MODULE 4: ABSORPTION

LECTURE NO. 4

4.7. Counter-current multi-stage absorption (Tray absorber)

In tray absorption tower, multi-stage contact between gas and liquid takes place. In each tray, the liquid is brought into intimate contact of gas and equilibrium is reached thus making an ideal stage. In ideal stage, average composition of liquid leaving the tray is in equilibrium with liquid leaving that tray. The most important step in design of tray absorber is the determination of number of trays. The schematic of tray tower is presented in figure 4.7. The liquid enters from top of the column whereas gas is added from the bottom. The efficiency of the stages can be calculated as:

$$Stage\ efficiency = \frac{Number\ \ of\ ideal\ stages}{Number\ \ of\ real\ stages} \qquad (4.18)$$

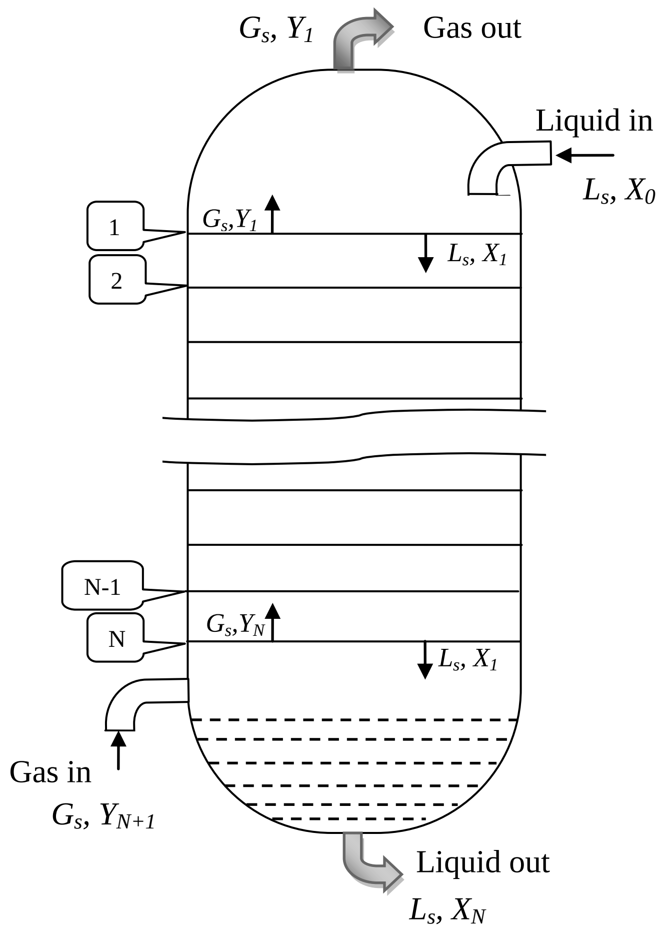

Figure 4.7: Schematic of tray tower.

The following parameters should be known for the determination of "number of stages"

(1) Gas feed rate

(2) Concentration of gas at inlet and outlet of the tower

(3) Minimum liquid rate; actual liquid rate is 1.2 to 2 times the minimum liquid rate.

(4) Equilibrium data for construction of equilibrium curve

Now, the number of theoretic stages can be obtained graphically or algebraically.

(A) Graphical Method for the Determination of Number of Ideal Stages

Overall material balance on tray tower

$$G_s(Y_{N+1} - Y_1) = L_s(X_N - X_0) \quad (4.19)$$

This is the operating line for tray tower.

If the stage (plate) is ideal, (X_n, Y_n) must lie on the equilibrium line, $Y^*=f(X)$

Top plate is located at P(X_0, Y_1) and bottom plate is marked as Q(X_N, Y_{N+1}) in X-Y plane. A vertical line is drawn from Q point to D point in equilibrium line at (X_N, Y_N). From point D in equilibrium line, a horizontal line is extended up to operating line at E (X_{N-1}, Y_N). The region QDE stands for N-th plate (refer Figure 4.8). We may get fraction of plates. In that situation, the next whole number will be the actual number of ideal plates. If the overall stage efficiency is known, the number of real plates can be obtained from Equation (4.18).

