29)$$

$$BV_3 = BV_2 - D_3 = \left[P - \frac{(P - SV)}{SOY} \times (2n-1)\right] - \left[\frac{(P - SV)}{SOY} \times (n-2)\right]$$

$$BV_3 = P - \left[\frac{(P - SV)}{SOY} \times (2n - 1 + n - 2)\right] = P - \frac{(P - SV)}{SOY} \times (3n-3) \quad \ldots\ldots \ (3.30)$$

The expressions for depreciation and book value for 4th year are given by;

$$D_4 = \frac{(n-4+1)}{SOY} \times (P - SV) = \frac{(P - SV)}{SOY} \times (n-3) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (3.31)$$

$$BV_4 = BV_3 - D_4 = \left[P - \frac{(P-SV)}{SOY} \times (3n-3)\right] - \left[\frac{(P-SV)}{SOY} \times (n-3)\right]$$

$$BV_4 = P - \left[\frac{(P-SV)}{SOY} \times (3n-3+n-3)\right] = P - \frac{(P-SV)}{SOY} \times (4n-6) \quad \text{…..… (3.32)}$$

Similarly the expressions for depreciation and book value for 5th year are given by;

$$D_5 = \frac{(n-5+1)}{SOY} \times (P-SV) = \frac{(P-SV)}{SOY} \times (n-4) \quad \text{…………………….…. (3.33)}$$

$$BV_5 = BV_4 - D_5 = \left[P - \frac{(P-SV)}{SOY} \times (4n-6)\right] - \left[\frac{(P-SV)}{SOY} \times (n-4)\right]$$

$$BV_5 = P - \left[\frac{(P-SV)}{SOY} \times (4n-6+n-4)\right] = P - \frac{(P-SV)}{SOY} \times (5n-10) \quad \text{…… (3.34)}$$

The expressions for book value in different years are presented above to find out the generalized expression for book value at end of any given year. Now referring to the expressions of book values BV_1, BV_2, BV_3, BV_4, BV_5 in above mentioned equations, it is observed that the variable terms are 'n', '(2n-1)', '(3n-3)', '(4n-6)' and '(5n-10)' respectively. These variable terms can also be written '(n+1-**1**)', '(2n+1-**2**)', '(3n+1-**4**)', '(4n+1-**7**)' and '(5n+1-**11**)' respectively. The numbers 1, 2, 4, 7, and 11 in these variable terms follow a series and it is observed that value of each term is equal to value of previous term plus the difference in the values of current term and previous term. On this note, the general expression for value of 'm' term of this series is given by;

$$\text{value of '}m\text{' term} = 1 + \frac{m(m-1)}{2} \quad \text{……………………………………… (3.35)}$$

Now the generalized expression for book value for any year 'm' is given by;

$$BV_m = P - \frac{(P-SV)}{SOY} \times \left(mn + 1 - \left[1 + \frac{m(m-1)}{2}\right]\right)$$

$$BV_m = P - \frac{(P-SV)}{SOY} \times \left[mn - \frac{m(m-1)}{2}\right] \quad \text{……………………………..… (3.36)}$$

In this method also, the annual depreciation during early years is more as compared to that in later years of the asset's useful life.

Sinking fund (SF) depreciation method:-

In this method it is assumed that money is deposited in a sinking fund over the useful life that will enable to replace the asset at the end of its useful life. For this purpose, a fixed amount is set aside every year from the revenue generated and this fixed sum is considered to earn interest at an interest rate compounded annually over the useful life of the asset, so that the total amount accumulated at the end of useful life is equal to the total depreciation amount i.e. initial cost less salvage value of the asset. Thus the annual depreciation in any year has two components. The first component is the fixed sum that is deposited into the sinking fund and the second component is the interest earned on the amount accumulated in sinking fund till the beginning of that year.

For this purpose, first the uniform depreciation amount (i.e. fixed amount deposited in sinking fund) at the end of each year is calculated by multiplying the total depreciation amount (i.e. initial cost less salvage value) over the useful life by sinking fund factor. After that the interest earned on the accumulated amount is calculated. The calculations are shown below.

The first component of depreciation i.e. uniform depreciation amount 'A' at the end of each year is given by;

$$A = (P - SV) \times (A/F, i, n) = (P - SV) \times \frac{i}{(1+i)^n - 1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots . \quad (3.37)$$

Where i = interest rate per year

Depreciation amount for 1st year is equal to only 'A' as this is the amount (set aside every year from the revenue generated) to be deposited in sinking fund at the end of 1st year and hence there is no interest accumulated on this amount.

