Lecture 1

Introduction, Motivation and Overview

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Points to Note:

- The topics covered in this course are “fairly generic”, with preferential emphasis on aerospace applications.

- This course is meant to equip the students with applied optimal control system and state estimation concepts.

- The topics covered are “fairly mathematical”. However, lucidity is maintained for better understanding and future usage.
Topics

- Review of necessary fundamentals
- Static optimization
- Calculus of variation (CV)
- Optimal control through CV
- Classical Numerical Techniques
- Linear Quadratic Regulator (LQR)
- Overview of Flight Dynamics
- LQR for Flight Control Applications

Topics

- Optimal Missile Guidance through LQR
- SDRE and $\theta - D$ Designs
- Discrete-time Optimal Control
- Dynamic Programming: HJB theory
- Approximate Dynamic Programming
- Adaptive Critic and SNAC Designs
- MPSP Design & Guidance Applications
- LQ Observer
Topics

- State Estimation using Kalman Filter
- Robust Control through Optimal Control and State Estimation
- Constrained Optimal Control
  - Control & State constrained problems
- Transcription & Pseudo-spectral Method
- Optimal Control of Distributed Parameter Systems

References

References

- Literature (Journal & Conference publications)

Introduction and Motivation

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Concepts and Definitions

- **System Variables**
  - **Input variables**
    - **Control inputs**: Manipulative input variables (usually known, computed precisely)
    - **Noise inputs**: Non-manipulative (usually unknown)
  - **Output variables**
    - **Sensor outputs**: Variables that are measured by sensors
    - **Performance outputs**: Variables that govern the performance of the system (Note: Sensor and performance outputs may or may not be same)
  - **State variables**: A set of variables that describe a system completely

Classification of System Study

- **System Study**
  - **Model Development**
    - (Includes parameter identification)
    - Beyond the scope of this course
  - **System Analysis**
    - Predicting the system behaviour: Can be done either by mathematical analysis or numerical simulation
  - **System Synthesis**
    - To force the system to behave as we would like it to
    - We will study in this course with emphasis on system synthesis
System Classification

Classification

Linear Systems

- Linear systems are systems that obey the "Principle of superposition":
  - Multiplying the input(s) by any constant $\alpha$ must multiply the outputs by $\alpha$.
  - The response to several inputs applied simultaneously must be the sum of the individual responses to each input applied separately.
Example: Static System

Equation of a straight line $y = mx + c$ is

"Not Necessarily Linear" unless $c = 0

\[ y = mx + c \]

(Not linear)  

\[ y = mx \]  

(Linear)

Example: Dynamical System

Example - 1 (Linear System)
\[
\dot{x} = 2x \\
1) \quad \alpha \dot{x} = \alpha (2x) = 2(\alpha x) \\
2) \quad \frac{d}{dt}(x_1 + x_2) = \dot{x}_1 + \dot{x}_2 = 2x_1 + 2x_2 = 2(x_1 + x_2)
\]

Example - 2 (Nonlinear System)
\[
\dot{x} = 2x + 3 \\
1) \quad \alpha \dot{x} = \alpha (2x + 3) \neq 2(\alpha x) + 3 \\
2) \quad \frac{d}{dt}(x_1 + x_2) = \dot{x}_1 + \dot{x}_2 = (2x_1 + 3) + (2x_2 + 3) \neq 2(x_1 + x_2) + 3
\]

Example - 3 (Nonlinear System)
\[
\dot{x} = 2\sin x \\
1) \quad \alpha \dot{x} = \alpha (2\sin x) \neq 2\sin (\alpha x) \\
2) \quad \frac{d}{dt}(x_1 + x_2) = \dot{x}_1 + \dot{x}_2 = 2\sin x_1 + 2\sin x_2 \neq 2\sin (x_1 + x_2)
\]
Nonlinear and Analogous Systems

- Nonlinear systems are systems that are “Not Necessarily Linear”