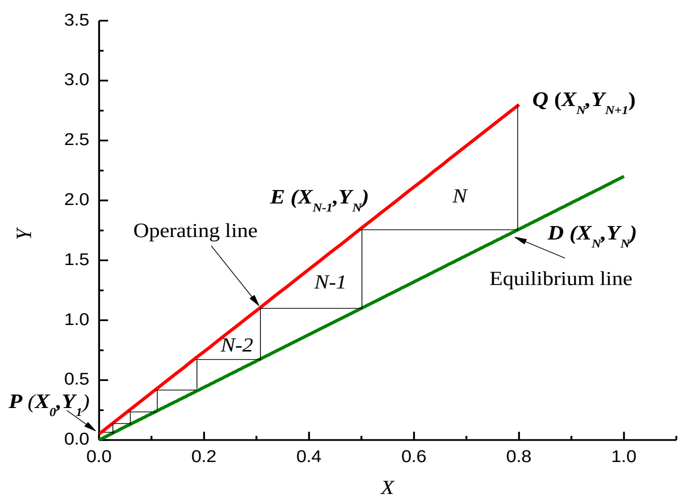

Figure 4.8: Graphical determination of number of ideal stages.

(B) Algebraic Determination of Number of Ideal Stages

If both operating line and equilibrium lines are straight, number of ideal stages can be calculated algebraically.

Let solute transfers from gas to liquid (Absorption)

Equilibrium line, $Y=\alpha X$

Point (X_N, Y_N) lies on the equilibrium line: $Y_N = \alpha X_N$

(4.20)

Operating line:

$$(Y_{N+1} - Y_1) = \frac{L_s}{G_s}(X_N - X_0) \quad (4.21)$$

$$(Y_{N+1} - Y_1) = \frac{L_s}{G_s}\left(\frac{Y_N}{\propto} - X_0\right)$$

$$(Y_{N+1} - Y_1) = \frac{L_s}{G_s \propto}(Y_N - \propto X_0)$$

$$(Y_{N+1} - Y_1) = \bar{A}(Y_N - \propto X_0) \quad (4.22)$$

where $\bar{A} = \frac{L_s}{G_s \propto} = \frac{Slope\ \ of\ \ operating\ \ line}{slope\ \ of\ \ equilibrium\ \ line} = absoption\ factor$

Now Equation 4.22 becomes,

$$(Y_{N+1} - Y_1) = (\bar{A}\, Y_N - \bar{A} \propto X_0)$$

$$(Y_{N+1} - \bar{A}\, Y_N) = (Y_1 - \bar{A} \propto X_0) \quad (4.23)$$

This Equation is linear first order "difference Equation" (non-homogeneous).

<u>Solution by finite difference method</u>

Corresponding homogeneous Equation: $Y_{N+1} - \bar{A}\, Y_N = 0$ (4.24)

Solution is $Y_N = K_1 Z^n$ (4.25)

$$K_1 Z^{N+1} - \bar{A} K_1 Z^N = 0 \quad (4.26)$$

$$Z = \bar{A} \quad (4.27)$$

Non-homogeneous Equation has a particular solution, which is constant. Assuming $Y_N = Y_{N+1}$, we have, $Y = K_2$

$$K_2 - \bar{A}\, K_2 = Y_1 - \propto \bar{A} X_0$$

$$K_2 = \frac{Y_1 - \propto \bar{A} X_0}{1 - \bar{A}} \quad (4.28)$$

The complete solution is as follows:

$$Y_N = \mathrm{K}_1 (\bar{A})^N + K_2 = \mathrm{K}_1 (\bar{A})^N + \frac{Y_1 - \propto \bar{A} X_0}{1 - \bar{A}} \quad (4.29)$$

Initial conditions:

N=0; $Y_0 = \alpha X_0$

$$\alpha X_0 = K_1 (\bar{A})^0 + \frac{Y_1 - \propto \bar{A} X_0}{1 - \bar{A}}$$

$$K_1 = \frac{\alpha X_0 - \propto \bar{A}X_0 - Y_1 + \propto \bar{A}X_0}{1-\bar{A}}$$

$$K_1 = \frac{\alpha X_0 - Y_1}{1-\bar{A}} \qquad (4.30)$$

$$Y_N = \frac{\alpha X_0 - Y_1}{1-\bar{A}}(\bar{A})^N + \frac{Y_1 - \propto \bar{A}X_0}{1-\bar{A}} \qquad (4.31)$$