Therefore $D_1 = A = A(1+i)^0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.38)$

Now book value at the end of 1st year is given by;

$$BV_1 = P - D_1 = P - A(1+i)^0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.39)$$

Depreciation amount for 2nd year is equal to uniform amount 'A' to be deposited at the end of 2nd year plus the interest earned on the amount accumulated till beginning of 2nd year i.e. on depreciation amount for 1st year. Thus depreciation for amount for 2nd year is given by;

$D_2 = A + D_1 \times i$

Now putting 'D_1' equal to 'A' in the above expression results in;

$D_2 = A + A \times i = A(1+i)$ ……………………………………..… (3.40)

Book value at the end of 2nd year is given by;

$BV_2 = BV_1 - D_2$

Now putting the expressions of 'BV_1' and 'D_2' from equation (3.39) and equation (3.40) respectively in above expression results in the following;

$BV_2 = BV_1 - D_2 = P - A(1+i)^0 - A(1+i) = P - A\left[(1+i)^0 + (1+i)^1\right]$ …… (3.41)

Depreciation amount for 3rd year is equal to uniform amount 'A' to be deposited at the end of 3rd year plus the interest earned on the amount accumulated till beginning of 3rd year i.e. on sum of depreciation amounts for 1st year and 2nd year. Thus depreciation for amount for 3rd year is given by;

$D_3 = A + (D_1 + D_2) \times i$

Putting the expressions of 'D_1' and 'D_2'from equation (3.38) and equation (3.40) respectively in above expression results in the following;

$D_3 = A + (D_1 + D_2) \times i = A + [A + A(1+i)] \times i = A + A \times i + A(1+i) \times i = A(1+i) + A(1+i) \times i$

On simplifying the above expression;

$D_3 = A(1+i) \times (1+i) = A(1+i)^2$ …………………………………… (3.42)

Now book value at the end of 3rd year is given by;

$BV_3 = BV_2 - D_3$

Putting the expressions of 'BV_2' and 'D_3' from equation (3.41) and equation (3.42) respectively in above expression results in;

$BV_3 = BV_2 - D_3 = P - A\left[(1+i)^0 + (1+i)^1\right] - A(1+i)^2 = P - A\left[(1+i)^0 + (1+i)^1 + (1+i)^2\right]$

………………….... (3.43)

Now the generalized expression for depreciation in any given year 'm' can be written as follows (referring to equations (3.38), (3.40) and (3.42));

$D_m = A(1+i)^{m-1}$ ……………………………………………………... (3.44)

Putting the value of 'A' from equation (3.37) in equation (3.44) results in the following;

$$D_m = A(1+i)^{m-1} = (P - SV) \times \frac{i}{(1+i)^n - 1} \times (1+i)^{m-1} \quad \text{………………….} \quad (3.45)$$

Similarly the generalized expression for book value at the end of any year '*m*' is given by (referring to equations (3.39), (3.41) and (3.43));

$$BV_m = P - A\left[(1+i)^0 + (1+i)^1 + (1+i)^2 + (1+i)^3 + \cdots\cdots\cdots + (1+i)^{m-2} + (1+i)^{m-1}\right]$$

……………..   (3.46)

The expression in the square bracket of equation (3.46) is in the form of a geometric series and its sum represents the factor known as uniform series compound amount factor (stated in Module-1). Therefore replacing the expression in square bracket of equation (3.46) with uniform series compound amount factor results in;

$$BV_m = P - A(F / A, i, m) \quad \text{…………………………………………..} \quad (3.47)$$

Putting the value of '*A*' from equation (3.37) and expression for uniform series compound amount factor in equation (3.47) results in the following generalized expression for book value at the end of any year '*m*' ($1 \leq m \leq n$).

$$BV_m = P - A(F / A, i, m) = P - (P - SV) \times \frac{i}{(1+i)^n - 1} \times \left[\frac{(1+i)^m - 1}{i}\right]$$

$$BV_m = P - (P - SV) \times \left[\frac{(1+i)^m - 1}{(1+i)^n - 1}\right] \quad \text{……………………………......} \quad (3.48)$$

In this method, the depreciation during later years is more as compared to that in early years of the asset's useful life.

Example -2

Using the information provided in previous example, calculate the annual depreciation and book value of the construction equipment using sum-of-years-digits method and sinking fund method. The interest rate is 8% per year.

Solution:

P = Rs.3500000

SV = Rs.500000

n = 10 years

For sum-of-years-digits method, the depreciation amount for a given year is calculated using equation (3.24).

SOY = sum of years' digits over the useful life = $\frac{10\times(10+1)}{2}=55$

Depreciation for 1st year = $D_1=\frac{(10-1+1)}{55}\times(3500000-500000)=\text{Rs.}545454.55$

Book value at the end of 1st year:

$BV_1=\text{Rs.}3500000-\text{Rs.}545454.55=\text{Rs.}2954545.45$

Depreciation for 2nd year = $D_2=\frac{(10-2+1)}{55}\times(3500000-500000)=\text{Rs.}490909.09$

Book value at the end of 2nd year:

$BV_2=\text{Rs.}2954545.45-\text{Rs.}490909.09=\text{Rs.}2463636.36$

Similarly the annual depreciation and book value at the end of other years for the construction equipment have been calculated and are presented in Table 3.2.

The book value at the end of different years can also be calculated by using equation (3.36). Using this equation, the book value at the end of 2nd year is calculated as follows;

$$BV_2=3500000-\frac{(3500000-500000)}{55}\times\left[2\times10-\frac{2\times(2-1)}{2}\right]=\text{Rs.}2463636.36$$

For sinking fund method, the depreciation amount for a given year is calculated using equation (3.45).