- Analogous Systems are systems having same mathematical form of the model.
  - However, their variables might have different physical meaning and their parameters might have different numerical values
  - Example: Spring-Mass-Damper and R-L-C systems are analogous

Nonlinear vs. Linear Systems

<table>
<thead>
<tr>
<th>Nonlinear Systems</th>
<th>Linear Systems</th>
</tr>
</thead>
<tbody>
<tr>
<td>• More realistic</td>
<td>• Approximation to reality</td>
</tr>
<tr>
<td>• Usually difficult to analyze and design</td>
<td>• Usually simpler to analyze and design</td>
</tr>
<tr>
<td>• Tools are under development</td>
<td>• A lot of tools are well-developed.</td>
</tr>
<tr>
<td>• Can have multiple equilibrium points</td>
<td>• Only single equilibrium point</td>
</tr>
<tr>
<td>• System stability depends on Initial condition (IC)</td>
<td>• Stability nature is independent of IC</td>
</tr>
<tr>
<td>• Limit cycles (self-sustained oscillations)</td>
<td>• (justifies the Transfer function approach, where “zero” ICs are assumed)</td>
</tr>
<tr>
<td>• Bifurcations (number of equilibrium points and their stability nature can vary with parameter values)</td>
<td>• No limit cycles</td>
</tr>
<tr>
<td>• Chaos (very small difference in I.C. can lead to large difference in output as time increases. That’s why predicting weather for a long time is erroneous!)</td>
<td>• No bifurcation</td>
</tr>
<tr>
<td>• Frequency and amplitude can be coupled</td>
<td>• No chaos</td>
</tr>
<tr>
<td></td>
<td>• Frequency and amplitude are independent</td>
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Classical vs. Modern Control

<table>
<thead>
<tr>
<th>Classical Control</th>
<th>Modern Control</th>
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<tbody>
<tr>
<td>Developed in 1920 – 1950</td>
<td>Developed in 1950 -1980</td>
</tr>
<tr>
<td>Frequency domain analysis &amp; design (transfer function based)</td>
<td>Time domain analysis and design (differential equation based)</td>
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<tr>
<td>Based on SISO models</td>
<td>Based on MIMO models</td>
</tr>
<tr>
<td>Deals with input and output variables</td>
<td>Deals with input, output and state variables</td>
</tr>
<tr>
<td>Well-developed robustness concepts (gain/phase margins)</td>
<td>Not well-developed robustness concepts</td>
</tr>
<tr>
<td>No controllability/observability inference</td>
<td>Controllability/Observability can be inferred</td>
</tr>
<tr>
<td>No optimality concerns</td>
<td>Optimality issues can be incorporated</td>
</tr>
<tr>
<td>Well-developed concepts and very much use in industry</td>
<td>Slowly gaining popularity in industry</td>
</tr>
</tbody>
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Benefits of Advanced Control Theory

- MIMO theory: Lesser assumptions and approximations
- Simultaneous disturbance rejection and command following (conflicting requirements)
- Robustness in presence of parameter variations, external disturbances, unmodelled dynamics etc.
- Fault tolerance
- Self-autonomy
Why Nonlinear Control?
Summary of Benefits

- Improvement of existing control systems (neglected physics can be accounted for)
- Explicit account of “hard nonlinearities” and “strong nonlinearities”
  - Hard nonlinearities: Discontinuity in derivatives (saturation, dead zones, hysteresis etc.)
  - Strong nonlinearities: Higher-order terms in Taylor series
- Can directly deal with model uncertainties
- Can lead to “design simplicity”
- Can lead to better “performance optimality”

Techniques of Nonlinear Control
Systems Analysis and Design

- Phase plane analysis
- Lyapunov theory
- Differential geometry (Feedback linearization)
- Intelligent techniques: Neural networks, Fuzzy logic, Genetic algorithm etc.
- Describing functions
- Optimal control theory (variational optimization, dynamic programming etc.)
Classical Control System

\[ R(s) + E(s) \rightarrow G(s) \rightarrow C(s) \]

Question: What is \( R(s) \)? How to design it??
Unfortunately, books remain completely silent on this!