When *N*=*N*+1;

$$Y_{N+1} = \frac{\alpha X_0 - Y_1}{1-\bar{A}}(\bar{A})^{N+1} + \frac{Y_1 - \propto \bar{A}X_0}{1-\bar{A}}$$

$$Y_{N+1} = \frac{\alpha X_0 - Y_1}{\left(\frac{1}{\bar{A}} - 1\right)}(\bar{A})^N + \frac{\frac{Y_1}{\bar{A}} - \propto X_0}{\left(\frac{1}{\bar{A}} - 1\right)}$$

$$\left(\frac{1}{\bar{A}} - 1\right)Y_{N+1} = (\alpha X_0 - Y_1)(\bar{A})^N + \left(\frac{Y_1}{\bar{A}} - \propto X_0\right)$$

$$(\alpha X_0 - Y_1)(\bar{A})^N = \left(\frac{1}{\bar{A}} - 1\right)Y_{N+1} - \left(\frac{Y_1}{\bar{A}} - \propto X_0\right)$$

$$(\bar{A})^N = \frac{\left(\frac{1}{\bar{A}} - 1\right)Y_{N+1} - \left(\frac{Y_1}{\bar{A}} - \propto X_0\right)}{(\alpha X_0 - Y_1)} = \left(\frac{Y_{N+1} - \propto X_0}{Y_1 - \propto X_0}\right)\left(1 - \frac{1}{\bar{A}}\right) + \frac{1}{\bar{A}} \qquad (4.32)$$

Taking logarithm in both the sides we get:

$$N ln\bar{A} = ln\left[\left(\frac{Y_{N+1} - \propto X_0}{Y_1 - \propto X_0}\right)\left(1 - \frac{1}{\bar{A}}\right) + \frac{1}{\bar{A}}\right]$$

$$N = \frac{ln\left[\left(\frac{Y_{N+1} - \propto X_0}{Y_1 - \propto X_0}\right)\left(1 - \frac{1}{\bar{A}}\right) + \frac{1}{\bar{A}}\right]}{ln\,\bar{A}} \text{ when } \bar{A} \neq 1 \qquad (4.33)$$

When $\bar{A} = 1$, Equation (4.23) becomes

$(Y_{N+1} - Y_N) = (Y_1 - \propto X_0)$ →Operating line Equation (4.34)

Put *N*=*N, N*-1, *N*-2, ………..3,2,1 and add to get

$$(Y_{N+1} - Y_1) = N(Y_1 - \propto X_0)$$

$$N = \frac{(Y_{N+1} - Y_1)}{(Y_1 - \propto X_0)} \qquad (4.35)$$

Let solute is transferred from liquid to gas (stripping).

$$N = \frac{ln\left[\left(\frac{X_0 - Y_{N+1}/\propto}{X_N - Y_{N+1}/\propto}\right)(1-\bar{A})+\bar{A}\right]}{ln\frac{1}{\bar{A}}} \quad \text{when } \bar{A} \neq 1 \qquad (4.36)$$

When $\bar{A} = 1$, Equation (4.23) becomes

$$N = \frac{(X_0 - X_N)}{(X_N - Y_{N+1}/\propto)} \qquad (4.37)$$

These four Equations (4.33, 4.35-4.37) are called "***Kremser Equations***".

Example Problem 4.2. It is desired to absorb 95% of acetone by water from a mixture of acetone and nitrogen containing 1.5% of the component in a countercurrent tray tower. Total gas input is 30 kmol/hr and water enters the tower at a rate of 90 kmol/hr. The tower operates at 27ºC and 1 atm. The equilibrium relation is Y=2.53X. Determine the number of ideal stages necessary for the separation using (a) graphical method as well as (b) Kremser analysis method.