The interest rate per year = i = 8%

Depreciation amount for 1st year:

$$D_1=(3500000-500000)\times\frac{0.08}{(1+0.08)^{10}-1}\times(1+0.08)^{1-1}=\text{Rs.}207088.47$$

Book value at the end of 1st year:

$BV_1=\text{Rs.}3500000-\text{Rs.}207088.47=\text{Rs.}3292911.53$

Depreciation amount for 2nd year:

$$D_2=(3500000-500000)\times\frac{0.08}{(1+0.08)^{10}-1}\times(1+0.08)^{2-1}=\text{Rs.}223655.55$$

Book value at the end of 2nd year:

$BV_2=\text{Rs.}3292911.53-\text{Rs.}223655.55=\text{Rs.}3069255.98$

Similarly the annual depreciation and book value at the end of other years are calculated in the same manner and are given in Table 3.2.

The book value at the end of a given year can also be calculated by using equation (3.48). Using this equation, the book value at the end of 2nd year is given by;

$$BV_2 = 3500000 - (3500000 - 500000) \times \left[\frac{(1+0.08)^2 - 1}{(1+0.08)^{10} - 1} \right] = \text{Rs.}3069255.99$$

Table 3.2 Depreciation and book value of the construction equipment using sum-of-years-digits method and sinking fund method

Year	Sum-of-years-digits method		Sinking fund method	
	Depreciation (Rs.) D_m	Book value (Rs.) BV_m	Depreciation (Rs.) D_m	Book value (Rs.) BV_m
0	-	3500000	-	3500000
1	545454.55	2954545.45	207088.47	3292911.53
2	490909.09	2463636.36	223655.55	3069255.98
3	436363.64	2027272.73	241547.99	2827707.99
4	381818.18	1645454.55	260871.83	2566836.16
5	327272.73	1318181.82	281741.58	2285094.58
6	272727.27	1045454.55	304280.90	1980813.68
7	218181.82	827272.73	328623.38	1652190.30
8	163636.36	663636.36	354913.25	1297277.06
9	109090.91	554545.45	383306.31	913970.75
10	54545.45	500000	413970.81	499999.94* = 500000

* This minor difference is due to effect of decimal points in the calculation.

The book value at the end of all years over the useful life of the construction equipment obtained from different depreciation methods (Example 1 and Example 2) are shown in Fig. 3.1.

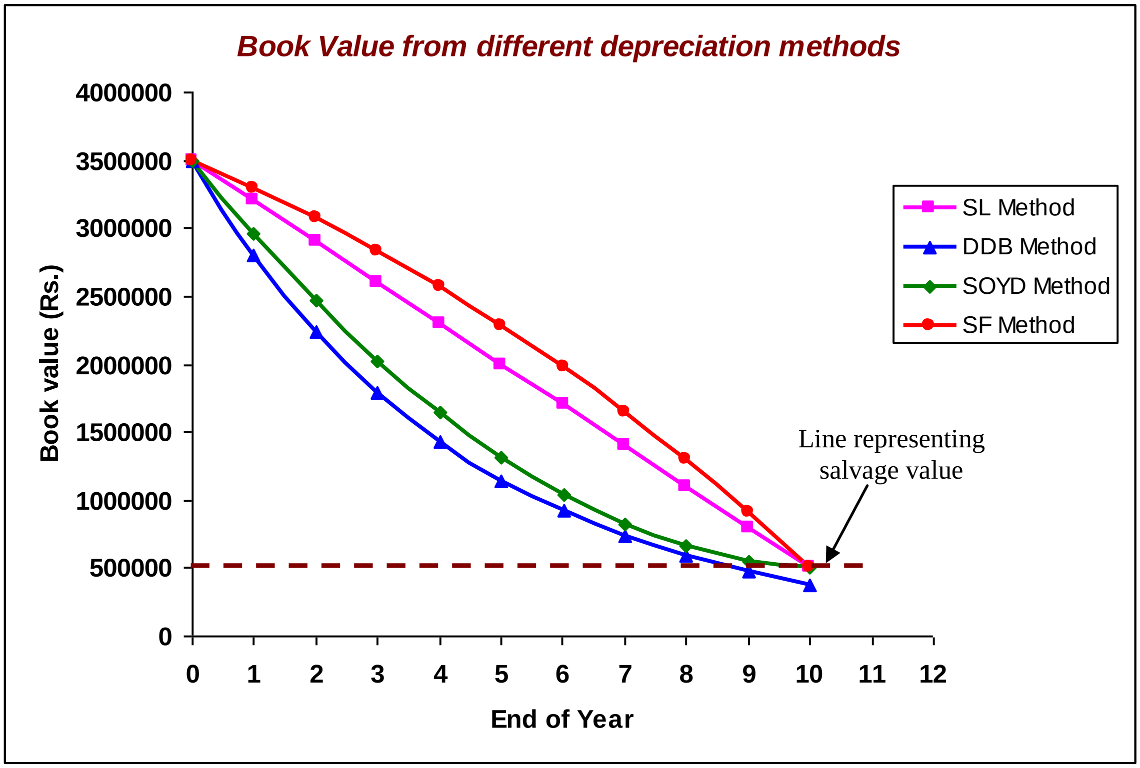

Fig. 3.1 Book value of the construction equipment using different depreciation methods

In the above figure, the line representing the salvage value is also shown. In double-declining balance method only, the calculated book value at the end of useful life i.e. 10th year is not same as the estimated salvage value.