Why Optimal Control?
Summary of Benefits

- A variety of difficult real-life problems can be formulated in the framework of optimal control.
- State and control bounds can be incorporated in the control design process explicitly.
- Incorporation of optimal issues lead to a variety of advantages, like minimum cost, maximum efficiency, non-conservative design etc.
- Trajectory planning issues can be incorporated into the guidance and control design.
**Difficulty of Optimal Control**

- **Fact:** Optimal control problems are *computationally very intensive* and hence are difficult to solve in real time!
- **Question:** Can the computational difficulty be avoided, so that optimal control design can be useful for real-time applications?
- **Answer:** Yes!
  - Linear Quadratic Regulator (LQR) problems
  - Nonlinear quadratic regulator for control affine systems
    - SDRE Method, $\theta - D$ Method
  - Pseudo-spectral methods
  - Adaptive-Critic methods (neural network based)
  - Model Predictive Static Programming (MPSP)

**Optimal control formulation:**

**Key components**

An optimal control formulation consists of:

- Performance index that needs to be optimized
- Appropriate boundary (initial & final) conditions
  - Hard constraints
  - Soft constraints
- Path constraints
  - System dynamics constraint (nonlinear in general)
  - State constraints
  - Control constraints
Optimal Control Design: Problem Statement

To find an "admissible" time history of control variable $U(t)$, $t \in [t_0, t_f]$ which:

1) Causes the system governed by $\dot{X} = f(t, X, U)$ to follow an "admissible trajectory"

2) Optimizes (minimizes/maximizes) a "meaningful" performance index

$$J = \varphi(t_f, X_f) + \int_{t_0}^{t_f} L(t, X, U) \, dt$$

3) Forces the system to satisfy "proper boundary conditions".

Meaningful Performance Index

1) Minimize the operational time

$$J = \left(t_f - t_0\right) = \int_{t_0}^{t_f} 1 \, dt$$

2) Minimize the control effort

$$J = \frac{1}{2} \int_{t_0}^{t_f} U^T RU \, dt$$

3) Minimize deviation of state from a fixed value $C$ with minimum control effort

$$J = \frac{1}{2} \int_{t_0}^{t_f} \left[ (X - C)^T Q (X - C) + U^T RU \right] \, dt \quad (Q \geq 0, R > 0)$$
Why State Estimation?

- State feedback control designs need the state information for control computation
- In practice all the state variables are not available for feedback. Possible reasons are:
  - Non-availability of sensors
  - Expensive sensors
  - Quality of sensor input is not acceptable due to noise (it’s an issue in output feedback control design as well)
- A state observer estimates the state variables based on the measurement of some of the output variables as well as the process information.

Main Aspects of Estimation

- **Prediction, Filtering and Smoothing**: improves noisy measurements, typically due to process noise and measurement noise.
- Can also take care of limited inaccuracy or errors in system modeling (i.e. relying on estimated states leads to a robust controller)
- Parameter estimation (for system identification)
Other Applications of Estimation

- Model development (parameter estimation, state-parameter estimation)
- Fault Detection and Identification (FDI)
- Estimation of states of other related systems (especially non-cooperative systems) to collect exogenous information that are relevant to the successful operation of the plant

Applications in Aerospace Engineering

- Missile Guidance and Control
  - Rapid and precise command response
  - System limitations (like tail-control and smaller fins)
  - Disturbance rejection (wind gust, motor burnout, stage separation etc.)
  - Prediction of target behaviour through state estimation
Applications in Aerospace Engineering

- Guidance and Control of Unmanned Air Vehicles (UAVs)
  - Way-point guidance
  - Automatic take-off and landing
  - Collision avoidance
  - Formation flying
  - Co-operative missions

Applications in Aerospace Engineering

- Aircraft Flight Control
  - Stability augmentation
  - Maneuver enhancement (within pilot limits)
  - Load alleviation
  - Structural mode control
  - Flutter margin augmentation
Thanks for the Attention....!!

questions ... ??