Solution 4.2:

Basis: 1 hour

G_{N+1}=30 kmol

y_{N+1}=0.015

L_0=90 kmol

Moles acetone in = 30×0.015 moles=0.45 moles

Moles nitrogen in = (30-0.45) moles=29.55 moles

Moles acetone leaving (95% absorbed) = 0.45×(1-0.95) moles=0.0225 moles

G_s=29.55 moles

L_s=90 moles

α=2.53 [as, Y=2.53X]

$$Y_1 = \frac{0.0225}{29.55} = 7.61 \times 10^{-4}$$

$$Y_{N+1} = 0.015$$

Rewriting Equation (4.19) (operating line) as

$$G_s(Y_{N+1} - Y_1) = L_s(X_N - X_0)$$

$$29.55 \times (0.015 - 7.61 \times 10^{-4}) = 90(X_N - 0)$$

$X_N = 4.68\times10^{-3}$

(a) Solution by graphical method

Construction of operating line PQ:

$P(X_0, Y_1) = P(0, 7.61\times10^{-4})$

$Q(X_N, Y_{N+1}) = Q(4.68\times10^{-3}, 0.015)$

Construction of equilibrium line ($Y=2.53X$):

X	0	0.001	0.002	0.003	0.004	0.005
Y	0	0.00253	0.00506	0.00759	0.01012	0.01265

From graphical construction (Figure 4.9), the number of triangles obtained is more than 7. Hence number of ideal stages is 8.

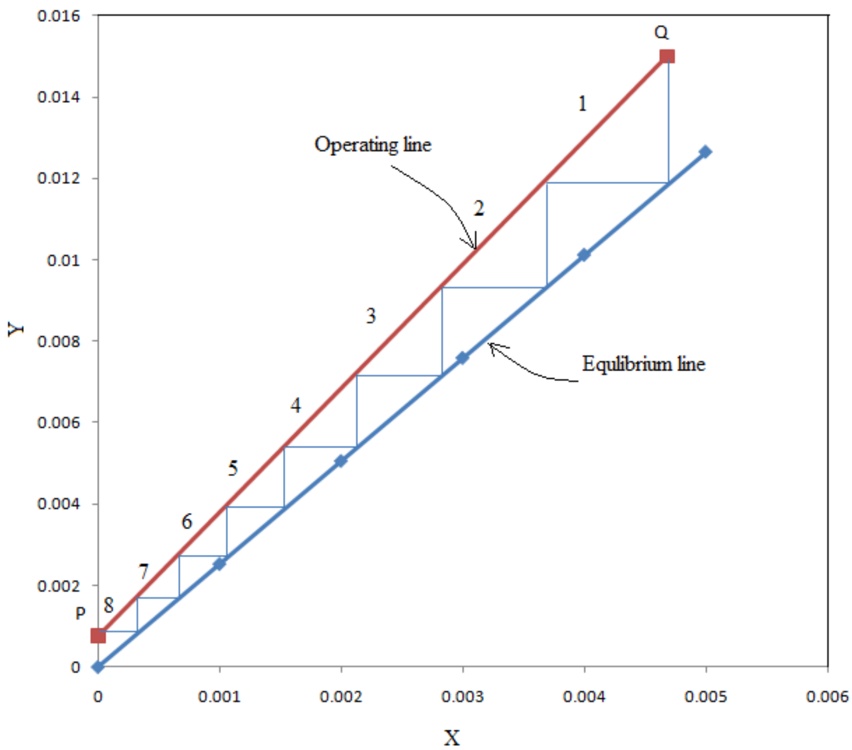

Figure 4.9: Graphical construction for determination of number of stages

(b) Solution by Kremser analysis method

As $\bar{A}\neq 1$, according to Kremser analysis method:

$$N=\frac{\ln\left[\left(\frac{Y_{N+1}-\propto X_0}{Y_1-\propto X_0}\right)\left(1-\frac{1}{\bar{A}}\right)+\frac{1}{\bar{A}}\right]}{\ln \bar{A}}$$

$$N=\frac{\ln\left[\left(\frac{0.015-2.53\times 0}{7.61\times 10^{-4}-2.53\times 0}\right)\left(1-\frac{1}{1.204}\right)+\frac{1}{1.204}\right]}{\ln 1.204}$$

$$N=\frac{\ln\left[(20)\left(\frac{0.204}{1.204}\right)+\frac{1}{1.204}\right]}{0.1856}$$

$$N=\frac{1.43966}{0.1856}=7.75$$

Number of ideal stages is 8.