Lecture-3

Switching between different depreciation methods:-

Switching from one depreciation method to another is done to accelerate the depreciation of book value of the asset and thus to have tax benefits. Switching is generally done when depreciation amount for a given year by the currently used method is less than that by the new method. The most commonly used switch is from double-declining balance (DDB) method to straight-line (SL) method. In double-declining balance method, the book value as calculated by using equation (3.17) never reaches zero. In addition the calculated book value at the end of useful life does not match with the salvage value. Switching from double-declining balance method to straight-line method ensures that the book value does not fall below the estimated salvage value of the asset.

In the following examples the procedure of switching from double-declining balance method to straight-line method is illustrated.

Example -3

The initial cost of an asset is Rs.1000000. It has useful life of 9 years. The estimated salvage value of the asset at the end of useful life is zero. Calculate the annual depreciation and book value using double-declining balance method and find out the year in which the switching is done from double-declining balance method to straight-line method.

Solution:

Initial cost of the asset = P = Rs.1000000

Useful life = n = 9 years

Salvage value = SV = 0

For double-declining balance (DDB) method, the constant annual depreciation rate 'd_m' is given by;

$$d_m = \frac{2}{n} = \frac{2}{9} = 0.222$$

The annual depreciation and the book value at the end different years are calculated using the respective equations stated earlier and are presented in Table 3.3.

Table 3.3 Depreciation and book value from double-declining balance method

Year	Depreciation (Rs.)	Book value (Rs.)
0	-	1000000
1	222000	778000
2	172716	605284
3	134373.05	470910.95
4	104542.23	366368.72
5	81333.86	285034.86
6	63277.74	221757.12
7	49230.08	172527.04
8	38301	134226.04
9	29798.18	**104427.86**

From the above table it is observed that the book value at the end of useful life is Rs.104427.86, which is more than the estimated salvage value i.e. 0. The asset is not completely depreciated. Thus switching is done from double-declining balance method to straight-line method and is shown in Table 3.4.

Table 3.4 Double-declining balance method and switching to straight-line method

Year	Depreciation amount (Rs.) (DDB method)	Depreciation amount (Rs.) (SL method)	Selected depreciation amount (Rs.)	Book value (Rs.)
0	-	-	-	1000000
1	222000	111111.11	222000	778000
2	172716	97250	172716	605284
3	134373.05	86469.14	134373.05	470910.95
4	104542.23	78485.16	104542.23	366368.72
5	81333.86	73273.74	81333.86	285034.86
6**	**63277.74**	**71258.72**	**71258.72**	213776.15
7	49230.08	71258.72	71258.72	142517.43
8	38301	71258.72	71258.72	71258.72
9	29798.18	71258.72	71258.72	0

**** Switching from DDB method to SL method**

In the above table, annual depreciation values from double-declining balance (DDB) method are also presented to compare with those obtained from straight- line method. For straight-line (SL) method the depreciation amount for a given year '*m*' is calculated by dividing the difference of the book value (at the beginning of that year) and salvage value by the number of years remaining from beginning of that year till the end of useful life and is shown below;

$$D_m = \frac{BV_{m-1} - SV}{(n-m+1)}$$

For illustration, the straight-line depreciation in 4th year (m = 4) is calculated as follows;

$$D_4 = \frac{470910.95 - 0}{(9-4+1)} = \text{Rs.}78485.16$$

From the annual depreciation values by straight-line method as shown in 3rd column of above table, it is observed that, the annual values are not uniform. This is because when switching is done from DDB method to SL method, the larger depreciation amount between the two the methods for a given year is subtracted from the previous year's book value to calculate the book value at the end of desired year and this book value is used for calculating the depreciation amount for next year in straight-line method. The larger annual depreciation values (i.e. selected depreciation amount) between DDB method and SL method are provided in 4th column of Table 3.4. The book value at the end of a given year presented in 5th column of above table is obtained by subtracting 'selected depreciation amount' of that year from the previous year's book value. From the above table it is observed that the annual depreciation amount in DDB method is greater than that in SL method from 1st year to 5th year and from 6th year onwards the annual depreciation is greater in SL method than that in DDB method. Thus switching from DDB method to SL method takes place in 6th year. With switchover from DDB method to SL method the book value at the end of useful life i.e. 9th year is equal to zero (same as the estimated salvage value) as compared to Rs.104427.86 without switching to straight-line method. In addition the total depreciation amount over the useful life of the asset (initial cost minus salvage value) with switchover from DDB method to SL method is Rs.1000000 (i.e. sum of values in 4th column of Table 3.4) as desired whereas the total depreciation amount without switchover is Rs.895572.14 (sum of values in 2nd column of Table 3.4).

Example -4

A piece of construction equipment has initial cost and estimated salvage value of Rs.1500000 and Rs.200000 respectively. The useful life of equipment is 10 years. Find out the year in which the switching from double-declining balance method to straight-line method takes place.

Solution:

Initial cost of the asset = P = Rs.1500000, Salvage value = SV = Rs.200000

Useful life = n = 10 years

The constant annual depreciation rate 'd_m' for double-declining balance (DDB) method is calculated as follows;

$$d_m = \frac{2}{n} = \frac{2}{10} = 0.2$$

The annual depreciation amount and the book value at the end different years using DDB method are presented in Table 3.5.

Table 3.5 Depreciation and book value using double-declining balance method

Year	Depreciation (Rs.)	Book value (Rs.)
0	-	1500000
1	300000	1200000
2	240000	960000
3	192000	768000
4	153600	614400
5	122880	491520
6	98304	393216
7	78643.20	314572.80
8	62914.56	251658.24
9	50331.65	201326.59
10	40265.32	**161061.27**

From the above table it is noted that the book value at the end of useful life i.e. 10th year is Rs.161061.27, which is less than the estimated salvage value i.e. Rs.200000. Thus the total depreciation amount over the useful life is more than the desired. Hence switching is done from double-declining balance method to straight-line method and is presented in Table 3.6.

In Table 3.6, annual depreciation by SL method, selected depreciation amount and book values are calculated in the same manner as that in the previous example. The annual depreciation for 10th year in DDB method is Rs.40265.32 and subtracting this amount from 9th year book value results in a book value (at the end of 10th year) less than the estimated salvage value (As seen from Table 3.5). Further the book value at the end of 9th year (i.e. Rs.201326.59) is greater than salvage value. Thus switching from DDB method to SL method takes place in 10th year. With switching from DDB method to SL method, the book value at the end of 10 years is equal to Rs.200000 (same as the estimated salvage value) as compared to Rs.161061.27 without switching to straight-line method.

Table 3.6 Double-declining balance method with switching to straight-line method

Year	Depreciation amount (Rs.) (DDB method)	Depreciation amount (Rs.) (SL method)	Selected depreciation amount (Rs.)	Book value (Rs.)
0	-	-	-	1500000
1	300000	130000	300000	1200000
2	240000	111111.11	240000	960000
3	192000	95000	192000	768000
4	153600	81142.86	153600	614400
5	122880	69066.67	122880	491520
6	98304	58304	98304	393216
7	78643.20	48304	78643.20	314572.80
8	62914.56	38190.93	62914.56	251658.24
9	50331.65	25829.12	50331.65	201326.59
10**	**40265.32**	**1326.59**	**1326.59**	200000

**** Switching from DDB method to SL method**

Further the total depreciation amount (i.e. initial cost less estimated salvage value) with switchover from DDB method to SL method is Rs.1300000 (i.e. sum of values in 4th column of Table 3.6) as desired whereas the total depreciation amount without switchover is Rs.1338938.73 (sum of values in 2^{nd} column of Table 3.6).

Lecture-4

Inflation:

Inflation is defined as an increase in the amount of money required to purchase or acquire the same amount of products and services that was acquired without its effect. Inflation results in a reduction in the purchasing power of the monetary unit. In other words, when prices of the products and services increase, we buy less quantity with same amount of money i.e. the value of money is decreased. For example, the quantity of items we purchase today at a cost of Rs.1000 is less than that was purchased 5 years ago. This is due to a general change (increase) in the price of the goods and services with passage of time. On the other hand deflation results in an increase in the purchasing power of the monetary unit with time and it rarely occurs. Due to effect of deflation, more can be purchased with same amount of money in future time period than that can be purchased today. The inflation rate (f) is measured as the rate of increase (per time period) in the amount of money required to obtain same amount of products and services. Till now, the interest rate *'i'* that was used in the economic evaluation of a single alternative or between alternatives by different methods as mentioned in earlier lectures was assumed to be inflation-free i.e. the effect of inflation on interest rate was excluded. This interest rate 'i' is also known as also real interest rate or inflation-free interest rate. It represents the real gain in money of the cash flows with time without the effect of inflation. However if inflation is there in the general market, then effect of inflation on the interest rate needs to be taken into account for the economic analysis. The interest rate that includes the effect of price inflation which is occurring in general economy is known as the market interest rate (i_c). It takes into account the adjustment for the price inflation in the market. Market interest rate is also known as inflated interest rate or combined interest rate as it combines the effect of both real interest rate and the inflation.

In addition to above parameters, it is also required to define two parameters namely actual monetary units and constant value monetary units while considering the effect of inflation in the cash flow of the alternatives. The monetary units can be Rupees, Dollars, Euros etc. The actual monetary units are also referred as future or inflated monetary units. The purchasing power of the actual monetary units includes the effect of inflation on the cash flows at the time it occurs. The constant value monetary units are also called as real or inflation-free monetary units. The constant value monetary units are expressed in terms of the same purchasing power for the cash flows with reference to a base period. Mostly in engineering economic studies, the base period is taken as '0' i.e. now. But it can be of any time period as required.

The effect of inflation on cash flows is demonstrated in the following example.

Example -5

The present (today's) cost of an item is Rs.20000. The inflation rate is 5% per year. How does the inflation affect the cost of the item for the next five years?

Solution:

The calculations are shown in Table 3.7.

Table 3.7 Effect of inflation on future cost of the item

Year	Increase in cost of item due to inflation @ 5% per year (Rs.)	Cost of the item in inflated monetary units i.e. inflated cost (Rs.)	Cost of the item in inflated monetary units expressed as function of today's cost (i.e. in inflation-free monetary units) and inflation rate, (Rs.)
0	-	20000	20000
1	20000 x 0.05 = 1000	20000 + 1000 = 21000	20000 x $(1 + 0.05)^1$ = 21000
2	21000 x 0.05 = 1050	21000 + 1050 = 22050	20000 x $(1 + 0.05)^2$ = 22050
3	22050 x 0.05 = 1102.50	22050+ 1102.5 = 23152.50	20000 x $(1 + 0.05)^3$ = 23152.50
4	23152.5 x 0.05 = 1157.63	23152.5 + 1157.62 = 24310.13	20000 x $(1 + 0.05)^4$ = 24310.13
5	24310.13 x 0.05 = 1215.50	24310.13 + 1215.5 = 25525.63	20000 x $(1 + 0.05)^5$ = 25525.63

From above table, the relationship between cost in actual monetary units (inflated) in time period *'n'* and cost in constant value monetary units (inflation-free) can be represented as follows;

$$\textit{Cost in actual monetary units (inflated)} = \textit{Cost in constant value monetary units} \times (1+f)^n \quad \ldots\ldots\ldots\ldots (3.49)$$

From equation (3.49), the expression for cost in constant value monetary units (inflation-free) is written as follows

$$\textit{Cost in constant value monetary units (inflation - free)} = \frac{\textit{Cost in actual monetary units (inflated)}}{(1+f)^n}$$
……………. (3.50)

In the above expressions, '*f*' is the inflation rate per year i.e. 5% and the base or reference period is considered as '0'. However the above relationship between constant value monetary units and actual monetary units can also be written for any base period *'b'*.

$$\textit{Cost in constant value monetary units (inflation - free)} = \textit{Cost in actual monetary units (inflated)} \times \left(\frac{1}{1+f}\right)^{n-b}$$
……….. (3.51)

In the above relationship, the base period *'b'* defines the purchasing power of constant value monetary units. As shown in Table 3.7, the actual cost (inflated) of the item in years 1, 2, 3, 4 and 5 are Rs.21000, Rs.22050, Rs.23152.50, 24310.13 and Rs.25525.63 respectively. However the cost of the item in inflation-free or constant value rupees in all the years is always Rs.20000 [i.e. $21000/(1+0.05)^1$, $22050/(1+0.05)^2$, etc.] i.e. equal to the cost at the beginning. In this example, effect of interest rate i.e. effect of time value of money is not considered.

Lecture-5

Equivalent worth calculation including the effect of inflation:-

For calculations of equivalent worth of cash flows of alternatives with price inflation in action, the interest rate to be adopted (i.e. real interest rate or combined interest rate) depends on whether the cash flows are expressed in inflated rupees or inflation-free rupees. When the cash flows are calculated in constant value monetary units, inflation-free interest rate (real interest rate) *'i'* is used. Similarly when the cash flows are estimated in actual monetary units, combined interest rate (inflated interest rate) *'i_c'* is used.

The relationship between present worth and future worth considering the time value of money is given by;

$$P = \frac{F_c}{(1+i)^n} \qquad \text{………………………………..} \quad (3.52)$$

P = present worth (at base period '0')

F_c = future worth in constant value monetary units (inflation-free)

In the above equation, future worth in constant value monetary units (inflation-free) can be represented in terms of future worth in actual monetary units (inflated) by using the relationship stated in equation (3.50). Equation (3.52) can be rewritten by including the inflation rate.

$$P = \frac{F_a}{(1+f)^n} \times \frac{1}{(1+i)^n} \qquad \text{…………………………..…} \quad (3.53)$$

In equation (3.53), F_a is the future worth in actual monetary units (inflated) and *'f'* is the inflation rate as stated earlier.

Equation (3.53) can be rewritten as follows;

$$P = F_a \times \frac{1}{(1+i+f+f\times i)^n} \qquad \text{………….……………} \quad (3.54)$$

$$P = F_a \times \frac{1}{[1+(i+f+f\times i)]^n} \qquad \text{…………………….....} \quad (3.55)$$

The expression $(i + f + f \times i)$ in the above equation that combines the effect of real interest rate '*i*' and inflation rate '*f*' is termed as combined interest rate or market interest rate (i_c) as stated earlier.

$$i_c = (i + f + f \times i) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. \quad (3.56)$$

Equation (3.55) can be rewritten as follows;

$$P = F_a \times \frac{1}{(1 + i_c)^n} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.57)$$

Using equation (3.57), the future worth in inflated monetary units can be calculated from known value of present worth '*P*'.

Equation (3.57) can also be written in functional representation as follows;

$$P = F_a(P/F, i_c, n) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.58)$$

Similarly the relationship between annual worth '*A*' and present worth '*P*' and that between 'A' and future worth 'F' of the cash flows considering the effect of inflation can be obtained.

From known values of combined interest rate 'i_c' and inflation rate '*f*', the real interest rate '*i*' can be obtained from equation (3.56) and is given as follows;

$$i_c = (i + f + f \times i) = f + i(1 + f)$$

$$i = \frac{(i_c - f)}{(1 + f)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots... \quad (3.59)$$

The real interest rate '*i*' (inflation-free) in equation (3.59) represents the equivalence between cash flows occurring at different periods of time with same purchasing power.

In the following examples the effect of inflation on equivalent worth of cash flows is illustrated.

Example -6

A person has now invested Rs.500000 at a market interest rate of 14% per year for a period of 8 years. The inflation rate is 6% per year.

i) What is the future worth of the investment at the end of 8 years?

ii) What is the accumulated amount at the end of 8 years in constant value monetary unit i.e. with the same buying power when the investment is made?

Solution:

The market or combined interest rate per year (i_c) and inflation rate per year (f) are14% and 6% respectively.

i) The future worth i.e. amount of money accumulated at the end of 8 years will include the effect of inflation and the interest amount accumulated will be calculated using market or combined interest rate.

$$FW = P(F/P, i_c, n)$$

Putting 'P', 'i_c', 'n' equal to Rs.500000, 14% and 8 years respectively in the above expression;

$$FW = 500000(F/P, 14\%, 8)$$

$$FW = 500000 \times 2.8526 = 1426300$$

Thus the future worth of the investment at the end of 8 years in actual monetary unit (inflated) is Rs.1426300.

ii) The future worth in constant value monetary unit i.e. with same buying power when the investment is made (i.e. now) can be obtained by excluding the effect of inflation from the inflated future worth. Thus the future worth in constant value monetary unit is calculated by putting the values of time period 'n', base period 'b', inflation rate 'f' and the future worth (inflated) as calculated above in equation (3.51). In this case, the value of base period 'b' is zero.

$$FW \text{ in constant value monetary unit (inflation - free)} = \text{'}FW\text{' in actual monetary unit (inflated)} \times \left(\frac{1}{1+f}\right)^{n-b}$$

$$FW \text{ in constant value monetary unit (inflation - free)} = 1426300 \times \left(\frac{1}{1+0.06}\right)^{8}$$

$$FW \text{ in constant value monetary unit (inflation - free)} = Rs.894861$$

Thus the future worth of the investment at the end of 8 years in constant value monetary unit (inflation-free) is Rs.894861.

The future worth of the investment with same buying power as now i.e. when the investment is made can also be calculated by using the real interest rate. The real interest rate can be calculated from market interest rate and inflation rate using equation (3.59).

$$i = \frac{(i_c - f)}{(1+f)}$$

$$i = \frac{(0.14 - 0.06)}{(1 + 0.06)} = 0.07547$$

i = 7.547%

Now the future worth of the investment with same buying power as now is calculated as follows;

$$FW = P(F/P,\ i,\ n).$$

$$FW = 500000(F/P,\ 7.547\%,\ 8) = 500000 \times 1.7897 = 894850$$

The calculated future worth of investment is same as that calculated earlier. The minor difference between the values is due to the effect of decimal points in the calculations.

Example -7

The cash flow of an alternative consists of payment of Rs.100000 per year for 6 years (uniform annual series). The real interest rate is 9% per year. The inflation rate per year is 5%. Find out the present worth of the uniform series when the annual payments are in actual monetary units (inflated).

Solution:

For determination of present worth of the uniform annual series payment, which is in actual monetary units (inflated), first the combined interest rate (i_c) is calculated from the known values of real interest rate (i) and inflation rate (f) using equation (3.56).

i = 9%, f = 5%

$$i_c = (i + f + f \times i)$$

$$i_c = (0.09 + 0.05 + 0.05 \times 0.09) = 0.1445$$

i_c = 14.45%

Now the present worth of the uniform series payment (in actual monetary units) with uniform annual amount 'A' (Rs.100000) is given by;

$$PW = A(P/A,\ i_c, n)$$

$$PW = 100000(P/A, 14.45\%, 6) = 100000 \times 3.8412 = 384120$$

PW = Rs.384120

The present worth of the uniform series payment (in actual monetary unit) can also be calculated by converting the uniform annual amount 'A' occurring in different years into constant value monetary units (inflation-free) and then determining the present worth of the series at real interest rate (i) by using 'P/F' factor.

The values of uniform annual amount '*A*' (Rs.100000) occurring in different years in constant value monetary units (inflation-free) are calculated as follows;

In year '1'

$$\frac{100000}{(1+f)^1}=\frac{100000}{(1+0.05)^1}=\text{Rs.}95238.1$$

In year '2'

$$\frac{100000}{(1+f)^2}=\frac{100000}{(1+0.05)^2}=\text{Rs.}90702.9$$

In year '3'

$$\frac{100000}{(1+f)^3}=\frac{100000}{(1+0.05)^3}=\text{Rs.}86383.8$$

In year '4'

$$\frac{100000}{(1+f)^4}=\frac{100000}{(1+0.05)^4}=\text{Rs.}82270.2$$

In year '5'

$$\frac{100000}{(1+f)^5}=\frac{100000}{(1+0.05)^5}=\text{Rs.}78352.6$$

In year '6'

$$\frac{100000}{(1+f)^6}=\frac{100000}{(1+0.05)^6}=\text{Rs.}74621.5$$

Now the present worth of above amounts at real interest rate i.e. 9% per year is calculated as follows;

$$PW=95238.1(P/F,i,1)+90702.9(P/F,i,2)+86383.8(P/F,i,3)+82270.2(P/F,i,4)+78352.6(P/F,i,5)+74621.5(P/F,i,6)$$

$$PW=95238.1(P/F,9\%,1)+90702.9(P/F,9\%,2)+86383.8(P/F,9\%,3)+82270.2(P/F,9\%,4)+78352.6(P/F,9\%,5)+74621.5(P/F,9\%,6)$$

$$PW=95238.1\times0.9174+90702.9\times0.8417+86383.8\times0.7722+82270.2\times0.7084+78352.6\times0.6499+74621.5\times0.5963$$

PW = Rs.384120

Thus the present worth of the uniform series payment (in actual monetary unit) is same as that calculated earlier.

Taxes:

In the economic comparison of mutually exclusive alternatives, it is possible to select the best alternative without considering the effect of taxes, as all the alternatives are evaluated on the same basis of not taking into account the effect of taxes on the cash flows. However the inclusion of taxes in the economic evaluation of alternatives results in improved estimate of cash flows and the estimated return on the investment. There are different types of taxes which are imposed by central/state governments on the firms. These are income taxes (on taxable income), property taxes (on value of assets owned), sales taxes (on purchase of goods/services), excise taxes (on sale of specific goods/services), etc. Among these taxes, income tax is the most important one, which is considered in the engineering economic analysis. Usually property taxes, excise taxes and sales taxes are not directly associated with income of a firm.

The calculation of income taxes is based on taxable income of the firm and the income tax to be paid by a firm is equal to taxable income multiplied with the applicable income tax rate. The taxable income is calculated at the end of each financial year. For obtaining the taxable income, all allowable expenses (excluding capital investments) and depreciation cost are subtracted from the gross income of the firm. It may be noted here that depreciation is not the actual cash outflow but it represents the decline in the value of the asset (depreciable) and is considered as an allowable deduction for calculating the taxable income. The taxable income is also known as net income before taxes. The net income after taxes is obtained by subtracting the income taxes from taxable income (i.e. net income before tax).

The expression for taxable income is shown below.

$$Taxable\ income = gross\ income - all\ expenses\ (excluding\ capital\ expenditures) - depreciation$$

The details about revenues, expenses and profit or loss of a firm are presented in Lecture 2 of Module 6. The capital investments are not included in the expenses for calculating the taxable income, as this does not affect the income of the firm directly; however the mode of financing the capital investment may have an effect on the taxes. The income tax to be paid is calculated as a percentage of the taxable income and is given as follows;

$$Income\ tax = taxable\ income \times income\ tax\ rate$$

The applicable annual tax rate is usually based on range or bracket of taxable income and the taxable income is charged as per the graduated tax rates wherein higher tax rates are applied to higher taxable income. A different marginal tax rate is applied to each of the taxable income bracket. Marginal tax rate indicates the rate that is applied on each additional unit of currency (Rupees, Dollars or Euros) of the taxable income. Higher marginal tax rate is applied to higher taxable income range. The taxable income in a certain bracket is charged at the specified marginal tax rate that is fixed for that particular bracket. Sometimes depending upon the taxable income, a single tax rate (i.e. flat tax rate) is applied to calculate the income tax. Capital gain and loss which represent the gain and loss on the disposal or sale of depreciable assets/property are also considered in the income tax analysis. The after-tax economic analysis of an alternative can be carried out by calculating the annual cash flows after taxes and after-tax rate of return. The after-tax rate of return depends on the before-tax rate of return and the tax rate. The annual cash flows after taxes is obtained by subtracting the taxes from annual cash flows before taxes.

The calculation of income tax using marginal tax rates for different taxable income brackets is presented in the following example.

Example -8

For a given financial year, the gross income of a construction contractor is Rs.11530000. The expenses excluding the capital investment and the depreciation are Rs.4170500 and Rs.2080000 respectively. Calculate the taxable income and income tax for the financial year with the following assumed marginal tax rates for different taxable income ranges.

Taxable income range (Rs.)	Income tax rate (%)
1 - 1000000	10
1000001 - 3000000	20
3000001 - 5000000	30
Over 5000000	35

Solution:

For the financial year, the taxable income of the construction contractor is calculated using the expression stated earlier.

$$\text{Taxable income} = \text{gross income} - \text{expenses (excluding capital expenditures)} - \text{depreciation}$$

$$\text{Taxable income} = \text{Rs.}11530000 - \text{Rs.}4170500 - \text{Rs.}2080000 = \text{Rs.}5279500$$

The income tax will be equal to the sum of income taxes calculated over the taxable income ranges at the specified tax rates. The construction contractor will pay 10% on first Rs.1000000 of taxable income, 20% on next Rs.2000000 (i.e. Rs.3000000 – Rs.1000000) of taxable income, 30% on next Rs.2000000 (i.e. Rs.5000000 – Rs.3000000) of taxable income and 35% on the remaining Rs.279500 (Rs.5279500 – Rs.5000000) of taxable income.

$$\text{Income tax} = 0.1 \times 1000000 + 0.2 \times 2000000 + 0.3 \times 2000000 + 0.35 \times 279500 = \text{Rs.}1197825$$

The taxable income and income tax for the financial year are Rs.5279500 and Rs.1197825 respectively. For the construction contractor with the above taxable income, the average tax rate fore the financial year is calculated as follows;

$$\text{Average tax rate} = \frac{\text{taxes paid}}{\text{taxable income}} = \frac{\text{Rs.}1197825}{\text{Rs.}5279500} = 0.2269 = 22.69\%$$